MOTION IN A PLANE

Important Points:

1. Scalar:

A physical quantity having only magnitude but no direction is called a **Scalar**.

Ex: Time, mass, distance, speed etc.

2. Vector:

A physical quantity having both magnitude and direction and which obeys the laws of vector addition is called a **Vector Quantity**.

Ex. Displacement, velocity etc.

3. Equal Vectors:

Vectors having same magnitude and which have same direction are called equal vectors.

4. Negative Vectors:

A vector which has the same magnitude as that of another and which is opposite in direction is called a **Negative Vector**.

5. Null Vector or Zero Vectors:

A vector whose magnitude is zero and which has no specific direction is called a Null Vector.

Ex. The cross product of two parallel vectors is a null vector.

6. Unit Vector:

It is a vector whose magnitude is unity. If $\vec{A}$ is a vector, the unit vector in the direction of $\vec{A}$ is written as $\hat{A} = \frac{\overline{A}}{|\overline{A}|}$

$\hat{i}, \hat{j}\ and\ \hat{k}$ are units vectors along x, y and z axis

7. Position Vector:

The position of a particle is described by a position vector which is drawn from the origin of a reference frame. The position vector of a particle 'P' in space is given by $\overline{A} = xi + yj + zk$

Its magnitude is given by $A = \sqrt{x^2 + y^2 + z^2}$

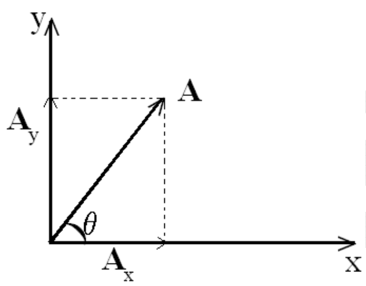
Unit vector of $\vec{A}$ is given by, $\hat{A} = \frac{\overline{A}}{|\overline{A}|} = \frac{xi + yj + zk}{\sqrt{x^2 + y^2 + z^2}}$

8. Resolution of Vectors:

Definition:

The process of dividing a vector into its components is called resolution of vector.

a) Rectangular components of a vector



Horizontal component $A_x = A\cos\theta$

Vertical component $A_y = A\sin\theta$

Resultant $A = \sqrt{A_x{}^2 + A_y{}^2}$

9. If α, β, γ are the angles made by $\vec{A}$ with X, Y and Z axes respectively, then

$\cos\alpha = \frac{A_x}{|\vec{A}|}$; $\cos\beta = \frac{A_y}{|\vec{A}|}$; $\cos\gamma = \frac{A_z}{|\vec{A}|}$ These are called direction cosines.

Also, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

10. Parallelogram Law of Vectors:

Statement:

If two vectors are represented both in magnitude and direction by the adjacent sides of a parallelogram drawn from a point, the diagonal passing through that point represents their resultant both in magnitude and direction.

Magnitude of Resultant:

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Where θ is the angle between the vectors.

Direction of resultant vectors $\bar{R}$:

$$Tan\,\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$ Where α is angle made by the $\bar{R}$ with $\vec{A}$

11. Triangle Law of Vectors:

Statement:

If two vectors are represented in magnitude and direction by the two sides of a triangle taken in order, then the third side of the triangle taken in reverse order represents their resultant in magnitude and direction.

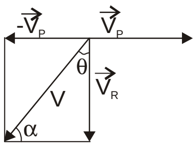
12. Relative Velocity of Rain Drops:

Formula: $\vec{V}_{rel} = \vec{V}_r - \vec{V}_m$

Where

V_m = Velocity of man

V_r = Velocity of rain



$$V_{rel} = \sqrt{V_r^2 + V_m^2 - 2V_r V_m \cos\theta}$$

$$\theta = angle\, between\ V_r\ \ and\ V_m$$

When rain drops fall vertically and person moves along horizontal,

$$V_{rel} = \sqrt{V_r^2 + V_m^2} \qquad \text{And} \qquad \tan\theta = \frac{V_m}{V_r}$$

