MECHANICAL PROPERTIES OF FLUIDS

Important Points:

1. Buoyancy:

Whenever a body is immersed in a liquid partly (or) wholly, it experiences an upward force called buoyant force (or) buoyancy.

2. A body of volume v density d_1 is fully immersed in a liquid of density d_2 then

Apparent weight = True weight – up thrust

$$= vd_3g - vd_1g$$

$$w' = w\left(1 - \frac{d_l}{d_B}\right) = vd_Bg\left(1 - \frac{d_l}{d_B}\right)$$

3. Stream Line Flow:

a) Every particle of the liquid follows the path of its preceding one.

b) The mass of the liquid entering the tube is equal to that of leaving the tube.

c) No radial flow.

4. Turbulent Flow:

The velocity of the particle crossing any particular point of the liquid is not constant in both magnitude and in direction.

5. Critical Velocity:

The velocity of the liquid below which the flow becomes streamline is called critical velocity.

Critical velocity (V_c) = $\frac{K\eta}{\rho d}$

ρ – Density of the liquid

d – Diameter of the pipe

If K < 1000, the flow is streamline.

If K > 3000 the flow is turbulent

If K is in between 1000 and 3000 the flow is unsteady.

6. Viscosity:

The property of a fluid by virtue of which it opposes the relative motion between its different layers is called viscosity.

7. Co-Efficient of Viscosity (h):

Co-efficient of viscosity is defined as the tangential force between two layers of a liquid of unit surface area and unit velocity gradient.

$$F = -\eta\ A\frac{dv}{dx}$$

Velocity gradient = $\frac{dv}{dx}\text{sec}^{-1}$

η Is called the co-efficient of viscosity

8. Units of η:

CGS: dy sec/cm^2 or poise

MKS: Ns/m^2 or poiseuille

1 poise = 0.1 N.S/m^2

1 deca poise = 10 poise

9. Viscous force is electromagnetic in nature.

10. Poiseuille's equation is given by

$$V = \frac{\pi \Pr^4}{8\eta L}$$

Where

V = Volume of liquid following per sec.

P = Pressure difference between the ends of the capillary tube.

L = Length of the capillary tube

r = Radius of capillary tube.

11. Equation of Continuity:

When a non-viscous liquid flows steadily through a tube of uniform bore the product of area of cross section and the velocity of the flow is same at every cross-section of the tube.

Mass flux = Avd = const.

Volume flux = AV = const.

12. Stoke's Formula:

a) Stoke's formula for viscous force is given by

$$F = 6\pi\eta r V_t$$

Where F = Viscous force, V_t = terminal velocity

b) $V_T = \frac{2}{9}\frac{r^2}{\eta}(\rho - \sigma)g$

Where

ρ = density of solid

σ = density of liquid

13. Bernoulli's Theorem:

When a non-viscous incompressible fluid flows between two points, then the sum of pressure energy, PE and KE per unit volume of the liquid is constant.

$$PV + mgh + \frac{1}{2} mv^2 = \text{constant.}$$

Per unit volume, $P + dhg + \frac{1}{2} dv^2 = \text{const}$

Per unit mass, $\frac{p}{d} + gh + \frac{1}{2}v^2 = \text{constant.}$

Ex: Spinning motion of a cricket ball (Magnus Effect), Aerodynamic lift, Atomizer etc.

14. Torricelli's Theorem:

The speed of a liquid coming out of a hole at a depth 'h' below the surface of a liquid is same as that of a particle fallen through the height 'h' under gravity. This is called Torricelli Theorem.

Velocity of efflux $V = \sqrt{2gh}$

15. **Surface Tension:**

The force acting on the surface of the liquid per unit length on its either side is called surface tension.

$$T = \frac{Force(F)}{length(l)}$$

Unit : N/m

Dimensional formula : $M^1L^0T^{-2}$

16. Surface Energy:

The potential energy per unit area of the surface film is called surface energy

$$W = T(A_2 - A_1)$$

17. Angle of Contact:

The angle between the tangent to the liquid surface at point of contact and the solid surface inside the liquid is known as angle of contact.

18. Wetting agents decrease the angle of contact below 90°.

Ex: detergents.

19. Non-wetting agents (water proofing) increase the angle of contact above 90°.Ex: rain coats.

20. Capillarity:-

The rise (or) fall of liquid in a capillary tube is called Capillarity.

