SURFACE AREAS AND VOLUMES

Introduction

What had been learnt in previous classes regarding surface areas and volumes of solids like cuboid, cube, right circular cylinder cone and sphere has been reviewed in the previous chapter. In this chapter, we shall discuss problems on conversion of one of these solid in another.

In our day-to-day life we come across various solids which are combinations of two or more such solids. For example, a conical circus tent with cylindrical base is a combinations of a right circular cylinder and a right circular cone also an ice-cream cone is a combination of a cone and a hemi – sphere. We shall discuss problems on finding surface areas and volumes of such solids. We also come across solids which are a part of a cone. For example, a bucket, a glass tumbler, a friction clutch etc. these solids are known as frustums of a cone. In the end of the chapter, we shall discuss problems on surface area and volume of frustum of a cone.

Some Useful Formulae

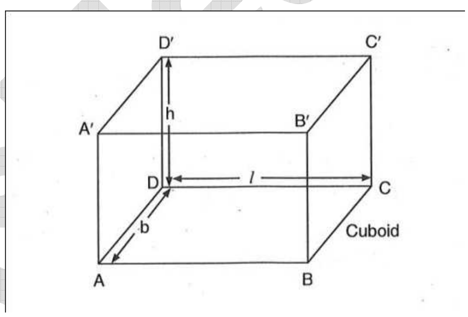
CUBOID: let l,b and h denote respectively the length, breadth and height of a cuboid. Then,

(i) Total surface area of the cuboid=$2(lb+bh+lh)$ square units

(ii) volume of the cuboid=area of the base x Height =Length x Breadth x Height

$= lbh$ cubic units

(iii) Diagonal of the cuboid $=\sqrt{l^2+b^2+h^2}$ units.



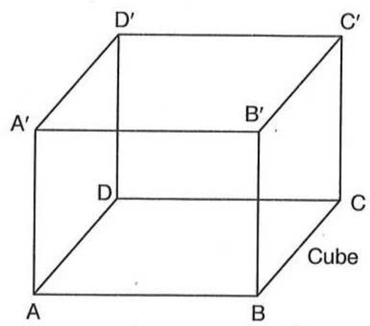
(iv) Area of four walls of a room $= lh+lh+bh+bh=2(l+b)h$ square units.

CUBE: If the length of each edge of a cube is 'a' units, then

(i) Total surface area of the cube = $6a^2$ square units

(ii) Volume of the cube = a^3 cubic units

(iii) Diagonal of the cube = $\sqrt{3}a$ units



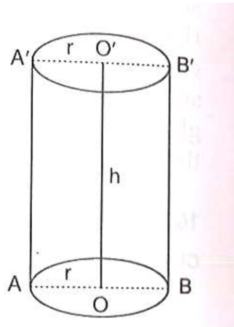
Right Circular Cylinder: For right circular cylinder of base radius r and height (or length) h, we have

(i) Area of each end = Area of base = πr^2

(ii) Curved surface area = $2\pi rh$

$= 2\pi r \times h$

= Perimeter of the base × Height



(iii) Total surface area = Curved surface area + Area of circular ends

$= 2\pi rh + 2\pi r^2$

$= 2\pi r(h + r)$

(iv) Volume $= \pi r^2 h$

= Area of the base × Height

Right Circular Hollow Cylinder: Let R and r be the external and internal radii of a hollow cylinder of height h. Then,