Where $\alpha = angle\ between\ \vec{V}_{rel}\ \&\ \vec{V}_r$

13. Motion of Boat in a River:

Resultant velocity of boat $\vec{V}_R = \vec{V}_B + \vec{V}_W$

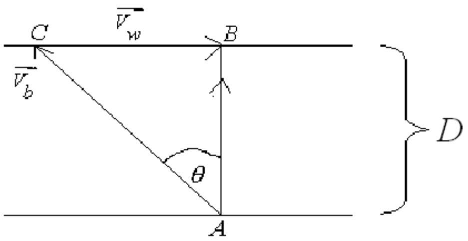
a) Shortest Path: $Sin\ \theta = \frac{V_w}{V_b}$

Where V_w = velocity of water and V_b = velocity of boat

Resultant velocity $(V_R) = \sqrt{V_b^2 - V_w^2}$

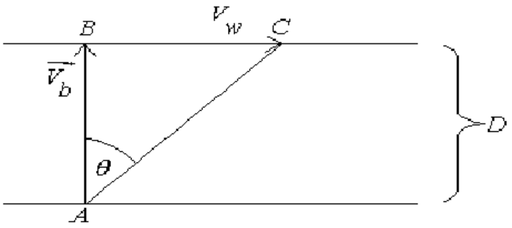
Time taken to cross the river $t = \frac{D}{\sqrt{V_b^2 - V_w^2}}$

Where D $\rightarrow$ width of the river



b) Shortest Time: Time taken to cross the river $t = \frac{D}{V_b}$

BC = Drift $= D\left(\frac{V_w}{V_b}\right)$



14. Oblique Projection:

a. A body which has uniform velocity in the horizontal direction and uniform acceleration in the vertical direction is called a projectile.

b. The path of a projectile is called Trajectory and it is a parabola.

c. For a projectile, the horizontal component of velocity ($u_x = u \cos\theta$) is same throughout its motion.

d. The vertical component ($u_y = u \sin\theta$) is subjected to acceleration due to gravity.

e. After a time 't',

Horizontal displacement x = $(u\cos\theta)t$

Vertical displacement $y = (u\sin\theta)t - \frac{1}{2}gt^2$

f. The velocity of projectile is minimum $(u\cos\theta)$ at the highest point of its path. Here vertical component of velocity is zero.

g. At any instant of time 't' the horizontal component of velocity is $v_x = u\cos\theta$.

The vertical component of velocity is $v_y = u\sin\theta - gt$

The resultant velocity is $v = \sqrt{v_x^2 + v_y^2}$.

The direction of velocity with the horizontal $Tan\,\alpha = \frac{v_y}{v_x}$.

h. Equation for the trajectory of the body $y = x\tan\theta - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2$

i. Time of ascent $(t_a) = \frac{u\sin\theta}{g}$

j. Time of descent $(t_d) = \frac{u\sin\theta}{g}$

k. Time of flight T = $t_a + t_d = \frac{2u\sin\theta}{g}$

l. Maximum height reached $H_{max} = \frac{u^2\sin^2\theta}{2g}$

m. Horizontal range R $= \frac{2u_x u_y}{g} = \frac{u^2\sin(2\theta)}{g}$

n. Range is maximum if the angle of projection $\theta = 45^0$

o. Range is same for two angles of projection 'θ' and $(90-\theta)$.

p. If the range and maximum height of a projectile are equal, the angle of projection $\theta = \tan^{-1}(4)$.

q. The relation between range and maximum height is $\frac{R}{H} = \frac{4}{\tan\theta}$

15. Horizontal Projection:

a. The path of a body projected horizontally from the top of a tower of height 'h' is a parabola.

b. The equation of the trajectory is given by $y = \left(\frac{g}{2u^2}\right)x^2$

c. The horizontal range R = u x t = $u\sqrt{\frac{2h}{g}}$

d. At any instant 't' the resultant velocity v = $\sqrt{v_x^2 + v_y^2}$

e. The angle made by resultant velocity with horizontal $\tan\alpha = \frac{v_y}{v_x}$.

f. A body is dropped from the window of the moving train. The path of the body is

i) Vertical straight line for an observer in the train.

ii) Parabolic for an observer outside the train.

g. From the top of a tower a stone is dropped and simultaneously another stone is projected horizontally with a uniform velocity. Both of them reach the ground simultaneously.