21. Importance in Daily Life:-

a) Ink absorbed by paper

b) Water soaked by plant

c) Oil raised in wick of a lamp

d) Water absorbed by towel

22. Formula to determine surface tension by capillarity method.

$$T = \frac{hrdg}{2\cos\theta}$$

h = capillary height.

r = radius of bore

d = density of liquid

θ = Angle of contact.

23. Excess pressure inside a soap bubble $P = \frac{4T}{R}$

Where 'R' is the radius of the bubble

24. Excess pressure inside a liquid drop $P = \frac{2T}{R}$

Where 'R' is the radius of the drop

Very Short Answer Questions

1. Define average pressure. Mention its units and dimensional formula. Is it a scalar or vector?

A. **Average Pressure**:

The normal force exerted by fluid at rest per unit area of the surface in contact with it, is called average pressure of liquid.

If *F* be the normal force acting on a surface of area *A* in contact with fluid ,then pressure exerted by liquid on this surface is P = F/A

Units: N/m^2 or Pascal (S.I.)

Dimension: $P = [ML^{-1}T^{-2}]$

2. Define co-efficient of viscosity. What are its units and dimensions?

A. Viscosity:

The property of a fluid by virtue of which one layer of fluid opposes the relative motion of the other is called viscosity.

Coefficient of Viscosity:

Definition:

The coefficient of viscosity of a liquid is the tangential viscous force per unit area per unit velocity gradient normal to the direction of flow.

Units: SI : $N.Sm^{-2}\,(or)\,Pa.S$

CGS: Poiseulli

Dimensional formula: $ML^{-1}T^{-1}$

3. What is the principle behind the carburetor of an automobile?

A. The working of carburetor of an automobile is based on Bernoulli's theorem. The carburetor of an automobile has a venture channel (nozzle) through which air flows with a large speed. The pressure is then lowered at the narrow neck and the petrol is sucked up in the chamber to provide the correct mixture of air fuel necessary for combustion.

4. What is Magnus Effect?

A. **Magnus effect:**

When a sphere or cylinder moves in still air while spinning about an axis perpendicular to the direction of its motion, its curved path is more curved than when it is moving without spinning. This is called Magnus effect.

5. Why are drops and bubbles are spherical?

A. For a given volume, sphere has minimum surface area due to surface tension. Hence small rain drops are spherical in nature. Similarly bubbles are also spherical.

6. Give an expression for the excess pressure in a liquid drop?

A. Excess pressure in a liquid drop $P = \frac{2T}{r}$

Where T = Surface tension and r = radius of drop.

7. Give an expression for the excess pressure in an air bubble inside the liquid?

A. Excess pressure in an air bubble inside the liquid $P = \frac{2T}{r}$

Where T = Surface tension and r = radius of air bubble.

8. Give an expression for the excess pressure for the soap bubble?

A. Excess pressure inside a soap bubble in air $P = \frac{4T}{r}$

Where T = Surface tension and r = radius of soap bubble.

9. What are water proofing agents and water wetting agents? What do they do?

A. Water Proofing Agents:

Water proofing agents increase the angle of contact above 90°. It doesn't wet the solid.

Wetting Agents:

Wetting agents decrease the angle of contact below 90°. These are used in detergents to clean the clothes.

10. What is Angle of Contact?

A. **Angle of Contact:**

The angle made by the tangent drawn to the surface of the liquid at the point of contact of the liquid with the solid surface, measured inside the liquid is known as the angle of contact.

11. Mention any two examples that obey Bernoulli's theorem and. justify them.

A. Applications:

1) Dynamic lift on aeroplane wing:

The aeroplane wing (or) airfoil is more curved at the top. Hence velocity is more and pressure is less at the top. At bottom velocity is less. Hence pressure is more at bottom. Due to this pressure difference aeroplane experiences a dynamic lift.

2) Magnus effect.

When a sphere or cylinder moves in still air while spinning about an axis perpendicular to the direction of its motion, its curved path is more curved than when it is moving without spinning. This is called Magnus effect.

12. When water flows through a pipe, which of the layers moves fastest and slowest?

A. The water layer which moves along the axis of the pipe is fastest. The water layer which is in contact with the pipe moves slowest.

13. "Terminal velocity is more if surface area of the body is more" Give reasons in support of your answer?

