MOTION IN A STRAIGHT LINE

Important Points:

1. An object is said to be at rest, if the position of the object does not change with time with respect to its surroundings.

2. An object is said to be in motion, if its position changes with time with respect to its surroundings.

3. Rest and Motion are Relative:

A person travelling in a bus is at rest with respect to the co-passenger and he is in motion with respect to the person on the road.

4. **Distance and Displacement:**

a) Distance is actual path and displacement is the shortest distance between its initial and final positions.

b) Distance is a scalar and displacement is a vector quantity.

5. Speed:

a) The rate of distance travelled by a body is called speed. Its SI unit is ms^{-1}.It is a scalar quantity.

$$\text{Speed} = \frac{\text{distance travelled}}{\text{time taken}}.$$

b) Instantaneous speed $= \underset{\Delta t \to 0}{Lt} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$.

c). Average speed = $\frac{\text{Total distance}}{\text{Total time}}$

6. Velocity:

a) The rate of change of displacement of a body is called velocity. Its SI unit is ms^{-1}.It is a vector quantity.

b) A body is said to move with uniform velocity, if it has equal displacements in equal intervals of time, however small these intervals may be.

c) The velocity of a particle at any instant of time or at any point of its path is called instantaneous velocity.

$$\vec{V} = \underset{\Delta t \to 0}{Lt} \frac{\Delta \vec{s}}{\Delta t} = \frac{\vec{ds}}{dt}$$

d) Average velocity = $\dfrac{\text{Total displacement}}{\text{Total time}}$

7. **Acceleration:**

a) The rate of change of velocity of a body is called acceleration.

$$a = \frac{v-u}{t}$$

b) The acceleration lies along the direction of change in velocity.

8. **Retardation:**

Any body travelling with decreasing velocity possesses retardation or deceleration.

9. Acceleration under earth's gravity is called acceleration due to gravity (g) and it is equal to 980 cms^{-2} or 9.8 ms^{-2}.

10. **Equations of Motion (Uniform Acceleration):**

1) $v = u + at$ 2) $S = ut + \frac{1}{2}at^2$ 3) $v^2 - u^2 = 2as$ 4) $S_n = u + a\left(n - \frac{1}{2}\right)$

Where S_n is the distance travelled by a body in the n^{th} second of its journey.

11. **Equations of motion of a freely falling body (u = 0 ; s = h and a = g):**

1) $v = gt$ 2) $h = \frac{1}{2}gt^2$ 3) $v^2 = 2gh$ 4) $S_n = g\left(n - \frac{1}{2}\right)$

12. **Equations of motion for a body projected vertically up** $(a=-g, s=h)$**:**

1) $v=u-gt$ 2) $h=ut-\frac{1}{2}gt^2$ 3) $v^2=u^2-2gh$ 4) $h_n=u-g\left(n-\frac{1}{2}\right)$

13. For a body thrown up vertically from the top of a tower of height ‘h’,

1) v = - u + gt   2) h = - ut + $\frac{1}{2}gt^2$ 3) $v^2=u^2-2gh$ 4) $h_n=-u+g\left(n-\frac{1}{2}\right)$

14. **For a body thrown vertically up with an initial velocity ‘u’**

a. Maximum Height:

Maximum vertical distance travelled is known as maximum height.

$$H=\frac{u^2}{2g}$$

b. Time of Ascent:

The time taken by a body to reach the maximum height is known as time of ascent.

$$t_a = \frac{u}{g}$$

c. Time of Descent:

The time in which body comes down from maximum height is known as time of descent.

$$t_d = \frac{u}{g}$$

d. Time of Flight:

The total time for which a body remains in air before reaching the ground is known as time of flight.