16. Uniform Circular Motion:

a) When a particle follows a circular path at constant speed, its motion is called uniform circular motion.

b) The line joining the centre of the circle and the position of the particle at any instant of time is called the radius vector.

c) The angle made by the radius vector in a given interval of time is called the angular displacement.

d) Angular displacement is measured in radians or degrees $1\,Radian = 57.5^0$.

e) The rate of change of angular displacement is called angular velocity (ω).

$$\omega = \frac{\theta}{t}\, rads^{-1}$$

f) Linear velocity $(\vec{V}) = \vec{\omega} \times \vec{r}$.

g) Rate of change of angular velocity is called angular acceleration (α). Unit is rad s^{-2}.

$$\alpha = \frac{\omega_2 - \omega_1}{t}$$

h) Linear acceleration $\vec{a} = \vec{\alpha} \times \vec{r}$.

i) Angular displacement (θ) and angular acceleration (a) are pseudo vectors.

j) Equations of motion are

i) $\omega = \omega_0 + \alpha t$.

ii) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

iii) $\omega^2 - \omega_0^2 = 2\alpha\theta$

k) The acceleration experienced by the body directed towards the centre is called normal or radial or centripetal acceleration

$a = \frac{v^2}{r}$ and $4\pi^2 n^2 r$ where n is the number of revolutions per second.

l) Centripetal force $= \frac{mv^2}{r} = mr\omega^2$

m) A pseudo force which is away from the centre is called the centrifugal force.

Centrifugal force = $-\frac{mv^2}{r} = mr\omega^2$

Very Short Answer Questions

1. The vertical component of a vector is equal to its horizontal component. What is the angle made by the vector with X-axis?

A. If θ is the angle made by a vector $\vec{A}$ with x-axis.

X-component of $\vec{A} = A_x = A\cos\theta$ and Y-component of $\vec{A} = A_y = A\sin\theta$

But, $A_x = A_y \Rightarrow A\cos\theta = A\sin\theta \Rightarrow \tan\theta = 1 \Rightarrow \theta = 45^0$

2. A vector $\vec{v}$ makes an angle θ with the horizontal. The vector is rotated through an angle α. Does this rotation change the vector $\vec{v}$?

A. Due to the rotation, the vector changes in its direction but the magnitude does not change.

3. Two forces of magnitudes 3 units and 5 units act at 60^0 with each other. What is the magnitude of their resultant?

A: Magnitude of the resultant $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$$\therefore R = \sqrt{(3)^2 + (5)^2 + 2\times3\times5\cos 60} = 7 \text{ Units}$$

4. $\vec{A} = \vec{i} + \vec{j}$. What is the angle between the vector and x-axis?

A. If 'θ' is the angle made by the vector $\vec{A}$ with x-axis

$$\cos\theta = \frac{A_x}{|\vec{A}|} = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^0$$

5. When two right angled vectors of magnitude 7 units and 24 units combine, what is the magnitude of their resultant?

A: Here $\theta = 90^0$

Magnitude of their resultant R = $\sqrt{(7)^2 + (24)^2 + 2\times7\times24\cos 90}$ = 25 Units

6. If $\vec{P} = 2\hat{i} + 4\hat{j} + 14\hat{k}$ and $\vec{Q} = 4\hat{i} + 4\hat{j} + 10\hat{k}$ find the magnitude of $\vec{P} + \vec{Q}$.

A. $\vec{P} + \vec{Q} = (2\hat{i} + 4\hat{j} + 14\hat{k}) + (4\hat{i} + 4\hat{j} + 10\hat{k}) = 6\hat{i} + 8\hat{j} + 24\hat{k}$

$\left|\vec{P} + \vec{Q}\right| = \sqrt{36 + 64 + 576} = \sqrt{676} = 26\,\text{Units}$

7. Can a vector of magnitude zero have nonzero components?

A. No. If the components of a vector are non-zero components, then its magnitude will never be equal to zero.

8. What is the acceleration of projectile at the top if its trajectory?

A: a = g and it is directed vertically downwards.

9. Can two vectors of unequal magnitude add up to give the zero vector? Can three unequal vectors add up to give the zero vector?

A: i) No. Two vectors of unequal magnitude cannot add up to give zero vector.

ii) Yes. Three vectors of unequal magnitude can add up to give zero vector.