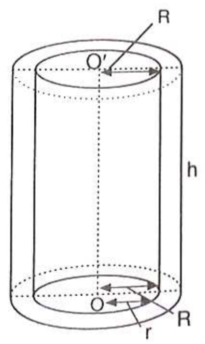
(i) Area of each end = $\pi(R^2 - r^2)$

(ii) Curved surface area of hollow cylinder

= External surface area + Internal surface area

$= 2\pi Rh + 2\pi rh$

$= 2\pi rh(R + r)$



(iii) Total surface area = $2\pi Rh + 2\pi rh + 2(\pi R^2 - \pi r^2)$

$= 2\pi h(R + r) + 2\pi(R + r)(R - r)$

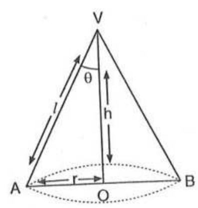
$= 2\pi(R = r)(R + h - r)$

(iv) Volume of material = External volume – Internal volume

$= \pi R^2 h - \pi r^2 h$

$= \pi h(R^2 - r^2)$

Right Circular Cone: For a right circular cone of height h, slant height l and radius of base r, we have



(i) $l^2 = r^2 + h^2$

(ii) Curved surface area $= \pi\, r\, l\ sq.\ units$

(iii) Total surface area = Curved surface area + Area of the base

$$= \pi\, rl + \pi\, r^2$$
$$= \pi\, r\,(l + r)\ sq.\ units$$

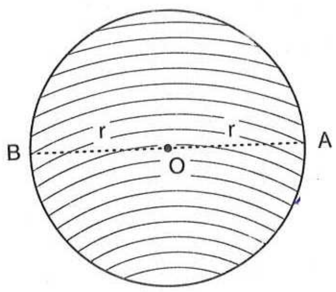
(iv)Volume $= \dfrac{1}{3}\pi\, r^2 h$

$$= \frac{1}{3}(Area\ of\ the\ base) \times Height$$

Sphere: For a sphere of radius r, we have

(i) Surface area = $4\pi\, r^2$

(ii) Volume = $\dfrac{4}{3}\pi\, r^3$



For a hemi – sphere of radius r, we have

(i) Surface area = $2\pi\, r^2$

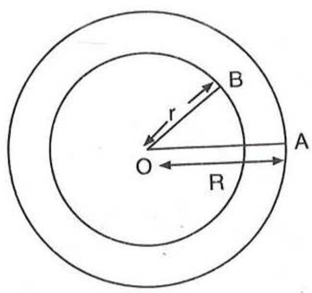
(ii) Total surface area = $2\pi\, r^2 + \pi\, r^2 = 3\pi\, r^2$

(iii) Volume = $\dfrac{2}{3}\pi\, r^3$

Spherical Shell: If R and r are respectively the outer and inner radii of a spherical shell, then

(i) Outer surface area $= \dfrac{4}{3}\pi\, R^2$

(ii) Volume of material = $\dfrac{4}{3}\pi\,(R^3 - r^3)$



Problems

1. Three cubes whose edges measure 3 cm, 4 cm and 5 cm respectively to form a single cube. Find its edge. Also, find the surface area of the new cube.

Solution: Let x cm be the edge of the new cube. Then,

Volume of the new cube = sum of the volumes of three cubes.

$\Rightarrow\ x^3 = 3^3 + 4^3 + 5^3 = 27 + 64 + 125$

$\Rightarrow\ x^3 = 216$

$\Rightarrow\ x^3 = 6^3 \Rightarrow x = 6\ cm$

$\therefore$ Edge of the new cube is 6 cm long.

Surface area of the new cube = $6x^2 = 6\times(6)^2\,cm^2 = 216cm^2$

2.Two cubes each of volume 64 cm³ are joined end to end. Find the surface area and volume of the resulting cuboid.

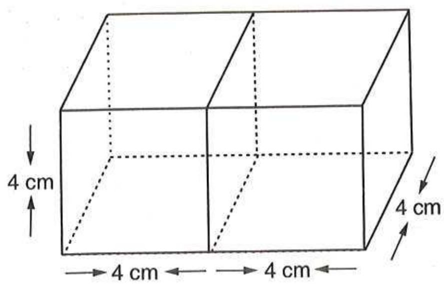
Solution: Let the length of each edge of the cube of volume 64 cm³ be x cm. Then,

Volume = 64 cm³

$\Rightarrow\ x^3 = 64$

$\Rightarrow\ x^3 = 4^3$

$\Rightarrow\ x = 4\ cm$



The dimensions of the cuboid so formed are :

