PROBABILITY

Key Concepts

1. **Random Experiment :** An experiment which can be repeated any number of times under essentially identical conditions and which is associated with a set of known results, is called a random experiment or trial if the result of any single repetition of the experiment is not certain and is any one of the associated set.

   Ex: Tossing a coin, throwing a die.

2. **Elementary Event:** Outcome of any single repetition of a random experiment is called an **Elementary Event**.

3. Elementary events are said to be equally likely if they have the same chance of happening.

4. A collection of elementary events is said to be an event.

5. Experimental probability of an event E is denoted by P(E) and defined as,

   $$P(E) = \frac{\textit{Number of trials in which the event happened}}{\textit{Total number of trails}}$$

6. Sum of the probabilities of all the elementary events of an experiment is 1.

7. Probability of an event which is impossible to occur is '0'. Such an event is called an impossible event.

8. Probability of an event which is certain to occur is 1. Such an event is called a sure event or certain event.

9. $P(\bar{E}) = 1 - P(E)$

Problems and Solutions

1) Find the probability of getting a head when a coin is tossed once. Also find the probability of getting a tail.

Sol. When we toss a coin, there are two possible outcomes "Getting Head", " Getting Tail.

So total number of outcomes = 2

P (Getting Head) = $\frac{\text{Number of times that head comes}}{\text{Total number of out comes}} = \frac{1}{2}$

Similarly P (Getting Tail) $= \frac{1}{2}$

2) We throw a die once (i) What is the probability of getting a number greater than 4 (ii) What is the probability of getting a number less than or equal to 4?

Sol. In rolling a die,

Sample space, S = $\{1,2,3,4,5,6\}$

Let A denote the event of getting a number greater than 4.

B denote the event of getting a number less than or equal to 4.

Then, A = $\{5,6\}$ *and* $B = \{1,2,3,4\}$

So, (i) P (A) = $\frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

(ii) P (B) = $\frac{n(B)}{n(S)} = \frac{4}{6} = \frac{2}{3}$

3) Once card is drawn from a well – shuffled deck of 52 cards. Calculate the probability that the card will (i) be an ace, (ii) not be an ace.

Sol. There are 4 aces in a deck of 52 cards.

So, n (S) = 52.

Let E denote the event of getting an ace.

So, n (E) =4

P(E) = $\frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

E denote the event of not getting an ace.

P $(\overline{E}) = 1 - P(E) = 1 - \frac{1}{13} = \frac{12}{13}$

4) Sangeeta and Reshma, play a tennis match. It is known that the probability of Sangaeeta winning the match is 0.62. What is the probability of Reshma winning the match?

Sol. Let 'A' denote the event that Sangeeta wins the match. In a match, either Sangeeta wins or Reshma wins. So, $\overline{A}$ denote the event that Reshma wins.

Given that P(A) = 0.62

So, probability of Reshma winning the match = P$(\overline{A}) = 1 - P(A)$ $P(\overline{A}) = 1 - 0.62 = 0.38$

5) If P (E) = 0.05, what is the probability of 'not E'?

Sol . Given P(E) = 0.05

So, probability of 'not E' = P$(\overline{E}) = 1 - P(E)$

= 1-0.05 = 0.95

6) A die is thrown once. Find the probability of getting

i) a prime number, ii) a number lying between 2 and 6;

iii) an odd number.

Sol. When a die is thrown, sample space

S = { 1, 2, 3, 4, 5, 6}

i) Let A denotes the event of getting a prime number. Then A = { 2,3,5}

So, required probability = P(A)

$= \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

ii) Let B denote the event of getting a number between 2 and 6.

Then, B = {3,4,5}

∴ Required probability = P (B) = $\frac{n(B)}{n(S)}$

$$= \frac{3}{6} = \frac{1}{2}$$

iii) Let C denotes the event of getting an odd number. Then C = {1,3,5}

So, required probability, P(C) = $\frac{n(C)}{n(S)}$

$$= \frac{3}{6} = \frac{1}{2}$$

7) A box contains 3 blue, 2 white and 4 red marbles. If a marble is drawn at random from the box, what is the probability that it will be (i) white? (ii) Blue? (iii) Red?

Sol. n (S) = Total number of marbles in the bag = 3+2+4=9

Let A, B and C denote the events of drawing a White, Blue, Red marbles respectively.