Short Answer Questions

1. State parallelogram law of vectors. Derive an expression for the magnitude and direction of the resultant vector?

A. **Statement:**

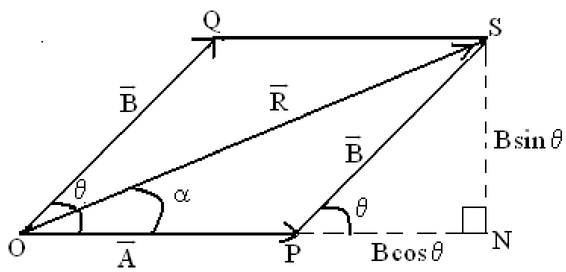
If two vectors are represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, then their resultant is represented both in magnitude and direction by the diagonal passing through the same point.

Explanation:

Let OP and OQ represent the two vectors $\vec{A}$ and $\vec{B}$ making an angle θ. The diagonal $\overrightarrow{OS}$ represents the resultant $\vec{R}$.

Magnitude (R):

Extend the line OP and draw a perpendicular SN. From the



In the right angled triangle ONS,

$OS^2 = ON^2 + NS^2 = (OP + PN)^2 + NS^2$

Or $R^2 = OP^2 + PN^2 + 2OP.PN + NS^2$ ------ (1)

From the triangle PNS,

$PN = PS \cos\theta = B \cos\theta$ and $NS = PS \sin\theta = B \sin\theta$ ----------- (2)

$PN^2 + NS^2 = B^2 \cos^2\theta + B^2 \sin^2\theta = B^2$ ------------ (3)

Using Eq. (2) and (3) in Equation (1)

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$$\therefore R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Direction:

Let the resultant $\vec{R}$ makes an angle α with $\vec{A}$.

In the right angled triangle OSN, $\tan\alpha = \frac{NS}{ON} = \frac{NS}{OP + PN}$

$$\text{Or } \tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\therefore \alpha = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right)$$

2. What is relative motion? Explain it?

A. Relative Motion:

The apparent motion (displacement (or) velocity (or) acceleration) which one body seems to possess, when viewed from another body, is said to be the relative motion of one body with respect to another body (or) It is the motion of one body with respect to another body.

a. The relative velocity of body 'A' with respect to 'B' is given by $\vec{V}_R = \vec{V}_A - \vec{V}_B$

b. The relative velocity of body 'B' with respect to 'A' is given by $\vec{V}_R = \vec{V}_B - \vec{V}_A$

c. $\vec{V}_A - \vec{V}_B$ and $\vec{V}_B - \vec{V}_A$ are equal in magnitude but opposite in direction

d. $|\vec{V}_R| = |\vec{V}_A - \vec{V}_B| = \sqrt{V_A^2 + V_B^2 - 2.V_A V_B.\cos\theta}$

e. For two bodies moving in the same direction, relative velocity is equal to the difference of velocities. ($\theta = 0°.\cos 0 = 1$)

$|\vec{V}_R| = V_A - V_B$

f. For two bodies moving in opposite direction, relative velocity is equal to the sum of their velocities. ($\theta = 180°;\cos 180 = -1$)

$\therefore |\vec{V}_R| = V_A + V_B$

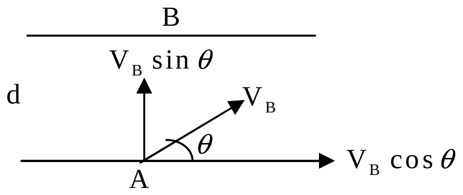
g. If they move at right angle to each other, then the relative velocity $= \sqrt{v_1^2 + v_2^2}$.

3. Show that a boat must move at an angle of 90^0 with respect to river water in order to cross the river in a minimum time?

A. Let AB = d be width of the river.

Suppose that a boat moves with a velocity V_B such that it makes an angle θ with the line AB as shown in figure.

$$d = (V_B \, Sin\theta) t$$



Time taken by the boat to cross the river is $t = \dfrac{d}{V_B \, Sin\theta}$

If $\theta = 90^o$, t is minimum (i.e.) Hence a boat must move at an angle of 90^0 with respect to river water in order to cross the river in a minimum time.

4. Define unit vector, null vector and position vector?

A. **Unit Vector:**

A vector whose magnitude is equal one is called a unit vector. A unit vector has no units and dimensions.

The unit vector $\vec{A}$ is given by $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$

Null Vector:

A vector whose magnitude is zero and which has no specific direction is called a null vector.

