GRAVITATION

Important Points:

1. Kepler's Laws

a) Law of Orbits:

Every planet revolves around the sun in an elliptical orbit with the sun at one of the foci.

b) Law of Areas:

The areal velocity of the radius vector drawn from the sun to the planet sweeps out equal areas in equal time intervals.

c) Law of time Periods:

The square of the time period of a planet around the sun is proportional to the cube of the semi-major axis. $T^2 \alpha R^3$

2. Kepler's second law obeys the law of conservation of angular momentum.

 $r^2 \omega = \text{const.}$ (or) r v = const.

Or $I \omega = const$

Where r = the distance between the sun and the planet.

V = the speed of the planet.

 ω = the angular velocity of the Planet.

3. Basic forces in Nature:

- i) Gravitational Force
 - ii) Electromagnetic Force
 - iii) Strong Nuclear Force
 - iv) Weak Nuclear Force

4. The ratio strengths of gravitational, weak nuclear, Electromagnetic and strong nuclear forces are respectively $1:10^{31}:10^{36}:10^{38}$

5. Newton's law of Gravitation:

The force of attraction between any two bodies is directly proportional to product of their masses and inversely proportional to square of distance between them $F = \frac{Gm_1m_2}{r^2}$

Where G is universal gravitational constant

6. Value and units of G:

 $G = 6.67 \text{ x } 10^{-11} \text{ Newton - } m^2 \text{ kg}^{-2}$

Dimensional formula : $G = M^{-1}L^3T^{-2}$

7. Relation between 'G' and 'g':

Relation between gravitational constant and acceleration due to gravity is given by

$$g = \frac{Gm}{R^2}$$

8. Variation of 'g':

a) Acceleration due to gravity at certain altitude (height)'h' is given by

$$g_h = g\left(\frac{R}{R+h}\right)^2 \cong g\left(1-\frac{2h}{R}\right)$$

b) Acceleration due to gravity at certain depth'd' is given by

 $g_d = g\left(1 - \frac{d}{R}\right)$

- c) Acceleration due to gravity at certain altitude ϕ is given by
 - $g_{\phi} = g R\omega^2 \cos^2 \phi$ Where ϕ latitude angle

9. Gravitational PE =
$$\frac{GMm}{2r} = \frac{mgr}{2}$$

10. Gravitational KE =
$$-\frac{GMm}{2r} = -\frac{mgr}{2}$$

11. Gravitational. Total energy $= -\frac{GMm}{2r} = -\frac{mgr}{2}$

13. Orbital Velocity:

It is the horizontal velocity with which a body should be projected in order that it may revolve round the Earth in an orbit.

$$V_0 = \sqrt{\frac{GM}{R+h}} \cong \sqrt{\frac{GM}{R}} \cong \sqrt{gR} = 7.92 \, km \, / \, \text{sec}$$

14. Escape Velocity (V_c) :

The minimum velocity with which a body should be projected to overcome the Earth's gravitational field is called the escape velocity.

$$V_e = \sqrt{\frac{GM}{R}} = \sqrt{2gR} \cong 11.2 \, km \, / \, s$$

15. Relation between ' v_e ' and ' v_0 ' is $V_e = \sqrt{2}v_0 = 1.414v_0$

16. Geo-Stationary or Polar Satellite:

- i. Time period is 24hrs.
- ii. Its velocity relative to the Earth is zero.

iii. Height of the geo-satellite from the surface of the Earth is about 36,500km and 42,500 Km

from the centre of the Earth.
$$h = \left(\frac{GM}{4\pi^2}T^2\right)^{1/3} - R$$

iv. Geo-satellites are used to know the shape and size of the Earth, communication purpose etc.

Very Short Answer Questions

1. State the units and dimensions of universal gravitational constant (G)?

A. Units : $N - m^2 / kg^2$

Dimensional formula: $\left\lceil M^{-1}L^{3}T^{-2}\right\rceil$

2. State vector form of Newton's law of gravitation?

A. The force of attraction of between the bodies of masses m_1 and m_2 separated by a distance r

is given by
$$F = \frac{Gm_1m_2}{r^2}\hat{r} = \frac{Gm_1m_2}{r^3}\bar{r}$$

- 3. If the gravitational force of Earth on the moon is F, what is the gravitational force of moon on Earth? Do these forces from the action reaction pair?
- A: i) The gravitational force of the moon on the Earth = F

ii) These two forces equal and opposite direction. Hence they from action and reaction pair.