Then, n (A) = Number of White marbles =2

N(B) = 3, n (C) = 4

So, i) Probability that the drawn marble is white = P(A) = $\frac{n(A)}{n(S)} = \frac{2}{9}$

ii) Similarly, P(B) = $\frac{n(B)}{n(S)} = \frac{3}{9} = \frac{1}{3}$

iii) P(C) = $\frac{n(C)}{n(S)} = \frac{4}{9}$

8) A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is (i) red? (ii) Not red?

Sol. n (S) = Number of total balls in the bag = 3+5=8

Let R denote the event that the ball drawn is red. Then n(R) = 3

So, i) Probability that the ball drawn is red, P(R) = $\frac{n(R)}{n(S)} = \frac{3}{8}$

ii) Probability that the ball drawn being not red, $P(\overline{R}) = 1 - P(R)$

$$= 1 - \frac{3}{8} = \frac{5}{8}$$

9) A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be (i) red? (ii) White? (ii) Not green?

Sol. n(S) = Number of total marbles in the box = 5+8+4=17.

Let R, W, G denote the events that the taken out marble is red, white and green respectively.

Then n (R) = 5, n(W) = 8, n (G) = 4.

i) Probability that the marble is red,

$$P(R) = \frac{n(R)}{n(S)} = \frac{5}{17}$$

ii) Probability that the marble is white,

$$P(W) = \frac{n(W)}{n(S)} = \frac{8}{17}$$

iii) Probability that the marble is not green,

$$P(\overline{G}) = 1 - P(G) = 1 - \frac{n(G)}{n(S)}$$

$$= 1 - \frac{4}{17} = \frac{13}{17}$$

10) A lot consists of 144 ball pens of which 20 are defective and the others are good. The shopkeeper draws one pen at random and gives it to Sudha. What is the probability that (i) She will buy it? (ii) She will not buy it?

Sol. n(S) = total number of pens in the lot = 144

i) Let 'E' denote the event that she will buy it.

Then n(E) = Number of good pens in the lot = 144 – 20 = 124

So, probability that she will buy it,

$$P(E)=\frac{n(E)}{n(S)}=\frac{124}{144}=\frac{31}{36}$$

ii) Probability that she will not buy it,

$$P(\overline{E})=1-P(E)=1-\frac{31}{36}=\frac{5}{36}$$

11) A box contains 12 balls out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball ? If 6 more black balls are put in the box, the probability of drawing black balls is now double of what it was before. Find x ?

Sol. Before:

Number of balls in the box = 12

Number of black balls in the box = x

Probability that black ball is drawn $=\frac{x}{12}$

After 6 more black balls are put in the box.

Number of balls in the box = 12+6=18

Number of black balls in the box = $x + 6$

Probability that black ball is drawn

$$=\frac{x+6}{18}$$

But given that

$$\frac{x+6}{18}=2\times\frac{x}{12}=\frac{x}{6}$$

$$\Rightarrow x+6=3x\Rightarrow 2x=6\Rightarrow x=3$$

12) Why is 0 ≤ P(E) ≤ 1 for any event E ?

Sol. We know that P(E) = $\frac{n(E)}{n(S)}$ for an event 'E'

Since $E \subseteq S$, n (E) ≤ n (S)

$\Rightarrow \frac{n(E)}{n(S)} \leq 1 \Rightarrow P(E) \leq 1$..................(1)

The least value that n (E) can take is '0"

So, P(E) $\geq 0 \rightarrow (2)$

From (1)&(2), $0 \leq P(E) \leq 1$

13) Find the probability that there are 53 Sundays in a leap year.

Sol. A leap year contains 52 weeks and 2 days.

So, 52 Sundays are guaranteed and for our event one of the two days should be Sunday.

S = {(Sun, Mon), (Mon, Tue), (Tue, Wed), (Wed, Thu), (Thu, Fri), (Fri, Sat), (Sat, Sun)}

Out of all the 7 possible outcomes only 2 outcomes (Sun, Mon) and (Sat, Sun) will favor our event.

So, required probability = $\frac{2}{7}$

14) If a student is selected randomly from a class, find the probability that his birthday is not in February in a leap year?

Sol. P (not in February)

=1 - P (in February)

$= 1 - \frac{Number\ of\ days\ in\ February}{Number\ of\ days\ in\ the\ year}$

$= 1 - \frac{29}{366} = \frac{337}{366}$