Position Vector:

The position of a particle is described by a position vector which is drawn from the origin of a reference frame. This helps to locate the particle in space.

Let the co-ordinates of the particle at P are x, y and z. The position vector of 'P' with respect to origin O given by $\overrightarrow{OP} = \vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$

Its magnitude, $\left|\overrightarrow{OP}\right| = \left|\vec{r}\right| = \sqrt{x^2+y^2+z^2}$

5. If $\left|\vec{a}+\vec{b}\right| = \left|\vec{a}-\vec{b}\right|$, prove that the angle between $\vec{a}$ and $\vec{b}$ is 90^0?

A: Let 'θ' be the angle between $\vec{a}$ and $\vec{b}$

$$\left|\vec{a}+\vec{b}\right| = \left|\vec{a}-\vec{b}\right|$$

$$\sqrt{a^2+b^2+2ab\cos\theta} = \sqrt{a^2+b^2-2ab\cos\theta}$$

$$a^2+b^2+2ab\cos\theta = a^2+b^2-2ab\cos\theta$$

Or $4ab\cos\theta = 0$

$\therefore \theta = 90^0$.

6. Show that the trajectory of an object thrown at certain angle with the horizontal is a parabola?

A. Let a body be projected with an initial velocity 'v_0' at an angle 'θ_0' with the horizontal, from the point O. The path of the body is called trajectory.

Initial horizontal component of velocity $v_x = v_0 \cos\theta_0$

Initial vertical component of velocity $v_y = v_0 \sin\theta_0$

The horizontal component is constant throughout the motion of the body. The vertical component changes both in magnitude and direction due to gravity.

Let the projectile is at the point P (x, y) after a time interval 't'.

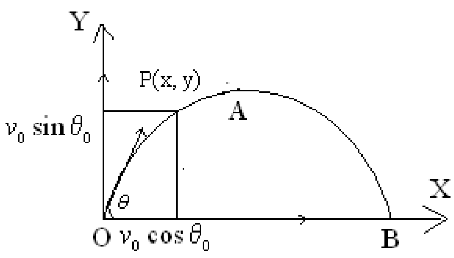
Along X-direction, using

$$s = ut + \frac{1}{2}at^2$$

x = ($v_0 \cos\theta_0$) t

Or $t = \dfrac{x}{v_0 \cos\theta_0}$ --------------- (1)

Along Y-direction, using $s = ut + \dfrac{1}{2}at^2$



$y = (v_0 \sin\theta_0)t - \dfrac{1}{2}gt^2$ -------------- (2)

From equations (1) and (2) ,

$$y = (v_0 \sin\theta_0)\frac{x}{v_0 \cos\theta_0} - \frac{1}{2}g\left(\frac{x}{v_0 \cos\theta_0}\right)^2$$

$$y = (\tan\theta_0)x - \left(\frac{g}{2v_0^{\,2} \cos^2\theta_0}\right)x^2$$

Let $\tan\theta_0 = A$ and $\dfrac{g}{2v_0^{\,2}\cos^2\theta_0}$ =B

.: $y = Ax - Bx^2$ Where A and B are constants

Hence the path of a projectile is a parabola.

7. Explain the terms the average velocity and instantaneous velocity. When are they equal?

A. Average Velocity:

It is defined as the ratio of displacement Δx to the time interval Δt.

$$V_{ave} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

It is independent of path followed by the particle between initial and final positions. It gives only the result of motion.

Instantaneous Velocity:

The velocity of a particle at a particular instant of time is known as instantaneous velocity.

$$V_{ins} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

In uniform motion the instantaneous velocity of a body is equal to the average velocity.

8. Show that the maximum height and range of a projectile are $\frac{u^2 \sin^2 \theta}{2g}$ *and* $\frac{u^2 \sin 2\theta}{g}$ respectively where the terms have their regular meanings?

A: **Maximum Height (H):**

The maximum vertical displacement of a projectile is called Maximum Height.

Initial velocity (u_y) = u sinθ

Final velocity (v_y) = 0

Acceleration (a) = -g

Displacement $= H_{max} = H$

Using, $v^2 - u^2 = 2as$

$0 - u^2 \sin^2 \theta = 2(-g)H$

Or $u^2 \sin^2\theta = 2gH$

$$\therefore H = \frac{u^2 \sin^2\theta}{2g}$$

Horizontal Range (R):

The maximum horizontal displacement of a projectile is called Range (R).