4. What would be the change in acceleration due to gravity (g) at the surface, if the radius of the Earth decreases by 2%, keeping the mass of the Earth constant?

A.
$$g = \frac{GM}{R^2} \Rightarrow \frac{\Delta g}{g} = -2\frac{\Delta R}{R}$$

$$\frac{\Delta g}{g} \times 100 = -2\left(\frac{\Delta R}{R} \times 100\right) = -2\left(-2\%\right) = 4\%.$$

If the radius decreases by 2%, then acceleration due to gravity also increases by 4%

- 5. As we go from one planet to another how will a) the mass and b) the weight of body change?
- A: a) Mass of the body remains same
 - b) As g changes, weight of the body also changes

- 6. Keeping the length of a simple pendulum constant, will the time period be the same on all planets? Supports your answer with reason?
- A: No As 'g' changes from planet to planet. Time period $\left[T \propto \frac{1}{\sqrt{g}}\right]$ also changes
- 7. Give the equation for the value of g at a depth 'd' from the surface of Earth. What is the value of 'g' at the centre of Earth?
- A: The equation of g at a depth'd' from surface of Earth, $g_d = g \left| 1 \frac{d}{R} \right|$

At the centre of the Earth $g_d = g\left(1 - \frac{R}{R}\right) = 0$ [:: d = R]

8. What are the factors that make 'g' the least at the equator and maximum at the poles?

A: Equator:

The factors that make 'g' the least at the equator are

i) Equatorial radius of the Earth is maximum.

ii) Latitude, $\phi = 0^0 \ [\because g_{\phi} = g - R\omega^2 \cos^2 \phi]$.

Poles:

The factors that make 'g' maximum at the poles are

i) Polar radius of the Earth is minimum.

ii) Latitude $\phi = 90^{\circ}$.

9. "Hydrogen is in abundance around the sun but not around the Earth". Explain?

A. The escape velocity on Earth is, which is less than r.m.s. velocity of hydrogen gas. But the escape velocity on Sun is, which is greater than the r.m.s. velocity of hydrogen gas. Hence the hydrogen is abundance around the sun and less around the Earth.

- 10. What is the time period of revolution of a geo-stationary satellite? Does it rotate from west to east or from east to west?
- A: Time period of revolution of a geostationary satellite is 24 hours. It rotates from west to east.

11. What are polar satellites?

A: A satellite that revolves in a polar orbit is called a polar satellite. A polar orbit passes over north and south poles of the Earth and has a smaller radius 500 – 800km

Short Answer Questions

1. State Kepler's laws of planetary motion

A: Kepler's Laws:

1) I Law (or) Law of Orbits:

All planets move in elliptical orbits around the Sun with the sun situated at one of the foci.

2) II Law (or) Law of Areas:

The line that joins any planet to the sun sweeps equal areas in equal intervals of time.

3) III Law (or) Law of Periods:

The square of the time period of revolution of a planet around the Sun is proportional to the cube of the semi major axis of the ellipse traced out by the planet i.e., $T^2 \propto a^3$

2. Deduce the relation between acceleration due to gravity (g) at the surface of a planet and gravitational constant (G)?

A. Relation Between g and G:

Consider a body of mass 'm' on the surface of a planet of radius R and mass M. The gravitational force acting on the body is given by-

F = mg - (1)

According to universal law of gravitation

$$F = \frac{GMm}{R^2} - (2)$$

Equating (1) & (2), $mg = \frac{GMm}{R^2}$ Or $g = \frac{GM}{R^2}$

- 3. How does the acceleration due to gravity (g) change for the same values of height (h) and depth (d)?
- A. a) For smaller heights, the variation of 'g' with height 'h' is $g_h = g\left(1 \frac{2h}{R}\right)$

The variation of 'g' with depth'd' is $g_d = g\left(1 - \frac{d}{R}\right)$

If
$$d = h$$
, $g_h = g\left(1 - \frac{h}{R} - \frac{h}{R}\right) = g_d - \frac{gh}{R}$

$$\therefore g_h < g_o$$

Hence for same value of d and h, the value of 'g' decreases more at height compared to depth.

b) For larger heights, the variation of 'g' with height 'h' is $g_h = g \left(\frac{R}{R+h}\right)^2$

The variation of 'g' with depth'd' is $g_d = g\left(1 - \frac{d}{R}\right)$

If
$$d = h$$
, $g_h = \frac{g}{4}$ and $g_d = 0$

 $\therefore g_h > g_d$

Hence for same value of d and h, the value of 'g' increases more at height compared to depth.