$$T = t_a + t_d = \frac{u}{g}+\frac{u}{g}=\frac{2u}{g}$$

15. Time of ascent is equal to time of descent, if air resistance force is neglected.

16. If a body is allowed to fall freely from the top of a tower of height 'h' and another is projected simultaneously from the foot of tower in the upwards with velocity u , then they meet after time

$$t = \frac{h}{u}$$

17. **Relative Velocity:**

Velocity of one body with respect to that of another body is called relative velocity. The velocity of `A` with respect to that of `B` is given by

$$\vec{V}_{AB} = \vec{V}_A \pm \vec{V}_B$$

If 'A' and 'B' are moving at an angle θ with one another

$$\left|\vec{V}_{AB}\right| = \sqrt{V_A^2 + V_B^2 - 2V_A V_B \cos\theta}$$

18. Position- Time Graph:

a) Position-time graph is plotted by taking time along X-axis and position of the particle on Y-axis.

b) Slope of the graph gives the velocity of the particle.

19. Acceleration - Time Graph:

a) The graph is plotted by taking time along X -axis and velocity of the particle on Y- axis

b) Slope of the graph gives the acceleration.

c) Area under the graph gives the distance travelled.

Very Short Answer Questions

1. The states of motion and rest are relative. Explain?

A. **Rest:**

A body is said to be at rest if it occupies the same position for any length of time, with respect to its surroundings.

Motion:

A body is said to be in motion if it occupies different positions at different time intervals with respect to its surroundings.

Ex: A person travelling in a bus is at rest with respect to the co-passenger and he is in motion with respect to the person on the road. Hence rest and motion are relative.

2. How is average velocity different from instantaneous velocity?

A. **Average Velocity:**

It is the ratio of its total displacement of the body and the total time taken for that displacement.

$$V_{average} = \frac{Total\ displacement}{Total\ time}$$

Instantaneous velocity:

It is the velocity of the body at any instant of time.

$$V_{Inst} = \underset{\Delta t \to 0}{Lt} \frac{\Delta x}{\Delta t} = \frac{ds}{dt}$$

3. Give an example where the velocity of an object is zero but its acceleration is not zero?

A. At the highest point of a vertically projected body, velocity is zero but it is still under acceleration due to gravity.

4. A vehicle travels half the distance L with speed v_1 and the other half with speed v_2. What is the average speed?

A: Average speed $= \frac{\text{Total distance}}{\text{Total time}}$

$$V_{ave} = \frac{L}{t_1 + t_2} = \frac{L}{\frac{L/2}{V_1} + \frac{L/2}{V_2}} = \frac{L}{\left(\frac{L}{2}\right)\left(\frac{1}{V_1} + \frac{1}{V_2}\right)}$$

$$\therefore V_{ave} = \frac{2V_1V_2}{V_1 + V_2}$$

5. **A lift coming down is just about to reach the ground floor. Taking the ground floor as origin and positive direction upwards for all quantities, which one of the following is correct?**

a) x < 0, v < 0, a > 0

b) x > 0, v < 0, a < 0

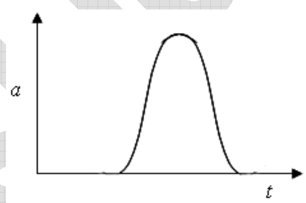
c) x > 0, v < 0, a > 0

d) x > 0, V > 0, a > 0

A: x < 0, v < 0, a > 0

6. **A uniformly moving cricket ball is hit with a bat for a very short time and is turned back. Show the variation of its acceleration with time taking the acceleration in the backward direction as positive?**

A:



7. **Give an example of one-dimensional motion where a particle moving along the positive x-direction comes to rest periodically and moves forward.**
The question may be corrected like this:
Give an example of one-dimensional motion where a particle moving along the positive x-direction comes to rest periodically.

A: A particle in simple harmonic motion comes to rest periodically at the extreme positions.

8. An object falling through a fluid is observed to have an acceleration given by a= g-bv where g is the gravitational acceleration and b is a constant. After a long time it is observed to fall with a constant velocity. What would be the value of this constant velocity?

A. If velocity is constant, acceleration is zero. a = g - bv = 0

$$\therefore v = \frac{g}{b}$$

9. If the trajectory of a body is parabolic in one frame, can it be parabolic in another frame that moves with a constant velocity with respect to the first frame? If not, what can it be?