L = Length = (4+4) cm = 8 cm, b = Breadth = 4 cm and, h = Height = 4 cm

$\therefore$ Surface area of the cuboid = $2(lb + bh + lh)$

$$= 2(8\times4 + 4\times4 + 8\times4)cm^2 = 160cm^2$$

Volume of the cuboid = $lbh = 8\times4\times4\ cm^3 = 128\ cm^3$

3. The dimensions of a metallic cuboid are : 100 cm $\times 80\ cm \times 64\ cm$. It is melted and recast into a cube. Find the surface area of the cube.

Solution: Let the length of each edge of the recasted cube be $a\ cm$.

Volume of the metallic cuboid = $100\times 80\times 64\ cm^3 = 512000\ cm^3$

The metallic cuboid is melted and is recasted into a cube.

$\therefore$ Volume of the cube = Volume of the metallic cuboid

$\Rightarrow\ a^3 = 512000$

$\Rightarrow\ a^3 = 8^3\times 10^3 = (8\times 10)^3$

$\Rightarrow\ a = 8\times 10cm = 80\ cm$

$\therefore$ Surface area of the cube = $6a^2\ cm^2 = 6\times(80)^2\ cm^2 = 38400\ cm^2$

4.The radii of the bases of two right circular solid cones of same height are r_1 *and* r_2 respectively. The cones are melted and recast into a solid sphere of radius R. Show that the height of each cone is given by h = $\frac{4R^3}{r_1^2 + r_2^2}$

Solution: Let h be the height of each one. Then,

Sum of the volumes of two cones = Volume of the sphere

$$\Rightarrow \frac{1}{3}\pi r_1^2 h + \frac{1}{3}\pi r_2^2 h = \frac{4}{3}\pi R^3$$

$$\Rightarrow (r_1^2 + r_2^2)h = 4R^3$$

$$\Rightarrow h = \frac{4R^3}{r_1^2 + r_2^2}$$

5. The diameter of a metallic sphere is 6 cm. It is melted and drawn into a wire having diameter of the cross – section as 0.2 cm. Find the length of the wire.

Solution: Diameter of metallic sphere = 6 cm.

$\therefore$ Radius of metallic sphere = 3 cm

Also, we have

Diameter of cross – section of cylindrical wire = 0.2 cm

∴ Radius of cross – section of cylindrical wire = 0.1 cm

Let the length of the wire be h cm. since metallic sphere is converted into a cylindrical shaped wire of length h cm.

∴ Volume of the metal used in wire = Volume of the sphere

$$\Rightarrow \pi \times (0.1)^2 \times h = \frac{4}{3} \times \pi \times 3^3$$

$$\Rightarrow \pi \times \left(\frac{1}{10}\right)^2 \times h = \frac{4}{3} \times \pi \times 27$$

$$\Rightarrow \pi \times \frac{1}{100} \times h = 36\pi$$

$$\Rightarrow h = \frac{36\pi \times 100}{\pi} cm = 3600 cm = 36\ metres$$

6.How many shots each having diameter 3 cm can be made form a cuboidal lead solid of dimensions 9 cm × 11 cm × 12 cm ?

Solution: Volume of the lead in cubical solid = $(9 \times 11 \times 12)\, cm^3 = 1188\, cm^3$

Suppose x shots can be made from the cubical solid. Then

Volume of lead in x spherical shots = Volume of the solid

$$\Rightarrow \left\{\frac{4}{3}\pi \times \left(\frac{3}{2}\right)^3\right\} x = 1188$$

$$\Rightarrow \left(\frac{4}{3} \times \frac{22}{7} \times \frac{27}{8}\right) x = 1188$$

$$\Rightarrow x = \frac{1188 \times 3 \times 7 \times 8}{4 \times 22 \times 27} = 84$$

Hence, 84 shots can be made from the cubical solid.