4. What is Orbital velocity? Obtain an expression for it?

A. Orbital Velocity:

The velocity to be given to a body in order to revolve round the Earth in circular orbit is known as orbital velocity.

Expression:

Consider a satellite of mass 'm' revolving around the Earth at a height 'h'. Let M be the mass and R be the radius of the Earth. Let ' v_o ' be the orbital velocity on the surface of the Earth. The gravitational force between Earth and satellite provides necessary

centripetal force.



If the satellite is very close to Earth, $v_o = \sqrt{gR} = 7.92$ Km/sec.

5. What is escape velocity? Obtain expression for it?

A. Escape Velocity:

The minimum velocity required by a body to escape from the gravitational influence of Earth is called escape velocity (V_e) of the body.

Expression:

Consider a body of mass 'm' on the surface of the Earth of mass 'M' and radius R. Let V be the escape velocity of the body.

The work done in bringing the body from infinity to the surface of the Earth is stored as gravitational potential energy.

Gravitational potential energy

 $=-\frac{GMm}{R}$ ----- (1)

The negative sign indicates that the body is bound to the Earth.

Kinetic energy
$$=\frac{1}{2}mV_e^2$$
 (2)

From (1) and (2)

$$\frac{1}{2}mV_e^2 = \frac{GMm}{R} \Longrightarrow V_e = \sqrt{\frac{2GM}{R}}$$

But, g

6.

A.

 $\therefore V_e = \sqrt{2gR} = 11.2 \text{ Km/sec}$

GM

What is a Geo Stationary satellite? State its uses?

Geo - Stationary or Polar Satellite:

A Satellite whose time period of revolution is equal to that of rotation of Earth (24 hours) is called **Geostationary Satellite**.

Uses:

These are used to

- 1) Know the shape and size of the Earth.
- 2) Study the changes in the atmosphere.
- 3) Study the upper regions of the atmosphere.
- 4) Identify the minerals and natural resources present inside the Earth.
- 7. If two places are at the same height from the mean sea level. One is a mountain and other is in air. At which place will 'g' be greater? State the reason for your answer

A:
$$g = \frac{GM}{R^2} = \frac{4}{3}\pi R\rho G$$
 $\left[\because M = V\rho = \frac{4}{3}\pi R^3\rho \right]$

Where ρ is the mean density of the Earth. Since $g \propto \rho$, the density of the mountain is greater than that of air. Therefore, g is more at the mountain.

- 8. The weights of an object is more at the poles than at the equator. At which of these can we get more sugar for the same weight? State the reason for your answer.
- A: Since the weight of the object is more at the poles than at the equator, $W_p > W_e \Rightarrow M_p g_p > m_e g_e$

Hence $g_p > g_e$ [:: $m_p = m_e$ = constant]

If weight, W = mg = constant, then $m \propto \frac{1}{g}$

As $g_p > g_e$, $m_e > m_p$ i.e., mass of sugar at equator is more than mass of sugar at poles. Hence at equator, we get more sugar for the same weight.

- 9. If a nut becomes loose and gets detached from a satellite revolving around the Earth, will it fall down to the Earth or will it revolve in the same orbit as the satellite? Give reason for your answer?
 - A. The nut is initially revolving around the Earth along with the satellite. Hence it revolves in the same orbit around the Earth even after it becomes loose.
- **10.** An object projected with a velocity greater than or equal to 11.2km.s⁻¹ will not return to Earth. Explain the reason
- A: The escape velocity on the surface of the Earth is 11.2km/s. hence an object is projected with a velocity greater than or equal to 11.2 km/s the object will not come back to the Earth.