A. According to stokes's law, the terminal velocity V_t is given by $V_t = \frac{2}{9}\frac{r^2 g(\rho - \rho_o)}{\eta}$

Where r = radius of the spherical body

η = coefficient of viscosity

g = Acceleration due to gravity

ρ = Density of the falling spherical body

ρ_o = Density of the fluid through which the spherical body is travelling

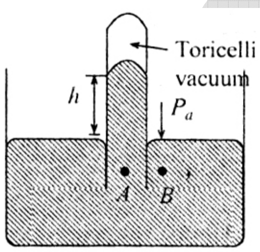
Hence if Terminal velocity is more if surface area of the body is more.

Short Answer Questions

1. What is atmospheric pressure and how is it determined using barometer?

A: The envelope of gases surrounding the earth is called atmosphere. The pressure exerted by the atmosphere on the earth's surface is called atmosphere pressure.

The atmospheric pressure at any point is numerically equal to the weight of a column of air of unit cross sectional area extending from that point to the top of the atmosphere



Barometer is used to measure atmospheric pressure. It consists of a long glass tube filled with mercury and then inverted in thought of mercury to stand vertically as shown in figure. The space above the mercury column in the tube is almost except for negligible mercury vapours. This space is called Torricelli vacuum.

The pressure inside the column at point A must be equal to the pressure at point B which is at the same level. Pressure at B = atmospheric pressure $= P_a$

$P_a = \rho g h =$ pressure at A

Where ρ is the density of mercury and h is the height of the mercury column in the tube.

In the experiment it is found that the mercury column in the barometer has a height of about 76cm at sea level equivalent to one atmosphere.

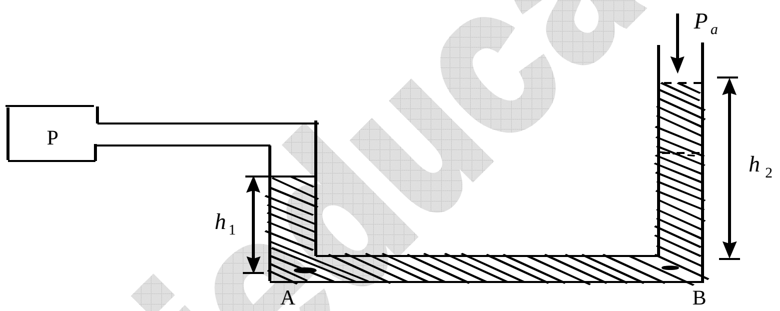
2. What is gauge pressure and how is a manometer used for measuring pressure differences?

A. **Gauge Pressure:**

The pressure difference of between the two points separated by a depth h is known as gauge pressure.

Manometer:

A manometer consists of a U-shaped tube filled with mercury for measuring a high gauge pressure or with light density liquid to measure low gauge pressure. One end of the tube is open while the other end is connected to the system whose pressure is to be measured. The pressure P at A is equal to pressure at point B. Let P be the absolute pressure and P_a be the atmospheric pressure, then $P - P_a$ gives gauge pressure



$$P + h_1\rho g = P_a + h_2\rho g \Rightarrow P - P_a = (h_2 - h_1)\rho g = h\rho g$$

P = absolute pressure, P_a = atmospheric pressure.

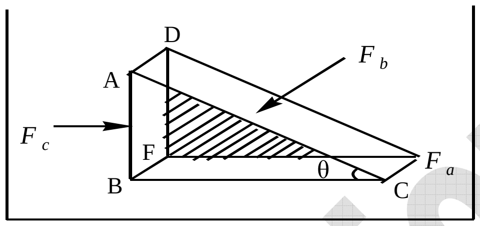
3. State Pascal's law and verify it with the help of an experiment?

A. **Pascal's Law:**

The pressure in a fluid at rest is same at all points if they are at the same height.

Verification:

Consider a very small element in the form of a right angled prism ABCDEF placed inside a fluid at rest. As the element is very small every part of it can be considered at the same depth from the liquid surface. The force exerted by the fluid at rest on the element has to be perpendicular to its surfaces. Thus the fluid exerts pressures P_a, P_b and P_c on this element of area corresponding to the normal forces F_a, F_b and F_c as shown in the figure BEFC, ADFC and ADEB denoted by A_a, A_b and A_c respectively. Then



$F_b \sin\theta = F_c ; F_b \cos\theta = F_a$ (By equilibrium)

$A_b \sin\theta = A_c ; A_b \cos\theta = A_a$ (By geometry)

Hence $\frac{F_a}{A_a} = \frac{F_b}{A_b} = \frac{F_c}{A_c} \Rightarrow P_a = P_b = P_c$

Hence pressure exerted is same in all directions in a fluid at rest.

