0166

Total No. of Questions - **30**
Total No. of Printed Pages - **4**

**Regd.
No.**

mnracademy.com

Part - III

MATHEMATICS, Paper - IA

(English Version)

MODEL QUESTION PAPER (FOR IPE 2020-21 ONLY)

Time : 3 Hours   *Max. Marks : 75*

Note: This question paper consists of three section A, B and C.

Section - A

Very short answer type questions.

(i)   **Answer all questions.**

(ii)   **Each question carries 2 marks.**   **10×2=20**

1.   If $A = \left\{ 0, \dfrac{\pi}{6}, \dfrac{\pi}{4}, \dfrac{\pi}{3}, \dfrac{\pi}{2} \right\}$ and $F : A \to B$ is a surjection defined by $f(x) = \cos x$, then find B.

2.   Find the domain of the real valued function $f(x) = \dfrac{1}{\log(2-x)}$.

3.   If $A = \begin{bmatrix} 2 & 3 & -1 \\ 7 & 8 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -4 & -1 \end{bmatrix}$ then find A+B.

4.   If $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$, find A^2.

5.   if $\bar{a} = 2\bar{i} + 5\bar{j} + \bar{k}$ and $\bar{b} = 4\bar{i} + m\bar{j} + n\bar{k}$ are collinear, then find *m* and *n*.

6.   Find the vector equation of the line passing through the point $2\bar{i} + 3\bar{j} + \bar{k}$ and parallel to the vector $4\bar{i} - 2\bar{j} + 3\bar{k}$.

Turn Over

7. If $\bar{a} = \bar{i} + 2\bar{j} - 3\bar{k}$ and $\bar{b} = 3\bar{i} - \bar{j} + 2\bar{k}$ then show that $\bar{a} + \bar{b}$ and $\bar{a} - \bar{b}$ are perpendicular to each other.

8. Prove that $\dfrac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \cot 36^\circ$.

9. Find the period of the function defined by $f(x) = \tan\left(x + 4x + 9x + \ldots\ldots + n^2 x\right)$.

10. If $\sinh x = 3$, then show that $x = \log_e\left(3 + \sqrt{10}\right)$.

Section - B

Short answer type questions. mnracademy.com   **5×4=20**

(i)   **Answer any FIVE questions.**

(ii)   **Each question carries four marks.**

11. If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then show that $(aI + bE)^3 = a^3 I + 3a^2 bE$ where 'I' is unit matrix of order 2.

12. Show that $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ is non-singular and find A^{-1}.

13. Let ABCDEF be regular hexagone with centre O, show that $\overline{AB} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{AF} = 3\overline{AD} = 6\overline{AO}$.

14. Find the equation of the plane passing through the point $\bar{a} = 2\bar{i} + 3\bar{j} - \bar{k}$ and perpendicular to the vector $3\bar{i} - 2\bar{j} - 2\bar{k}$ and the distance of this plane from the origin.

15. If the vectors $\bar{a} = 2\bar{i} - \bar{j} + \bar{k}$, $\bar{b} = \bar{i} + 2\bar{j} - 3\bar{k}$ and $\bar{c} = 3\bar{i} + p\bar{j} + 5\bar{k}$ are coplanar, then find 'P'.

16. If A is not an integral multiple of $\dfrac{\pi}{2}$, then prove that

     (i) $\tan A + \cot A = 2\csc 2A$

     (ii) $\cot A - \tan A = 2\cot 2A$

17. Find the range of $7\cos x - 24\sin x + 5$.

18. Prove that $\dfrac{\cosh x}{1 - \tanh x} + \dfrac{\sinh x}{1 - \coth x} = \sinh x + \coth x$ for $x \neq 0$.

Turn Over

2

19. Prove that $\cot\dfrac{A}{2} + \cot\dfrac{B}{2} + \cot\dfrac{C}{2} = \dfrac{s^2}{\Delta}$.

20. If $\sin\theta = \dfrac{a}{b+c}$ then show that $\cos\theta = \dfrac{2\sqrt{bc}}{b+c}\cos\dfrac{A}{2}$.

Section - C

Long Answer type questions.   **5×7=35**

(i)   **Answer any FIVE questions.**

(ii)   **Each question carries seven marks.**

21. If $f = \{(1,\,2),\,(2,\,-3),\,(3,\,-1)\}$ then find (i) 2f (ii) 2+f (iii) f² (iv) $\sqrt{f}$

22. If $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$, then find $A^3 - 3A^2 - A - 3I$, where I is unit matrix of order 3.

23. Solve the following system of equations by Cramer's rule

    $x + y + z = 1$, $2x + 2y + 3z = 6$, $x + 4y + 9z = 3$.

24. Solve the following system of equations by Matrix Inversion method

    $2x - y + 3z = 9$, $x + y + z = 6$, $x - y + z = 2$.

25. Find the vector equation of the plane passing through points $4\bar{i} - 3\bar{j} - \bar{k}$, $3\bar{i} + 7\bar{j} - 10\bar{k}$ and $2\bar{i} + 5\bar{j} - 7\bar{k}$ and show that the point $\bar{i} + 2\bar{j} - 3\bar{k}$ lies in the plane.

26. If $\bar{a} = 7\bar{i} - 2\bar{j} + 3\bar{k}$, $\bar{b} = 2\bar{i} + 8\bar{k}$ and $\bar{c} = \bar{i} + \bar{j} + \bar{k}$, then compute $\bar{a} \times \bar{b}$, $\bar{a} \times \bar{c}$ and $\bar{a} \times (\bar{b} + \bar{c})$. Verify whether the cross product is distributive over vector addition.

27. If $[\,b\ c\ d\,] + [\,c\ a\ d\,] + [\,a\ b\ d\,] = [\,a\ b\ c\,]$. Then show that the points with position vectors a, b, c and d are coplanar.

28. If A, B, C are angles in a triangle, then prove that

    $\sin A + \sin B - \sin C = 4\sin\dfrac{A}{2}\sin\dfrac{B}{2}\cos\dfrac{C}{2}$.

29. If $\cot\dfrac{A}{2} : \cot\dfrac{B}{2} : \cot\dfrac{C}{2} = 3 : 5 : 7$, then show that a:b:c = 6:5:4.

30. If $a = 13$, $b = 14$, $c = 15$, show that $R = \dfrac{65}{8}$, $r = 4$, $r_1 = \dfrac{21}{2}$, $r_2 = 12$ and $r_3 = 14$.