Long Answer Questions

- 1. Define gravitational potential energy and derive an expression for it associated with two particles of masses m_1 and m_2 .
- A: When two or more bodies interact with each other due to gravitational forces, some work must be done in assembling together in their respective places. That work done is called potential energy of the system.

For a system of two masses, potential energy of the system is defined as the amount of work done in bringing these two masses from infinity to their respective places



Consider a space, where there is no gravitational field. now work is done to bring a body of mass m_1 to a point A. now, m_1 produces a gravitational field in the space. To bring another body of mass m_2 from infinity to point B at a distance r from m_1 some work is required. This work is done by the gravitational force of attraction of m_1 acting on m_2

Gravitational force acting on m_2 , which is at P is $F = \frac{Gm_1m_2}{x^2}$

Let the body m_2 is displaced by a distance dx to point Q work done

$$dW = \overrightarrow{F}.\overrightarrow{dx} = Fdx\cos\theta = \frac{Gm_1m_2}{x^2}dx \quad [\because \theta = 0^0]$$

 \therefore Total work done to bring the mass m_2 from infinity to point B is

$$W = \int dW = \int_{\infty}^{r} \frac{Gm_1m_2}{x^2} dx = \frac{-Gm_1m_2}{r}$$

We know that work done by conservation force is equal to negative of change in potential energy

i.e.,
$$W = -(U_r - U_t) \Longrightarrow W = U_t - U_r$$

Since P.E. at infinity is zero $W = U_t = U_{(say)}$

$$\therefore U = -\frac{Gm_1m_2}{r}$$

- 2. Derive expression for the variation of acceleration due to gravity?
 - (a) Above and (b) Below the surface of the Earth.

A. Height:

Let M be the mass, R be the radius and be the density of the Earth .

On the surface of the Earth
$$g = \frac{GM}{R^2}$$

At a height 'h' from the surface of the Earth $g_h = \frac{GM}{(R+h)^2} - -(2)$

From equations (1) & (2), $g_h = g \left(\frac{R}{R+h}\right)^2$

If h << R,
$$g_h = g\left(1 - \frac{2h}{R}\right)$$

Hence the acceleration due to gravity decreases with the increase of height.

Depth:

Let M be the mass, R be the radius and be the density of the Earth .

On the surface of the Earth g = $\frac{GM}{R^2}$

But mass of the Earth (M) = $\frac{4}{3}\pi R^3 \rho$

$$\therefore g = \frac{4}{3} \pi RG \rho \dots (1)$$

The gravitational force at a depth 'd' from the surface of the Earth is only due to the inner solid sphere of radius (R-d).

$$\therefore g_d = \left(\frac{4}{3}\pi G\rho\right)(R-d) \dots (2)$$

From equations (1) & (2), $\frac{g_d}{g} = \frac{(R-d)}{R}$

$$\therefore g_d = g \left[1 - \frac{d}{R} \right]$$

Thus the value of 'g' decreases with i.e. increases of depth.

3. State Newton's universal law of gravitation. Explain how the value of the gravitational constant (G) can be determined by Cavendish method?

A. Newton's Universal Law of Gravitation:

Every particle in the universe attracts every other particle with a force, which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. This force acts along the line joining the two particles.

Let two particles of masses m_1 and m_2 are separated by a distance 'r'. From the law of gravitation, the force of attraction F between them is



Where G is called universal gravitational constant.

Cavendish Method:

Two small lead spheres each of mass 'm' are attached at the ends of a bar AB .The bar AB is suspended from a rigid support by a fine quartz fiber in the horizontal plane. Two big lead spheres, each of mass 'M', are brought close to small ones but on opposite sides as shown in figure. The small spheres move towards the big ones due to gravitational force of attraction, $F = \frac{GMm}{d^2}$, where d is the distance from the centre of big to its neighboring small

sphere.



This produces a torque which deflects the rod. If L be the length of the rod, then the deflecting gravitational torque = $F \times L = \frac{GMmL}{d^2}$

Restoring torque of the suspension wire = $\tau \theta$

Where θ is the angle of twist of the suspended wire,

At equilibrium, deflecting gravitational torque = restoring torque

$$\frac{GMmL}{d^2} = \tau \theta \quad Or \quad G = \frac{\tau \theta d^2}{MmL}$$

Thus the value of G can be determined $G = 6.67 \times 10^{-11} Nm^2 kg^{-2}$.