4. Explain hydraulic lift and hydraulic brakes?

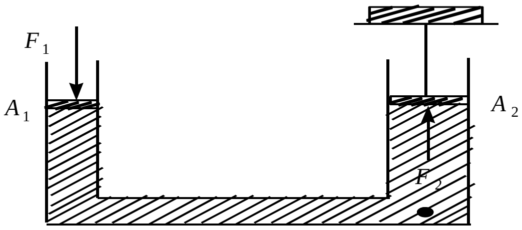
A. **Pascal's Law:**

An external pressure is applied on any part of a fluid contained in a vessel is transmitted undiminished and equally in all directions.

1) Hydraulic Lift:

It consists of two pistons of unequal cross sectional areas separated by the space filled with a liquid. A piston of small cross section A_1 is used to exert a force F_1 directly on the liquid .The pressure developed due to this $P = F_1/A_1$ is transmitted through out the liquid.

The larger piston of area A_2 experiences an upward force of P x A_2. Hence the piston is capable of supporting a large force $F_2 = PA_2 = F_1 A_2/A_1$. By changing the force at A1 the platform can be moved up or down. Thus the applied force has been increased by a factor of A_2/A_1 this factor is called mechanical advantage of the device.

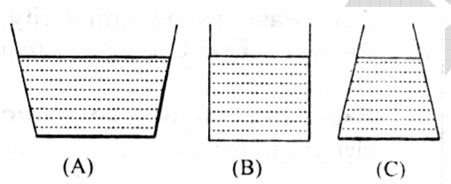


2) Hydraulic Brake:

When a brake pedal is pressed with foot, the lever system works. Piston is pushed into the master cylinder. The pressure developed is transmitted through the brake oil to act on a piston of larger area. A large force acts on the piston and is pushed down expanding the brakes shoes against brake lining. Thus a small force on the pedal produces a large retarding force on the wheel.

5. What is hydrostatic paradox?

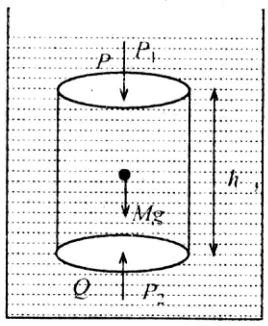
A:



Consider three vessels A, B and C of different shapes as shown in figure. All these vessels have the same area of base and all of them are filled with water to the same depth. The pressure at the base of each vessel is same regard less of the shapes of the vessels pressure directly proportional to depth and by applying Pascal's law it can be seen that pressure is independent of the size and shape of containing vessel. This is quite puzzling and contradictory to common sense because the three vessels are of different shapes and hold different amounts of water this is known as hydrostatics paradox.

6. Explain how pressure varies with depth?

A: Consider a liquid of density ρ contained in a vessel as shown in figure. To calculate the pressure different between two points P and Q separated by a vertical distance ‘h’. Consider an imaginary cylinder of liquid of cross sectional area A, such that point P and Q lie on its upper and lower circular faces respectively.



As the fluid is at rest the resultant horizontal forces should be zero and the resultant vertical forces should balance the weight of the imaginary cylinder. The forces acting in the vertical direction are due to the fluid pressure at the top (P_1A) acting downward, at the bottom (P_2A) acting upward.

$\therefore P_2A - P_1A = mg = (Ah)\rho g$ (Since mass = (vol) density m = $Ah\rho g$)

Where ρ is the density of the fluid $\Rightarrow P_2 = P_2 = \rho gh$

This equation gives the different of pressure between the point P and Q when the effect of gravity is considered

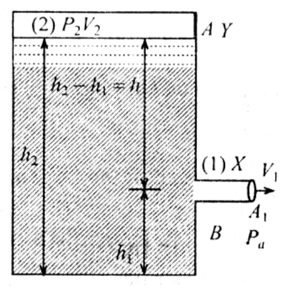
If the point P lies at the surface of the liquid and the point Q is at a depth ‘h’ below it. Then pressure at P is $P_1 = P_a$

Where P_a is atmospheric pressure. Pressure at Q, $P_2 = P_a + h\rho g$

$\therefore$ Pressure exerted by a column of liquid of depth ‘h’ is $P = P_2 - P_1 = h\rho g$

7. What is Torricelli’s law? Explain how the speed of efflux is determined with an experiment?

A: Toricelli’s law (or) Law of efflux: The velocity of efflux of a liquid through an orifice is equal to velocity acquired by the body in falling freely from the free surface of a liquid to the orifice.