A. No. The trajectory can be a vertical straight line.

10. A spring with one end attached to a mass and the other to a rigid support is stretched and released. When is the magnitude of acceleration a maximum?

A. Acceleration is maximum at the extreme points of its motion.

Short Answer Questions

1. Can the equations of kinematics be used when the acceleration varies with time? If not, what form would these equations take?

A. For uniform accelerated motion,

$$V = V_0 + at \;;\; x = V_0 t + \frac{1}{2} at^2 \;;\; V^2 = V_0^2 + 2ax$$

If acceleration varies with time,

(i) **To calculate velocity after time t:**

Instantaneous acceleration $a = \frac{dv}{dt}$

$$\therefore\ dv = a\ dt$$

$\int_{v_0}^{v} dv = \int_{0}^{t} a\ dt$, where v_0 is velocity at time $t = 0$

Here 'a' is a function of time.

(ii) **To calculate the velocity at position** $'x'$**;**

Instantaneous acceleration $a = \frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt} = \frac{dv}{dx}.v$

$$\therefore\ vdv = a\ dx$$

$\int_{v_0}^{v} v\ dv = \int_{x_0}^{x} a\ dx$, where is velocity at the position x_0

Here 'a' is function of position 'x'

Also, $v = \frac{dx}{dt} = \frac{dx}{dv}.\frac{dv}{dt} = \frac{dx}{dv}.a$

$$\therefore\ v.dv = a.dx$$

Then, $\int_{v_0}^{v} v\ dv = \int_{x_0}^{x} a\ dx$

2. A particle moves in a straight line with uniform acceleration. Its velocity at time t = 0 is v_1 and at time t = t is v_2. The average velocity of the particle in this time interval is $(v_1 + v_2)/2$. Is this correct? Substantiate your answer?

A: The given statement is correct.

Let a particle be moving with uniform acceleration ‘a’ along a straight. Its velocity is v_1 at time t = 0 at origin and it reached a position in a time t and its velocity became v_2.Then, average velocity over time interval‘t’ is given by

$$v_{ave} = \frac{\Delta x}{\Delta t} = \frac{x-0}{t-0} = \frac{x}{t}$$

But $x = \frac{V_2^2 - V_1^2}{2a}$ and $a = \frac{V_2 - V_1}{t}$

$$\therefore x = \frac{\left(V_2^2 - V_1^2\right)t}{2\left(V_2 - V_1\right)}$$

$$\therefore \frac{x}{t} = \frac{\left(V_2 + V_1\right)\left(V_2 - V_1\right)}{2\left(V_2 - V_1\right)}$$

Or $V_{ave} = \frac{V_1 + V_2}{2}$

3. Can the velocity of an object be in a direction other than the direction of acceleration of the object? If so, give an example?

A. Yes. Velocity may be in a direction other than direction of its acceleration.

Ex: If a body is projected vertically upwards, the velocity of body is directed vertically upwards but the acceleration due to gravity is always directed vertically downwards

4. A parachutist flying in an aeroplane jumps when it is at a height of 3 km above ground. He opens his parachute when he is about 1 km above ground. Describe his motion?

A: Assume that aeroplane is flying horizontally. Now he is like a freely falling body in vertically downward direction and has uniform velocity equal to plane velocity in horizontal direction till he opens parachute at a height 1 km above the ground.

After parachute is opened, air resistance (Viscous drag) and buoyancy also acts on him along with gravitational force. In the presence of these forces the net acceleration will become zero after falling through certain height, there he attains constant velocity called **Terminal Velocity**. Further he falls on to the ground with the same terminal.

5. A bird holds a fruit in its beak and flies parallel to the ground. It lets go of the fruit at some height. Describe the trajectory of the fruit as it falls to the ground as seen by (a) The Bird (b) a person on the ground?

A: When the bird dropped a fruit while flying horizontally, it falls on to the ground after some time. The path traversed by fruit depends on the choice of reference frame.

(a) The trajectory of fruit with respect to bird is a straight line vertically downward as it appears to be falling freely for the bird.