Problems

- 1. Two spherical balls each of mass 1kg are placed 1cm apart. Find the gravitational force of attraction between them?
- A: $m_1 = m_2 = 1kg$; $r = 1cm = 10^{-2}m$

From Newton law of gravitation,

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 1 \times 1}{10^{-4}} = 6.67 \times 10^{-7} N$$

- 2. The mass of a ball is four times the mass of another ball. When these balls are separated by a distance of 10cm, the gravitational force between them is $6.67 \times 10^{-7} N$. Find the masses of the two balls?
- A: $m_1 = m$; $m_2 = 4m$; $r = 10 \ cm = 10^{-1}m$; $F = 6.67 \times 10^{-7} N$

Using Newton law of gravitation, $F = \frac{Gm_1m_2}{r^2}$

$$6.67 \times 10^{-7} = \frac{6.67 \times 10^{-11} \times m \times 4m}{10^{-2}} \Longrightarrow m = 5k_0$$

 $\therefore m_1 = 5 \ kg$, $m_2 = 20 kg$

- 3. Three spherical balls of mass 1kg, 2kg and 3kg are placed at the corners of an equilateral triangle of side 1m. Find the magnitude of gravitational force exerted by the 2kg and 3kg masses on the 1kg mass.
- A. Force between 1kg and 2kg is-



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Similarly force between 1kg and 3kg is-

$$F_2 = \frac{Gm_1m_2}{r^2} = \frac{G \times 1 \times 3}{(1)^2} = 3G$$

In equilateral triangle $\theta = 60^{\circ}$, $F_1 = 2G$ and $F_2 = 3G$

:. Resultant force (F) = $\sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$$=\sqrt{2(G)^2+3(G)^2+2(G)3(G)\cos 60^6}$$

$$=\sqrt{4G^2+9G^2+6G^2}$$

$$\therefore F = G\sqrt{19}$$

4. At a certain height above the Earth's surface, the acceleration due to gravity is 4% of its value at the surface of the Earth. Determine the height?

A:
$$\frac{g^{1}}{g} = \frac{R^{2}}{(R+h)^{2}} \Rightarrow \frac{4}{100} = \frac{R^{2}}{(R+h)^{2}}$$

 $h = 4R = 4 \times 6400 = 25600 \text{km}$

5. A satellite is orbiting the Earth at a height of 1000km. Find its orbital speed.

A: h = 1000km, R = 6400km

Orbital velocity $V_0 = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{6.67 \times 10^{-7} \times 6 \times 10^{24}}{(6400+1000)10^3}} = \sqrt{\frac{40.02 \times 10^{13}}{7400 \times 10^3}}$

$$\therefore V_0 = 0.7354 \times 10^4 m = 7.354 km$$

6. A Satellite orbits the Earth at a distance equal to the radius of the Earth .Find its(i) Orbital Speed and (ii) Period of Revolution.

A.
$$M = 6 \times 10^{24} kg$$
; R= 6400km

(i) Speed of satellite
$$V_o = \sqrt{\frac{GM}{R+h}}$$

$$V_o = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{2 \times 6400 \times 10^3}} = 5.592 \, km \, / \, s$$

(ii)
$$T = \frac{2\pi (R+h)}{V_o}$$
 Or $T = \frac{2\pi \times 2 \times 6400 \times 10^3}{5.592 \times 10^9} = 4 hours$

7. The gravitational force of attraction between two objects decreases by 36% when the distance between them is increased by 4m. Find the original distance between them.

A.
$$\frac{F_1}{F_2} = \left(\frac{r_2}{r_1}\right)^2$$
 Or $\frac{100}{64} = \frac{r+4}{r} \Rightarrow \frac{10}{8} = \frac{r+4}{r}$

 $\therefore r = 16 m$

8. For identical masses of m are kept at the corners of a square of side a. Find the gravitational force exerted on one of the masses by the other masses.