Consider a liquid of density ρ in a tank up to point Y and orifice (hole) is located at point X which is a depth h from the free surface of liquid

If A_1, A_2 and V_1, V_2 are the areas of cross section and velocities of liquid at points X and Y then. According to equation of continuity

$A_1V_1 = A_2V_2$, $V_2 = \left(\dfrac{A_1}{A_2}\right)V_1$ since $A_2 >> A_1$ velocity of liquid at the top can be taken as at rest $V_2 = 0$

At point $XP_1 =$ atmospheric pressure P_a; $h_1 = h_1$; $V_1 = V_1$

At point $YR_2 = P$ $h_2 = h_2$; $V_2 = 0$

Using Bernoulli's theorem $P_a + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P + \rho g h_2$ (since $h_2 - h_1 = h$)

$$V_1 = \sqrt{2gh + \frac{2(P - p_a)}{\rho}}$$

If the tank is open to atmosphere then $P = P_a$

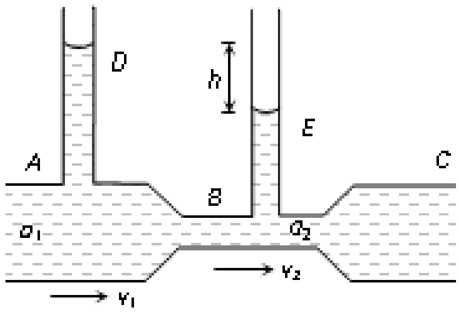
Then velocity of efflux $= V_1 = \sqrt{2gh}$

8. What is Venturimeter? Explain how it is used?

A. Venturimeter:

It is used for measuring the rate of flow of liquid through pipes. It is based on Bernoulli's theorem.

It consists of two identical coaxial tubes *A* and *C* connected by a narrow co-axial tube *B*. Two vertical tubes *D* and *E* are mounted on the tubes *A* and *B* to measure the pressure of the flowing liquid.



When the liquid flows in the tube *ABC*, the velocity of flow in part *B* will be larger than in the tube *A* or *C*. So the pressure in part *B* will be less than that in tube *A* or *C*.

Let a_1 and a_2 are area of cross section of tube A and B respectively.

v_1, v_2 = Velocity of flow of liquid through A and B respectively.

P_1, P_2 = Liquid pressure at A and B respectively.

$\therefore P_1 - P_2 = h\rho g$ ...(i) [ρ = density of flowing liquid]

From Bernoulli's theorem for horizontal flow of liquid.

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

From (i) and (ii) $h\rho g = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2}\rho\left[\frac{V^2}{a_2^2} - \frac{V^2}{a_1^2}\right]$ $[\because V = a_1 v_1 = a_2 v_2]$

$\therefore V^2 = \frac{2a_1^2 a_2^2 hg}{a_1^2 - a_2^2}$ Or $V = a_1 a_2 \sqrt{\frac{2hg}{a_1^2 - a_2^2}}$

9. What is Reynolds number? What is its significance?

A. **Reynolds Number:**

It is defined as the ratio of the inertial force per unit area to the viscous force per unit area for a flowing fluid.

$$N_R = \frac{Inertial\ force\ per\ unit\ area}{Viscous\ force\ per\ unit\ area}$$

If a liquid of density ρ is flowing through a tube of radius r and cross section A then mass of liquid flowing through the tube per second $\frac{dm}{dt}$ = volume flowing per second × density = $Av \times \rho$

$$\text{Inertial force per unit area} = \frac{dp/dt}{A} = \frac{v(dm/dt)}{A} = \frac{vAv\rho}{A} = v^2\rho$$

Viscous force per unit area $F/A = \frac{\eta v}{r}$

So by the definition of Reynolds number

$$N_R = \frac{Inertial\ force\ per\ unit\ area}{Viscous\ force\ per\ unit\ area} = \frac{v^2\rho}{\eta v/r} = \frac{v\rho r}{\eta}$$

Value of Reynold's Number:

(i) 0 to 1000 - Streamline Flow.

(ii) 1000 to 3000 - Flow Is Unstable

(iii) Above 3000 - Turbulent flow.