7.A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Solution: Let the height of the cylinder be h cm. Then,

Volume of the cylinder = Volume of the sphere

$$\Rightarrow \pi \times 6^2 \times h = \frac{4}{3} \times \pi \times (4.2)^3$$
$$\Rightarrow h = \frac{4 \times 4.2 \times 4.2 \times 4.2}{3 \times 6 \times 6}$$
$$\Rightarrow h = 4 \times 0.7 \times 0.7 \times 1.4\ cm$$

8.Metallic spheres of radii 6 cm, 8 cm and 10 cm respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Solution: Let the radius of the resulting sphere be r cm. Then,

Volume of the resulting sphere = Sum of the volumes of three spheres of radii 6 cm, 8cm and 10 cm

$$\Rightarrow \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 6^3 + \frac{4}{3}\pi \times 8^3 + \frac{4}{3}\pi \times 10^3$$
$$\Rightarrow r^3 = 216 + 512 + 1000$$
$$\Rightarrow r^3 = 1728$$
$$\Rightarrow r^3 = 12^3$$
$$\Rightarrow r = 12\ cm.$$

9. How many spherical bullets can be made out of a solid cube of lead whose edge measures 44 cm, each bullet being 4 cm in diameter.

Solution: Let the total number of bullets be x.

Radius of a spherical bullet $= \frac{4}{2} cm = 2\ cm$

Now, Volume of a spherical bullet $= \frac{4}{3}\pi \times (2)^3\ cm^3 = \left(\frac{4}{3} \times \frac{22}{7} \times 8\right) cm^3$

$\therefore$ Volume of x spherical bullets = $\left(\frac{4}{3} \times \frac{22}{7} \times 8 \times x\right) cm^3$

Volume of the solid cube = $(44)^3$ cm^3

Clearly, Volume of x spherical bullets = Volume of cube

$$\Rightarrow \frac{4}{3}\times\frac{22}{7}\times 8\times x = (44)^3$$

$$\Rightarrow \frac{4}{3}\times\frac{22}{7}\times 8\times x = 44\times 44\times 44$$

$$\Rightarrow x = \frac{44\times 44\times 44\times 3\times 7}{4\times 22\times 8} = 2541$$

Hence, total number of spherical bullets = 2541

10.How many spherical lead shots each 4.2 cm in diameter can be obtained from a rectangular solid of lead with dimensions 66 cm, 42 cm, 21 cm. (Use $\pi = 22/7$).

Solution: Let the number of lead shots be x

Volume of lead in the rectangular solid = $(66\times 42\times 21)\ cm^3$

Radius of a lead shot = $\frac{4.2}{2}\ cm = 2.1\ cm$

Volume of a spherical lead shot = $\frac{4}{3}\times\frac{22}{7}\times(2.1)^3\ cm^3$

$\therefore$ Volume of x spherical lead shots = $\left\{\frac{4}{3}\times\frac{22}{7}\times(2.1)^3\times x\right\} cm^3$

$\therefore$ Volume of x spherical lead shots = volume of lead in rectangular solid

$$\therefore \left\{\frac{4}{3}\times\frac{22}{7}\times(2.1)^3\times x\right\} = 66\times 42\times 21$$

$$\Rightarrow x = \frac{66\times 42\times 21\times 3\times 7}{4\times 22\times(2.1)^3} = \frac{66\times 42\times 21\times 21\times 1000}{4\times 22\times 21\times 21\times 21} = 1500$$

Hence, the number of spherical lead shots is 1500.

11.Solid cylinder of brass 8 m high and 4 m diameter is melted and recast into a cone of diameter 3 m. Find the height of the cone.