(b)The trajectory of fruit with respect to a person on the ground is parabola.

6. A man runs across the roof of a tall building and jumps horizontally on to the (lower) roof of an adjacent building. If his speed is $9\,m\,s^{-1}$ and the horizontal distance between the buildings is 10 m and the height difference between the roofs is 9m, will he be able to land on the next building? (Take $g = 10\,m\,s^{-2}$)

A: In this interval horizontal displacement covered is $x = u\sqrt{\frac{2(h_2 - h_1)}{g}}$

$$\therefore\ x = 9\sqrt{\frac{2\times 9}{10}} = 12.07\ m$$

Since the horizontal separation is 10 m between the two buildings, which is less than $12.07\ m$.Thus he can safely land on to the building with his effort.

7. A ball is dropped from the roof of a tall building and simultaneously another ball is thrown horizontally with some velocity from the same roof. Which ball lands first? Explain your answer?

A. Both the balls reach the ground simultaneously.

Let 'h' be the height of the building. For the dropped ball, time taken to reach the ground -

$$t_1 = \sqrt{\frac{2h}{g}} \quad ---(1)$$

For the horizontally thrown ball the initial vertical component of velocity is zero. Let the time taken to reach the ground is t_2 .

From, $s = ut + \frac{1}{2}at^2$

$$h = \frac{1}{2}gt_2^2$$

$$\therefore \quad t_2 = \sqrt{\frac{2h}{g}} \quad -----(2)$$

From (1) and (2) $\quad t_1 = t_2$

Hence both reach the ground at the same time.

8. A ball is dropped from a building and simultaneously another ball is projected upward with some velocity. Describe the change in relative velocities of the balls as a function of time?

A: Let ball (1) be dropped from the top edge of building and ball (2) be projected vertically upwards with velocity 'u' from the same point.

At t = 0, velocity of ball (1) with respect to ball (2) is $u_{12} = 0 - (-u) = u$

Acceleration of ball (1) with respect to ball (2) is $a_{12} = a_1 - a_2 = g - g = 0$

The relative velocity of ball (1) with respect to ball (2) after time interval 't' is

$$V_{12} = u_{12} + a_{12} = u + (0)t = u = \text{constant}$$

Thus the relative velocity of the balls is always constant during their motion. Hence the change in their relative velocity is zero in course of time, which is equal to the velocity of projection of second ball.

9. A typical raindrop is about 4mm in diameter. If a raindrop falls from a cloud which is at 1 km above the ground, estimate its momentum when it hits the ground?

A: The momentum of rain drop just before reaching the ground is

$$P = \text{mass} \times \text{velocity} = \text{mv}$$

But, mass $m = volume \times density = \frac{4}{3}\pi r^3 \rho$ and $V = \sqrt{2gh}$

$$\therefore P = \frac{4}{3}\pi r^3 \rho \sqrt{2gh}$$

$$r = \frac{4mm}{2} = 2mm = 2 \times 10^{-3} m$$

$$P = \frac{4}{3} \times \frac{22}{7} \times \left(2 \times 10^{-3}\right)^3 \times 10^3 \times \sqrt{2 \times 9.8 \times 10^3} = \frac{4}{3} \times \frac{22}{7} \times 8 \times 10^{-6} \times 14 \times 10$$

$$= 0.00469 \text{ kg ms}^{-1}$$

10. Show that the maximum height reached by a projectile launched at an angle of 45° is one quarter of its range?

A. Maximum height $H = \frac{u^2 \sin^2 \theta}{2g}$

Horizontal range $R = \frac{u^2 \sin 2\theta}{g}$

$$Now, \frac{H}{R} = \frac{u^2 \sin^2 \theta}{2g} \times \frac{g}{u^2 \sin 2\theta} = \frac{\tan \theta}{4}$$

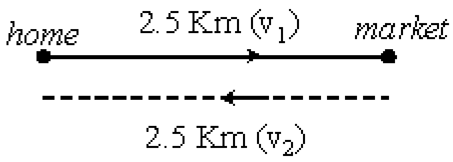
If , $\theta = 45^0$, $H = \frac{R}{4}$

Hence maximum height reached is one quarter of its range.