10. Explain dynamic lift with examples.

A. Dynamic Lift:

The force that acts on a body by virtue of its motion through a fluid is called **Dynamic Lift**.

Examples:

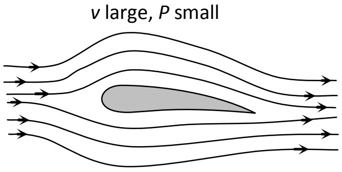
1. Aerodynamic Lift:

Aeroplane wings are so designed that the velocity of air flow above the wing is higher than the velocity of air flow under the wing. According to Bernoulli's

Principle, this difference of air speeds, creates pressure difference, due to which an upward force called "dynamic lift" acts on the plane.

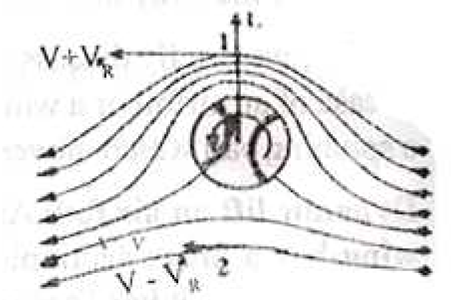
Dynamic lift = pressure difference x area of the wing

$$= (P_2 - P_1) \text{ x A} = \frac{1}{2}\rho\left[V_2^2 - V_1^2\right]xA$$



2. Magnus Effect:

The deviation of spinning cricket ball from its usual path is called Magnus effect.



Dynamic lift on a spinning ball

If the cricket ball is moving from left to right and also spinning about a horizontal axis perpendicular to the direction of motion, then relative to the ball air will be moving from right to left. The resultant velocity of air above the ball will be v + rw while below it

v - rw. Hence according to Bernoulli's principle pressure above the ball will be less than below it.

Due to this pressure difference the ball experiences an upward force called "dynamic lift".

11. Explain Surface Tension and surface energy?

A. **Surface Tension:**

The tangential force acting per unit length on either side of an imaginary line drawn on the liquid surface is called Surface Tension.

$$Surface\,Tension = \frac{Force(F)}{Length(L)}$$

Surface Energy:

The work done to increase the surface area of the liquid film by unit amount is called surface energy. W =T ($A_2 - A_1$) or $T = \frac{W}{A}$ J/m^2

12. Explain how surface tension can be measured experimentally?

A: Surface tension can be measured directly by using an apparatus as shown in figure. A common balance is taken such that one arm of the balance is attached to the a flat vertical glass plate, below which a vessel of some liquid is kept. The plate is balanced by weight on the other side with horizontal edge just over liquid just touches the glass plate and pulls it down a little due to surface tension. Weights are added till the plate just clears the liquid. Suppose the additional weight (W = Mg) is required to pull the plate from the liquid then

$$(2l)S = W = mg \Rightarrow S = \frac{W}{2l} = \frac{Mg}{2l}$$

Long Answer Questions

1. State Bernoulli's principle. From conservation of energy in a fluid flow through a tube, derive Bernoulli's equation. Give an application of derive Bernoulli's theorem?

A. **Statement:**

When a no viscous, irrotational and incompressible fluid flows steadily, the sum of the pressure energy, kinetic energy and potential energy per unit volume remains constant at all points in the path of the flow.

$$P+\rho gh+\frac{1}{2}\rho v^2 = cons\tan t$$

Proof:

Consider a steady, non viscous and incompressible flow of a fluid of density 'ρ 'through a tube as shown. Let v_1, P_1 and A_1 be the velocity, pressure and area of cross section at first end. Let v_2, P_2 and A_2 be the corresponding values at second end

The amount of work done is $W = F.ds = PA(v\ dt)$ $\left[\because P=\frac{F}{A}\ and\ \ v=\frac{ds}{dt}\right]$

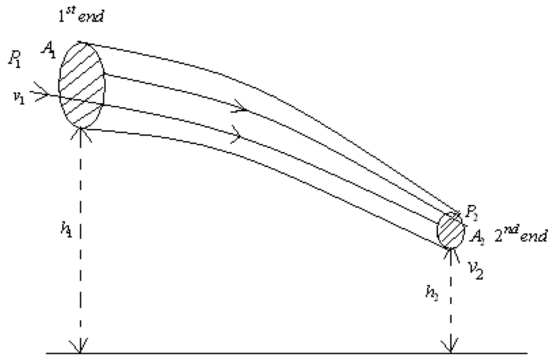
Work done by the fluid on the tube at first end is $W_1 = P_1A_1v_1(dt)$

