

Total No. of Questions – 24

Regd.

Total No. of Printed Pages - 4

No.

Part – III

MATHEMATICS, Paper – I(A)
(English Version)

Time : 3 Hours]

[Max. Marks : 75

Note : This question paper consists of **three** Sections – A, B and C.

SECTION – A

10 × 2 = 20

I. Very Short Answer Type questions :

- (i) Answer **all** the questions.
 (ii) Each question carries **two** marks.

1. If $A = \{-2, -1, 0, 1, 2\}$ and $f: A \rightarrow B$ is a surjection defined by $f(x) = x^2 + x + 1$, then find B.

2. If $f(x) = 2x - 1$, $g(x) = \frac{x+1}{2}$ for all $x \in \mathbb{R}$, then find $(g \circ f)(x)$.

3. If $\begin{bmatrix} x-3 & 2y-8 \\ z+2 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -2 & a-4 \end{bmatrix}$, then find the values of x , y , z and a .

4. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ find the rank of this matrix.

5. Let $\vec{a} = 2\vec{i} + 4\vec{j} - 5\vec{k}$, $\vec{b} = \vec{i} + \vec{j} + \vec{k}$ and $\vec{c} = \vec{j} + 2\vec{k}$. Find the unit vector in the opposite direction of $\vec{a} + \vec{b} + \vec{c}$.
6. Find the vector equation of the plane passing through the points $\vec{i} - 2\vec{j} + 5\vec{k}$, $-5\vec{j} - \vec{k}$ and $-3\vec{i} + 5\vec{j}$.
7. If the vectors $\lambda\vec{i} - 3\vec{j} + 5\vec{k}$ and $2\lambda\vec{i} - \lambda\vec{j} - \vec{k}$ are perpendicular to each other, find λ .
8. If $\sin \theta = \frac{4}{5}$ and θ is not in the first quadrant, find the value of $\cos \theta$.
9. If θ is not an integral multiple of $\frac{\pi}{2}$, prove that $\tan \theta + 2\tan 2\theta + 4\tan 4\theta + 8\cot 8\theta = \cot \theta$.
10. Show that $\tan^{-1}\left(\frac{1}{2}\right) = \frac{1}{2} \log_e 3$.

SECTION - B

5 × 4 = 20

II. Short Answer Type questions :

- (i) Answer any **five** questions.
- (ii) Each question carries **four** marks.

11. If $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$, then show that the adjoint of A is $3A'$. Find A^{-1} .

12. Show that the line joining the pair of points $6\vec{a} - 4\vec{b} + 4\vec{c}$, $-4\vec{c}$ and the line joining the pair of points $-\vec{a} - 2\vec{b} - 3\vec{c}$, $\vec{a} + 2\vec{b} - 5\vec{c}$, intersect at the point $-4\vec{c}$, when \vec{a} , \vec{b} , \vec{c} are non-coplanar vectors.

13. If $\vec{a} = 2\vec{i} + \vec{j} - \vec{k}$, $\vec{b} = -\vec{i} + 2\vec{j} - 4\vec{k}$ and $\vec{c} = \vec{i} + \vec{j} + \vec{k}$, then find $(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c})$.
14. Prove that $\left(1 + \cos \frac{\pi}{10}\right) \left(1 + \cos \frac{3\pi}{10}\right) \left(1 + \cos \frac{7\pi}{10}\right) \left(1 + \cos \frac{9\pi}{10}\right) = \frac{1}{16}$.
15. Give $p \neq \pm q$, show that the solutions of $\cos p\theta + \cos q\theta = 0$ form two series each of which is in A.P. Also, find the common difference of each A.P.
16. Prove that $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$
17. In ΔABC if $a = (b + c) \cos \theta$, then prove that $\sin \theta = \frac{2\sqrt{bc}}{b+c} \cos \frac{A}{2}$.

SECTION - C

5 × 7 = 35

III. Long Answer Type questions :

- (i) Answer any **five** questions.
- (ii) Each question carries **seven** marks.

18. Let $f: A \rightarrow B$, $g: B \rightarrow C$ be bijections, then prove that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.
19. Using mathematical induction, prove that for all $n \in \mathbb{N}$,

$$a + ar + ar^2 + \dots \text{ upto } n \text{ terms} = \frac{a(r^n - 1)}{(r - 1)}, r \neq 1.$$

20. Show that
$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ac - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix} = (a^3 + b^3 + c^3 - 3abc)^2.$$

21. $x - y + 3z = 5$, $4x + 2y - z = 0$, $-x + 3y + z = 5$, solve the system of equations by using Cramer's rule.

22. Show that the volume of a tetrahedron with \bar{a} , \bar{b} and \bar{c} as coterminous edges is $\frac{1}{6} |[\bar{a} \ \bar{b} \ \bar{c}]|$.
23. If $A + B + C = 0$, then prove that $\sin 2A + \sin 2B + \sin 2C = -4\sin A \sin B \sin C$.
24. In a ΔABC , if $a = 13$, $b = 14$, $c = 15$, show that $R = \frac{65}{8}$, $r = 4$, $r_1 = \frac{21}{2}$, $r_2 = 12$ and $r_3 = 14$.