Problems

1. A man walks on a straight road from his home to a market 2.5 km away with a speed of $5\,km\ h^{-1}$. Finding the market closed, he instantly turns and walks back home with a speed of $7.5\,km\ h^{-1}$. What is the (a) Magnitude of Average Velocity and (b) Average Speed of the man over the time interval 0 to 50 min?

A:



Time taken to go from home to market is $t_1 = \frac{S}{V_1} = \frac{2.5}{5}$

$$t_1 = \frac{1}{2} hr = 30\text{min}$$

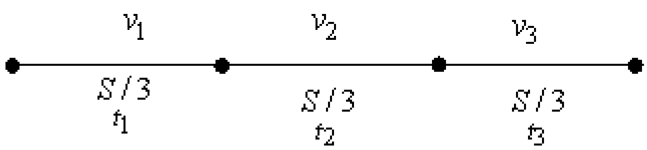
Time taken by him to get back from market to home is $t_2 = \frac{s}{V_2} = \frac{2.5}{7.5} = \frac{1}{3} hr$ =20 min

$$(a)\ magnitude\ of\ average\ velocity = \frac{total\ displacement}{total\ time} = \frac{0}{50} = 0$$

$$(b)\ average\ speed = \frac{total\ distnace}{total\ time} = \frac{2.5+2.5}{\left(\frac{1}{2}+\frac{1}{3}\right)} = 6\ kmph$$

2 A car travels the first third of a distance with a speed of 10 kmph, the second third at 20 kmph and the last third at 60 kmph. What is its mean speed over the entire distance?

A:



Mean speed $V_{av} = \frac{S_1+S_2+S_3}{t_1+t_2+t_3}$

$$V_{av} = \frac{\frac{S}{3}+\frac{S}{3}+\frac{S}{3}}{\frac{S}{3V_1}+\frac{S}{3V_2}+\frac{S}{3V_3}}$$

$$\therefore V_{av} = \frac{3V_1V_2V_3}{V_1V_2+V_2V_3+V_3V_1} = \frac{3(10)(20)(60)}{(10\times 20)+(20\times 60)+(60\times 10)}$$

Or $V_{av} = 18 kmph$

3. A bullet moving with a speed of $150\ ms^{-1}$ strikes a tree and penetrates 3.5 cm before stopping. What is the magnitude of its retardation in the tree and the time taken for it to stop after striking the tree?

A: $u = 150 m/s$; $v = 0$; $s = 3.5\times 10^{-2} m$

Acceleration = -a

i) **From,** $v^2 - u^2 = 2(-a)s$

$$0^2 - (150)^2 = 2\times(-a)\times 3.5\times 10^{-2}$$

$$a = \frac{150\times 150}{2\times 3.5\times 10^{-2}} = 3.214\times 10^5 m/s$$

ii) Let the time taken for it to stop is 't'

From, $v = u + (-a)t$

$$0 = 150 - 3.214\times 10^5 t$$

$$\Rightarrow t = \frac{150}{3.214\times 10^5} = 4.667\times 10^{-4} s$$

4. A motorist drives north for 30 min at 85 km/h and then stops for 15min. He continues travelling north and covers 130 km in 2 hours. What is his total displacement and average velocity?

A: Distance travelled in first case along north is $S_1 = v_1 t_1 = 85\times\frac{30}{60} km = 42.5 km$

Distance travelled in second case along north is $S_2 = 130 km$

The magnitude of total displacement is $S = S_1 + S_2 = (42.5+130) km = 172.5 km$

Total time he spend $t = \frac{30}{60} + \frac{15}{60} + 2 =$ 2hr 45min

Magnitude of average velocity $= \frac{Net\ displacement}{Total\ time} = \frac{172.5}{\left(\frac{1}{2}+\frac{1}{4}+2\right)} = \frac{172.5}{\left(\frac{11}{4}\right)} kmph$

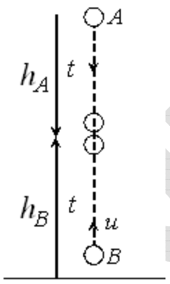
= 62.72 kmph

5. A ball A is dropped from the top of a building and at the same time an identical ball B is thrown vertically upward from the ground. When the balls collide the speed of A is twice that of B. At what fraction of the height of the building did the collision occur?