Initial velocity $(u_x) = u\cos\theta$

Acceleration $(a_x) = 0$

Time of flight (t) = $T = \dfrac{2u\sin\theta}{g}$

Displacement s = R

Using, $s = ut + \dfrac{1}{2}at^2$

$$R = (u\cos\theta)T = \frac{(u\cos\theta)(2u\sin\theta)}{g} = u^2\frac{(2\sin\theta\cos\theta)}{g}$$

$$\therefore \quad R = \frac{u^2\sin 2\theta}{g}$$

9. If the trajectory of a body is parabolic in one reference frame, can it be parabolic in another reference frame that moves at constant velocity with respect to the first reference frame? If the trajectory can be other than parabolic, what else it can be?

A. No, it may or may not be parabolic.

Explanation:

1) When an object is thrown out from a train moving with constant velocity with respect to ground, then it appears to the observer in the train as vertically thrown up body with a trajectory of straight line. Whereas for an observer on the ground the path of the body is parabolic.

2) If this body is observed from another train moving with constant velocity other than the first train, then its trajectory appears as parabolic only.

10. **A force $2\hat{i}+\hat{j}-\hat{k}$ Newton acts on a body which is initially at rest. At the end of 20 seconds, the velocity of the body is $4\hat{i}+2\hat{j}+2\hat{k}$ ms^{-1}. What is the mass of the body?**

A. Force $\vec{F}=2\hat{i}+\hat{j}-\hat{k}$ and $|\vec{F}|=\sqrt{4+1+1}=\sqrt{6}$

Initial velocity $\vec{u}=0$

Final velocity $\vec{v}=4\hat{i}+2\hat{j}+2\hat{k}$ and $|\vec{v}|=2\sqrt{6}$

Time t= 20

$$F=ma=m\frac{v-u}{t}\Rightarrow\sqrt{6}=m\frac{2\sqrt{6}}{20}$$

Mass of the body, m = 10Kg

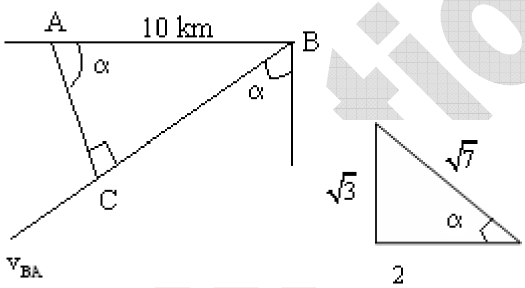
Problems

1. **Ship A is 10 km due west of ship B. Ship A is heading directly north at a speed of 30 km/h, while ship B is heading in a direction 60^0 west of north at a speed of 20 km/h.**
(i) Determine the magnitude of the velocity of ship B relative to ship A.
(ii) What will be their distance of closest approach?

A. (i) Velocity of ship B relative to A $\overline{V}_{BA} = \overline{V}_B - \overline{V}_A$

$$|\overline{V}_{BA}| = \sqrt{V_A{}^2 + V_B{}^2 + 2V_AV_B\mathrm{Cos}120^0} = \sqrt{30^2 + 20^2 + 2\mathrm{x}20\mathrm{x}30\mathrm{x}\left(\frac{-1}{2}\right)} = 10\sqrt{7}\ \text{kmph}$$

(ii) $\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta} = \dfrac{20\sin 120}{30 + 20\cos 120} = \dfrac{\sqrt{3}}{2}$



The closest approach is AC given by AC = AB cos $\theta = 10\mathrm{x}\dfrac{2}{\sqrt{7}} = 7.56$ km nearly.