Work done on the fluid by the tube at second end is $W = F.ds = PA(v\ dt)$

Net work done due to pressure difference (or) pressure energy

$= \quad W_1 - W_2$

$= P_1A_1v_1(dt) - P_2A_2v_2(dt)$



The net potential energy of the system = $mgh_1 - mgh_2$

The total work done due to pressure difference and height difference is

W= Pressure energy + P.E

W = $P_1A_1v_1(dt) - P_1A_2v_2(dt) + mgh_1 - mgh_2 \rightarrow (1)$

According to work- energy theorem, $W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \rightarrow (2)$

From equations (1) & (2)

$$P_1A_1v_1(dt) - P_2A_2v_2(dt) + mgh_1 - mgh_2 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$P_1A_1v_1(dt) + mgh_1 + \frac{1}{2}mv_1^2 = P_2A_2v_2(dt) + mgh_2 + \frac{1}{2}mv_2^2$$

Divide with 'm' on both sides.

$$\frac{P_1A_1v_1(dt)}{m} + gh_1 + \frac{1}{2}v_1^2 = \frac{P_2A_2v_2(dt)}{m} + gh_2 + \frac{1}{2}v_2^2$$

But $m = \rho_1A_1v_1(dt_1) = \rho_2A_2v_2(dt_2)$

$$\frac{P_1}{\rho_1} + gh_1 + \frac{1}{2}v_1^2 = \frac{P_2}{\rho_2} + gh_2 + \frac{1}{2}v_2^2$$

$$\frac{P}{\rho} + gh + \frac{1}{2}v^2 = cons\tan t$$

This equation is known as Bernoulli's equation.

Application:

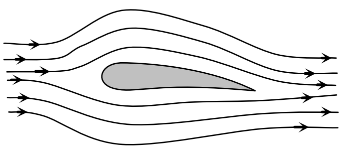
Aerodynamic Lift:

Aeroplane wings are so designed that the velocity of air flow above the wing is higher than the velocity of air flow under the wing. According to Bernoulli's

Principle, this difference of air speeds, creates pressure difference, due to which an upward force called "dynamic lift" acts on the plane.

Dynamic lift = pressure difference x area of the wing

$$= (P_2 - P_1) \text{ x } A = \frac{1}{2}\rho\left[V_2^2 - V_1^2\right] xA$$



2. **Define coefficient of viscosity. Explain Stokes law and explain the conditions under which a rain drop attains terminal velocity. Give the expression for v_t.**

A. **Coefficient of Viscosity (η):**

It is defined as the tangential viscous force acting per unit area per unit velocity gradient.

$$\eta = \frac{F}{A\left(\frac{\Delta v}{\Delta x}\right)} = \frac{Force}{Area \times \text{Velocity gradient}}$$

Stroke's Formula:

The viscous force of high viscous liquids like castor oil, glycerin is given by

$$F_v = 6\pi\eta rv$$

Where r is the radius of the spherical body

η = coefficient of viscosity.

V_T = the velocity of the body.

Terminal Velocity:

The constant velocity acquired by body after falling through a long column of a fluid is called the terminal velocity.

Condition for Terminal Velocity:

Consider a spherical body of radius r and density ρ falling through a fluid of density. The forces acting on the body are

i) Weight W = mg = $\frac{4}{3}\pi r^3 \rho g$ (Downwards)

ii) Viscous force $(F_v) = 6\pi\eta r v$ (Upwards)

iii) Buoyant force $F_B = \frac{4}{3}\pi r^3 \sigma g$ (Upwards)

If 'v_t' is the terminal velocity of the body, then $W = F_v + F_B$

Or $\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + 6\pi\eta r v_t$

Or $\frac{4}{3}\pi r^3 (\rho - \sigma) g = 6\pi\eta r v_t$

$$\therefore v_t = \frac{2}{9}\frac{r^2 g(\rho - \sigma)}{\eta}$$

Problems

1. **Calculate the work done, in blowing a soap bubble of diameter 0.6 cm against the surface tension forces. Surface Tension of soap solution is equal to 2.5 x 10^{-2} N/m?**