Solution: We have,

	Cylinder	Cone
Radii	$r_1 = 2m$	$r_2 = 1.5m$
Heights	$h_1 = 8m$	$h_2 = ?$
Volumes	V_1	V_2

Clearly, Volume of the cone = Volume of the cylinder

i.e., $V_1 = V_2$

$$\Rightarrow \quad \frac{1}{3}\pi r_2^2 h_2 = \pi r_1^2 h_1$$

$$\Rightarrow \quad r_2^2 h_2 = 3r_1^2 h_1$$

$$\Rightarrow \quad h_2 = \frac{3r_1^2 h_1}{r_2^2} \Rightarrow h_2 = \frac{3\times 2^2 \times 8}{(1.5)^2} m \Rightarrow h_2 = \frac{96}{2.25} m = 42.66m$$

Hence, the height of the cone is 42.66 m.

12.The barrel of a fountain – pen, cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen will be used up on writing 330 words on an average. How many words would use up a bottle of ink containing one fifth of a litre?

Solution: We have,

Volume of a barrel = $\left(\frac{22}{7}\times 0.25\times 0.25\times 7\right) cm^3 = 1.375\, cm^3$

Volume of ink in the bottle = $\frac{1}{5}\, litre = \frac{1000}{5}\, cm^3 = 200\, cm^3$

$\therefore$ Total number of barrels that can be filled from the given volume of ink = $\frac{200}{1.375}$

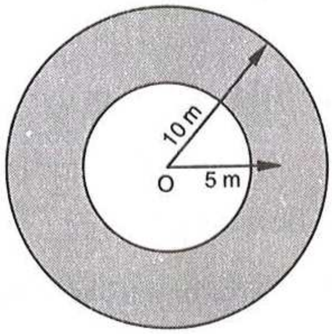
So, required number of words = $\frac{200}{1.375}\times 330 = 48000$

13.A well with 10 m inside diameter is dug 14 m deep. Earth taken out of it is spread all a round to a width of 5 m to form an embankment. Find the height of embankment.

Solution: We have,

Volume of the earth dugout = $(\pi r^2 h)\ m^3$

$\Rightarrow$ Volume of the earth dugout = $\frac{22}{7}\times 5\times 5\times 14\ m^3 = 1100\ m^3$



Area of the embankment (Shaded region) = $\pi(R^2 - r^2) = \pi(10^2 - 5^2)m^2 = \frac{22}{7}\times 75\ m^2$

$\therefore$ Height of the embankment = $\frac{Volume\ of\ the\ earth\ dugout}{Area\ of\ the\ embankment} = \frac{1100}{\frac{22}{7}\times 75} = \frac{7\times 1100}{22\times 75} = 4.66\ m.$

14.A cylindrical pipe has inner diameter of 7 cm and water flows through it at 192.5 litres per minute. Find the rate of flow in kilometers per hour.

Solution: We have,

Volume of water that flows per hour = $(192.50\times 60)\ \ litres$

$= (192.50\times 60\times 1000)\ cm^3$

Inner diameter of the pipe = 7 cm

$\Rightarrow$Inner radius of the pipe $\frac{7}{2}\ cm = 3.5\ cm$

Let h cm be the length of the column of water that flows in one hour.

Clearly, water column forms a cylinder of radius 3.5 cm and length h cm.

$\therefore$ Volume of water that flows in one hour = Volume of the cylinder of radius 3.5 cm and length h cm

$$= \left(\frac{22}{7} \times (3.5)^2 \times h \right) \ cm^3$$

From (i) and (ii), we have

$$\frac{22}{7} \times 3.5 \times 3.5 \times h = 192.50 \times 60 \times 1000$$

$$\Rightarrow \quad h = \frac{192.50 \times 60 \times 1000 \times 7}{22 \times 3.5 \times 3.5} \ cm = 300000 \ cm = 3 \ km$$

Hence, the rate of flow of water is 3 km per hour.

15.The rain water from a roof of 22 m ×20 m drains into a cylindrical vessel having diameter of base 2 m and height 3.5 m. If the vessel is just full, find the rain fall in cm.