2. **If θ is the angle of projection, R is the range, h is the maximum height, T is the time of flight then show that (a) tan $\theta = 4h/R$ and (b) h = $gT^2/8$?**

A: (a) $h = \dfrac{u^2\sin^2\theta}{2g}$ and $R = \dfrac{u^2\sin 2\theta}{g} = \dfrac{u^2 2\sin\theta\cos\theta}{g}$

$$\therefore \frac{h}{R} = \frac{\tan\theta}{4} \quad \textbf{or} \quad \tan\theta = \frac{4h}{R}$$

(b) $h = \dfrac{u^2\sin^2\theta}{2g}$ and $T = \dfrac{2u\sin\theta}{g} \Rightarrow T^2 = \dfrac{4u^2\sin^2\theta}{g^2}$

$$\therefore \frac{h}{T^2} = \frac{g}{8} \quad \text{or} \quad h = \frac{gT^2}{8}$$

3. A projectile is fired at an angle of 60^0 to the horizontal with an initial velocity of 800 m/s;

(i) Find the time of flight of the projectile before it hits the ground.

(ii) Find the distance it travels before it hits the ground (Range).

(iii) Find the time of flight for the projectile to reach its maximum height.

A: $\theta = 60^0, u = 800\, m/s$, $g = 9.8\ m/s^2$

$$\text{(i)}\quad T = \frac{2u\sin\theta}{g} = \frac{2x800x\frac{\sqrt{3}}{2}}{9.8} = 141.4\,\text{sec}$$

$$\text{(ii)}\quad R = \frac{u^2\sin 2\theta}{g} = \frac{(800)^2 x \sin 120}{9.8} = 56.555\ \text{km}$$

$$\text{(iii)}\quad t = \frac{T}{2} = \frac{u\sin\theta}{g} = 70.7\ \text{sec}$$

4. For a particle projected slantwise from the ground, the magnitude of its position vector with respect to the point of projection, when it is at the highest point of the path is found to be $\sqrt{2}$ times the maximum height reached by it. Show that the angle of projection is $\tan^{-1}$ (2).

A. Position vector with respect to the point of projection $\vec{r} = \frac{R}{2}\hat{i} + H\hat{j}$

$$|\vec{r}| = \sqrt{\left(\frac{R}{2}\right)^2 + H^2}$$

But, $|\vec{r}| = \sqrt{2}H$ (given)

$$\therefore\ \sqrt{2}H = \sqrt{\frac{R^2}{4} + H^2}$$

Or $2H^2 = \frac{R^2}{4} + H^2$ Or $R = 2H$

$$\frac{u^2 \sin 2\theta}{g} = 2\frac{u^2 \sin^2 \theta}{2g}$$

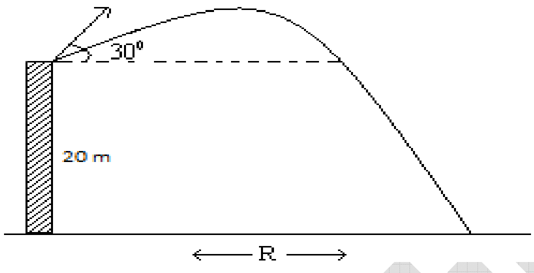
Or $\tan\theta = 2$ (or) $\theta = \tan^{-1}(2)$

5. **An object is launched from a cliff 20 m above the ground at an angle of 30^0 above the horizontal with an initial speed of 30 m/s. How far horizontally does the object travel before landing on the ground? (g = 10 m/s^2)**

A. $h = -(u \sin\theta)T + 1/2\, gT^2$

$$20 = (-30 \times \sin 30)t + \frac{1}{2} \times 10t^2$$

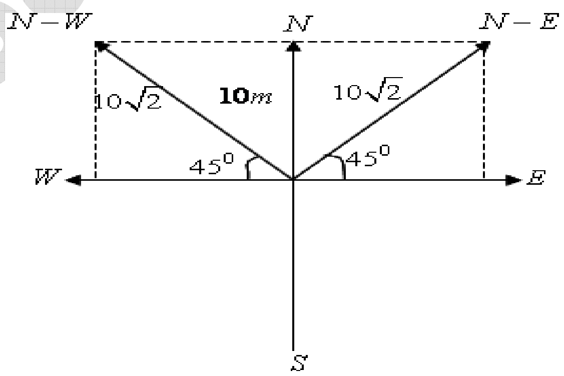
Solving the above we get, t = 4 Sec



Horizontal range R= $(u \cos\theta)\, t = (30 \cos 30)\, 4 = 60\sqrt{3}\, m$

6. **O is a point on the ground chosen as origin. A body first suffers a displacement of $10\sqrt{2}$m North - East, next 10 m North and finally $10\sqrt{2}$m North - West. How far it is from the origin?**