A. Let‘t’ be the time after which they collide

Let h_A is distance travelled by A and h_B is the distance travelled by B before they collide.

Let u be the initial speed of B. Let v_A and v_B be the speed of A and B at time of collision.



From $v = u + at$,

$v_A = 0 + gt \Rightarrow v_A = gt ----(1)$

and $v_B = u - gt ---(2)$

But $v_A = 2v_B$

$gt = 2(u - gt)$ or $t = \frac{2u}{3g}$ ....(3)

From, $s = ut + \frac{1}{2}at^2$

$h_A = 0(t) + \frac{1}{2}gt^2 = \frac{1}{2}gt^2$ ___(4)

$h_B = u(t) - \frac{1}{2}gt^2 ---(5)$

$$\frac{h_B}{h_A + h_B} = \frac{ut - \frac{1}{2}gt^2}{\frac{1}{2}gt^2 + \left(ut - \frac{1}{2}gt^2\right)} = \frac{ut - \frac{1}{2}gt^2}{ut}$$

But, $t = \frac{2u}{3g}$

Required fractions $= 1 - \frac{1}{2}\frac{g}{u}\left(\frac{2u}{3g}\right) = \left(1 - \frac{1}{3}\right) = \frac{2}{3}$

6 **Drops of water fall at regular intervals from the roof of a building of height 16m. The first drop strikes the ground at the same moment as the fifth drop leaves the roof. Find the distances between successive drops?**

A. Let 't' be the time interval between fall of two successive drops. Then from fig. and from the given information in the problems

Time of fall of 1st drop = 4t

Time of fall of 2nd drop = 3t

Time of fall of 3rd drop = 2t

Time of fall of 4th drop = t

Total height from which drops fall is given by 'h'

Clearly distance through which 1st drop fall is also 'h'

So $h = 0 + \frac{1}{2}g(4t)^2$

$$h = \frac{1}{2}g(4t)^2 \Rightarrow \frac{1}{2}gt^2 = \frac{h}{16} \quad ---(1)$$

Distance through which 1st drop fall is $h_1 = h$

Distance through which 2nd drop fall is $h_2 = \frac{1}{2}g(3t)^2$

$$h_2 = 9\left(\frac{1}{2}gt^2\right)$$

Distance through which 3rd drop fall is $h_3 = \frac{1}{2}g(2t)^2$

$$h_3 = \frac{4h}{16}$$

Distance through which 4th drop fall is $h_4 = \frac{1}{2} g(t)^2$

Gap b/w 1st and 2nd drop $= h_1 - h_2 = h - \frac{9h}{16} = \frac{7h}{16} = \frac{7}{16} \times 16 = 7m$

Gap b/w 2nd and 3rd drop $= h_2 - h_3 = \frac{9h}{16} - \frac{4h}{16}$

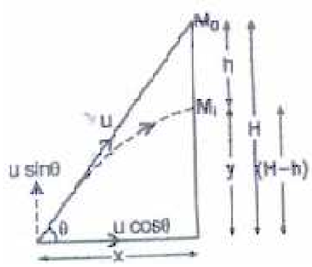
$$= \frac{5}{16} h = \frac{5}{16}(16) = 5m$$

Gap b/w 3rd and 4th drop $= h_3 - h_4 = \frac{4h}{16} - \frac{h}{16} = \frac{3h}{16} = \frac{3}{16} \times 16 = 3m$

Gap b/w 4th drop and 5th drop $= h_4 = \frac{h}{16} = 1m$

7. A hunter aims a gun at a monkey hanging from a tree some distance away. The monkey drops from the branch at the moment he fires the gun hoping to avoid the bullet. Explain why the monkey made a wrong move?