A: D = 0.6 cm, r = 0.3 cm =0.3 x 10^{-2} m, T = 2.5 x 10^{-2} N/m

Work Done

= 2 X 2.5 X 10^{-2} X 4× 3.14 X (0.3 X $10^{-2})^2$ = 5.65 X 10^{-6} J

2. **How high does methyl alcohol rise in a glass tube of diameter 0.06cm? (Surface tension of methyl alcohol =** $0.023Nm^{-1}$ **and density** $=0.8gmcm^{-3}$**. Assume that the angle of contact is zero)**

A: Diameter of glass tube = D = 0.06cm = $0.06\times10^{-2}m$

Radius of glass tube $=a=0.03\times10^{-2}m$

Surface tension $=S=0.023Nm^{-1}$

Density $=\rho=0.8gm/cm^3=0.8\times10^3kg/m^3$

Capillary rise $=h=\dfrac{2S}{\rho ga}=\dfrac{2\times0.023}{0.8\times10^3\times9.8\times0.03\times10^{-2}}=0.029m=2.9cm$

3. **What should be the radius of a capillary tube if the water has to rise to a height of 6 cm in it? Surface tension of water is** $7.2\ X\,10^{-2}\ N/m.$ $\left(g=10m/\sec^2, d=10^3kg/m^3\right)$

A: h = 6cm= $6\ X\,10^{-2}\ m$, $T\ =\ 7.2\ X\,10^{-2}\ N/m$,$\theta\ =\ 0^0$, d = 1000 kg/m^3 , r = ?

$$T=\frac{hrdg}{2}\Rightarrow\quad r=\frac{2T}{hdg}=\frac{2\ X\ 7.2\ X\ 10^{-2}}{6\ X\ 10^{-2}\ X\ 1000\ X\ 10}=0.24\text{ mm}$$

4. **Find the depression of the meniscus in the capillary tube when a capillary tube of diameter 0.4 mm is dipped in a beaker containing mercury. Density of mercury is 13.6 X 10^3 kg/m^3. Angle of contact is 130^0 and surface tension of mercury is 0.49 N/m**

(g = 9.8 m/sec^2)

A: D = 0.4 mm = 0.4 X 10^{-3} m, r = 0.2 X 10^{-3} m,

d = 13.6 X 10^3 kg/m^3, ,cos (130) = - 0.6428

T = 0.49 N/m, h = ?

$$T = \frac{hrdg}{2\cos\theta} \Rightarrow h = \frac{2T\cos\theta}{rdg} = \frac{2 \text{ X } 0.49 \ (-0.6428)}{0.2 \text{ X } 10^{-3} \text{ X } 13.6 \text{ X } 10^3 \text{ X } 9.8} \Rightarrow h = -0.024\ m$$

5. **If the diameter of a soap bubbles 10mm and its surface tension is $0.04 Nm^{-1}$, find the excess pressure inside the bubble?**

A: Diameter of soap bubble = $D = 10mm = 10\times10^{-3} m$

Radius of soap bubble $= a = 5\times10^{-3} m$

Surface tension $= S = 0.04 Nm^{-1}$

Excess pressure inside the bubble $= \frac{4S}{a} = \frac{4\times0.04}{5\times10^{-3}} = 0.032 Pa$

6. **If the work done by an agent to form a bubble of radius R is W, then how much energy is required to increase its radius to 2 R?**

A : $R_1 = R$, $R_2 = 2R$

$W_1 = W$, $W_2 = ?$, $W_2 - W_1 = ?$

$W = 4\pi R^2 T(n^{1/3} - 1) \Rightarrow W \propto R^2$

$$\frac{W_1}{W_2} = \left(\frac{R_1}{R_2}\right)^2 \Rightarrow W_2 = \left(\frac{R_2}{R_1}\right)^2 W_1$$

$W_2 = 4W \Rightarrow W_2 - W_1 = 4W - W = 3W$

7. **If two soap bubbles have radii R_1 and R_2 in vacuum. If they coalesce under isothermal conditions, what is the new radius of the bubble. If T is surface tension of the soap solution?**

A: $P_1V_1 + P_2V_2 = PV \Rightarrow \frac{4T}{R_1}\frac{4}{3}\pi R_1^3 + \frac{4T}{R_2}\frac{4}{3}\pi R_2^3 = \frac{4T}{R}\frac{4}{3}\pi R^3 \Rightarrow R^2 = R_1^2 + R_2^2 \Rightarrow R = \sqrt{R_1^2 + R_2^2}$