A. $\vec{S}_1 = 10\sqrt{2}m$ due $N - E$ $= (10\hat{i} + 10\hat{j})m$



$\vec{S}_2 = 10m \; due \; N \;\; = 10\hat{j}$

$\vec{S}_3 = 10\sqrt{2}\,m \;\; due \; N - W \;\; = -10\hat{i} + 10\hat{j}$

$\vec{S}_1 + \vec{S}_2 + \vec{S}_3 = 10\hat{i} + 10\hat{j} + 10\hat{j} - 10\hat{i} + 10\hat{j}$

$= 30\hat{j} = 30m \; due \; north$

7. From a point on the ground a particle is projected with initial velocity u, such that its horizontal range is maximum. Find the magnitude of average velocity during its ascent?

A. For maximum range, Angle of projection, $\theta = 45^0$

$\vec{u} = (u\cos\theta)\hat{i} + (u\sin\theta)\hat{j}$ (At point of projection)

$\vec{v} = (u\cos\theta)\hat{i}$ (At highest point)

Average velocity, $\vec{v}_{av} = \dfrac{\vec{u} + \vec{v}}{2}$

$$\vec{v}_{av} = \frac{(u\cos\theta)\hat{i} + (u\sin\theta)\hat{j} + (u\cos\theta)\hat{i}}{2} = \frac{(2u\cos\theta)\hat{i} + (u\sin\theta)\hat{j}}{2} = \frac{u}{\sqrt{2}}\hat{i} + \frac{u}{2\sqrt{2}}\hat{j}$$

$$|\vec{v}_{av}| = \sqrt{\left(\frac{u}{\sqrt{2}}\right)^2 + \left(\frac{u}{2\sqrt{2}}\right)^2} = \frac{\sqrt{5}u}{2\sqrt{2}}$$

8. A particle is projected from the ground with some initial velocity making an angle of 45^0 with the horizontal. It reaches a height of 7.5 m above the ground while it travels a horizontal distance of 10 m from the point of projection. Find the initial speed of projection (g = 10 m/s²)

A: $\theta = 45^0$; x = 10 m ; y = 7.5 m ; g = 10 m/s²

$$y = (\tan\theta)x - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2$$

$$7.5 = (\tan 45^0)(10) - \left(\frac{10}{2u^2\cos^2 45}\right)(10)^2$$

Solving we get $u = 20m/s$

9. Wind is blowing from the south at 5 ms^{-1}. To a cyclist it appears to be blowing from the east at 5 ms^{-1}. Find the velocity of the cyclist?

A. Velocity of wind, $\overrightarrow{v_w} = 5\hat{j}$

Velocity of wind relative to cyclist, $\overrightarrow{v_{wc}} = 5\hat{i}$

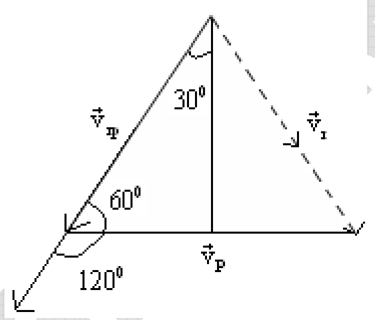
$\overrightarrow{v_{wc}} = \overrightarrow{v_w} - \overrightarrow{v_c}$

$\overrightarrow{v_c} = \overrightarrow{v_w} - \overrightarrow{v_{wc}} = 5\hat{j} - (-5\hat{i}) = +5\hat{j} + 5\hat{i}$

$|\vec{v}_c| = 5\sqrt{2}$ ms^{-1} towards north east.

10. A person walking at 4 m/s finds rain drops falling slantwise in to his face with a speed of 4 m/s at an angle of 30^0 with the vertical. Show that the actual speed of the rain drops is 4m/s?

A. Velocity of person $|\vec{v}_p| = 4m/s$



Velocity of rain drops relative to person, $|\vec{v}_{rp}| = 4m/s$

$$|\vec{v}_r| = \sqrt{v_{rp}^2 + v_p^2 + 2v_{rp}v_p \cos 120^0} = \sqrt{16+16+2(4)(4)\left(-\frac{1}{2}\right)} = \sqrt{16} = 4 \text{ m/s}$$