A.



If there were no gravity the bullet would reach height H in the time t taken by it to travel the horizontal distance X, *i.e.*

$H = u \sin\theta \times t$ with $t = \frac{X}{u\cos\theta}$

However, because of gravity the bullet has an acceleration g vertically downwards, so in time t the bullet will reach a height

$y = u\sin\theta \times t - \frac{1}{2}gt^2 = H - \frac{1}{2}gt^2$

This is lower than H by $\frac{1}{2}gt^2$ which is exactly the amount the monkey falls freely in this time. So the bullet will hit the monkey regardless of the initial velocity of the bullet so long as it is great enough to travel the horizontal distance to the tree before hitting the ground. However, for large u lesser will be the time of motion; so the monkey is hit near its initial position and for smaller u it is hit just before it reaches the floor. Bullet will hit the monkey only if

$$y > 0, i.e., \quad H - \frac{1}{2}gt^2 > 0$$

Or $$H > \frac{1}{2}gt^2 \quad or \quad H > \frac{1}{2}g \times \frac{x^2}{u^2\cos^2\theta}$$

or $u > \frac{x}{\cos\theta}\sqrt{\frac{g}{2H}}$

But, $\cos\theta = \frac{x}{\sqrt{H^2 + x^2}}$

Or $u > \sqrt{\frac{g}{2H}\left(x^2 + H^2\right)} = u_0$

If $u < u_0$, the bullet will hit the ground before reaching the monkey.

8. A food packet is dropped from an aero plane, moving with a speed of 360 kmph in a horizontal direction, from a height of 500m. Find (i) Its time of descent

(ii) The horizontal distance between the point at which the food packet reaches the ground and the point above which it was dropped?

A: $u_x = 360kmph = 360 \times \frac{5}{18} = 100m/s\,; h = 500m\,; g = 10m/s^2$

(i) $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 500}{10}} = 10s$

(ii) Horizontal distance $s_x = u_x t = 100 \times 10 = 1000m$

9. A ball is tossed from the window of a building with an initial velocity of $8\,ms^{-1}$ at an angle of 20^0 below the horizontal. It strikes the ground 3s later. From what height was the ball thrown? How far from the base of the building does the ball strike the ground?

A: $u = 8m/s,\ t = 3s,\ g = 9.8m/s^2;\ u_x = u\cos 20^0$ and $u_y = u\sin 20^0$

i) From, $s_y = u_y t + \frac{1}{2}a_y t^2$

$$h = \left(u\sin 20^0\right)t + \frac{1}{2}gt^2$$

$$h = 8\times\sin 20^0\times 3 + \frac{1}{2}\times 9.8\times 3^2 = 52.3m$$

ii) $S_x = u_x \times t$

$$u\left(\cos 20^0\right)\times 5 = 8\times 0.9396\times 3 = 22.55m$$

10. Two balls are projected from the same point in directions 30^0 and 60^0 with respect to the horizontal. What is the ratio of their initial velocities if they

(a) Attain the same height? (b) Have the same range?

A. $H = \dfrac{u^2\sin^2\theta}{2g}$ *and* $R = \dfrac{u^2\sin 2\theta}{g}$

a) $H_1 = H_2$

$$\frac{u_1^2\sin^2 30^0}{2g} = \frac{u_2^2\sin^2 60^0}{2g}$$

$$u_1^2\left(\frac{1}{2}\right)^2 = u_2^2\left(\frac{\sqrt{3}}{2}\right)^2 \Rightarrow u_1 : u_2 = \sqrt{3}:1$$

b) $R_1 = R_2$

$$\frac{u_1^2\sin 2(30)}{g} = \frac{u_2^2\sin 2(60)}{g}$$

$$u_1^2\left(\frac{\sqrt{3}}{2}\right) = u_2^2\left(\frac{\sqrt{3}}{2}\right) \Rightarrow u_1 : u_2 = 1:1$$