A:
$$m_1 = m_2 = m_3 = m_4 = m_4$$

Force between m_1 and m_2 , $F_{21} = \frac{Gm^2}{a^2}$

Similarly force between m_2 and m_3 , $F_{23} = \frac{Gm^2}{a^2}$

Resultant force between F_{21} and F_{23} is $F_R = \sqrt{F_{21}^2 + F_{23}^2}$



 F_R and F_{24} are parallel forces. So resultant of them is

$$F = F_R + F_{24} = \sqrt{2} \frac{Gm^2}{a^2} + \frac{Gm^2}{2a^2} = \frac{Gm^2}{a^2} \left[\sqrt{2} + \frac{1}{2}\right]$$

9. Two spherical balls of mass 1kg and 4kg are separated by a distance of 12cm. Find the distance from 1kg at which the gravitational force on any mass become zero?

A.
$$m_1 = 1Kg$$
; $m_2 = 4Kg$; d = 12cm

$$\frac{Gmm_1}{x^2} = \frac{Gmm_2}{(d-x)^2} \qquad \text{Or} \quad \frac{1}{x^2} = \frac{4}{(12-x)^2}$$

$$\therefore x = 4cm$$

10. Three uniform spheres each of mass m and radius R are kept in such a way that each touches the other two. Find the magnitude of the gravitational force on any one of the spheres due to the other two?

A:
$$m_1 = m_2 = m_3 = m$$
; r = 2R

Force between 1 and 3 spheres $F_1 = \frac{Gm^2}{(2R)^2} = F(say)$

Force between 1 and 2 spheres $F_2 = \frac{Gm^2}{(2R)^2} = F$

In equilateral triangle $\theta = 60^{\circ}$



Resultant force between F_1 and F_2 is $F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$$=\sqrt{F^2 + F^2 + 2F^2 \cos 60^0} = \sqrt{3}F$$

$$\therefore F_R = \sqrt{3} \frac{Gm^2}{4R^2}$$

11. Two satellites are revolving round the Earth at different heights. The ratio of their orbital speeds is 2:1. If one of them is at height of 100km, what is the height of the other satellite?

A:
$$(V_0)_1: (V_0)_2 = 2:1; h_1 = 100 km; h_2 = ?$$

$$\frac{(V_0)_1}{(V_0)_2} = \sqrt{\frac{R+h_2}{R+h_1}} \qquad \qquad \left[\because V_0 = \sqrt{\frac{Gm}{R+h}}\right]$$

$$\Rightarrow \frac{2}{1} = \sqrt{\frac{R+h_2}{R+100}} \Rightarrow \frac{4}{1} = \frac{R+h_2}{R+100} \Rightarrow h_2 = 3R+400 = 3 \times 6400 + 100$$

:.
$$h_2 = 19600 km$$

12. A satellite is revolving round in a circular orbital with a speed of $8kms^{-1}$ at a height where the value of acceleration due to gravity is $8ms^{-1}$. How high is the satellite from the Earth's surface? (Radius of planet = 600nm)

A:
$$V_0 = 8kms^{-1}$$
; $g = 8ms^{-2} = 8 \times 10^{-3} ms^{-2}$; R = 6000 km

Orbital velocity $V_0 = \sqrt{g(R+h)} \Longrightarrow 8 = \sqrt{8 \times 10^{-3}(R+h)}$

$$\Rightarrow 64 = 8 \times 10^{-3} (R+h) \Rightarrow R+h = 8000$$

- $\Rightarrow h = 8000 R = 8000 6000 = 2000 km$
- 13. a) Calculate the escape velocity of a body from the Earth's surface. B) the Earth were made of wood, its mass would be 10% of its current mass. What would be the escape velocity if the Earth were made of wood?

A:
$$M = 6 \times 10^{24} kg$$
; $g = 9.8 m s^{-2}$; $R = 6400 \text{ km} = 6.4 \times 10^6 m$

a) Escape velocity
$$V_e = \sqrt{2gR} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6} = 11.2 km s^{-1}$$

b)
$$M_{wood} = \frac{10}{100} M_e = 6 \times 10^{23} kg$$

$$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{23}}{6.4 \times 10^6}}$$

$$rac{1}{2}V_e = \sqrt{12.51 \times 10^6} = 3.536 \, km s^{-1}$$