Solution: We have,

r = Radius of cylindrical vessel = 1 m, h = Height of cylindrical vessel = 3.5 m

$\therefore$ Volume of cylindrical vessel $= \pi r^2 h = \frac{22}{7} \times 1^2 \times 3.5 \ m^3 = 11 \ m^3$

Let the rain fall be x m. Then,

Volume of the water = Volume of a cuboid of base 22 m × 20 m and height x metres

$$= (22 \times 20 \times x) \ m^3$$

Since the vessel is just full of the water that drains out of the roof into the vessel.

$\therefore$ Volume of the water = Volume of the cylindrical vessel

$\Rightarrow \quad 22 \times 20 \times x = 11$

$\Rightarrow \quad x = \frac{11}{22 \times 20} = \frac{1}{40} m = \frac{100}{40} cm = 2.5 \ cm$

16.Determine the ratio of the volume of a cube to that of a sphere which will exactly fit inside the cube.

Solution: Let the radius of the sphere which fits exactly into a cube be r units. Then, Length of each edge of the cube = $2r$ units

Let V_1 *and* V_2 be the volumes of the cube and sphere respectively. Then,

$$V_1 = (2\,r)^3 \text{ and } V_2 = \frac{4}{3}\pi\, r^3$$

$$\therefore \quad \frac{V_1}{V_2} = \frac{8r^3}{\frac{4}{3}\pi\, r^3} = \frac{6}{\pi} \Rightarrow V_1 : V_2 = 6 : \pi$$

17. If the diameter of cross – section of a wire is decreased by 5% how much percent will the length be increased so that the volume remains the same ?

Solution: Let r be the radius of cross – section of wire and h be its length. Then,

Volume = $\pi\, r^2 h$

5% of diameter of cross – section = $\frac{5}{100} \times 2r = \frac{r}{10}$

$\therefore$ New diameter = $2r - \frac{r}{10} = \frac{19r}{10}$

$\Rightarrow$ New radius = $\frac{19r}{20}$

Let the new length be h_1. Then,

Volume = $\pi \left(\frac{19r}{20}\right)^2 h_1$

From (i) and (ii), we obtain

$$\pi\, r^2 h = \pi \left(\frac{19r}{20}\right)^2 h_1 \Rightarrow h = \frac{361}{400} h_1 \Rightarrow h_1 = \frac{400}{360} h$$

$\therefore$ Increase in length = $h_1 - h = \frac{400h}{361} - h = \frac{39h}{361}$

$\Rightarrow$ Percentage increase in length = $\frac{h_1 - h}{h} \times 100 = \frac{39h}{h} \times 100 = \frac{3900}{360} = 10.8\%$

Hence, the length of the wire increases by 10.8%

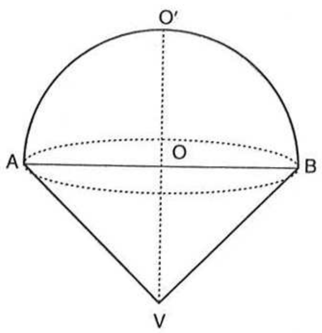
18.A cylindrical container of radius 6 cm and height 15 cm is filled with ice – cream. The whole ice – cream has to be distributed to 10 children in equal cones with hemispherical tops. If the height of the conical portion is four times the radius of its base, find the radius of the ice – cream cone.

Solution: Let the radius of the base of the conical portion be r cm. Then, height of the conical portion = $4r$ cm.

Let V be the volume of cone with hemispherical top. Then,

V = Volume of the cone + Volume of the hemispherical top

$$=\left(\frac{1}{3}\pi r^2 \times 4r+\frac{2}{3}\pi r^3\right) cm^3=\left(\frac{6}{3}\pi r^3\right) cm^3=\left(2\pi r^3\right) cm^3$$



Volume of 10 cones with hemispherical tops = 10 V = $(10\times 2\pi r^3)\ cm^3 = 20\pi r^3\ cm^3$

Volume of the cylindrical container = $(\pi\times 6^2\times 15)\ cm^3 = 540\pi\ cm^3$

Clearly,

Volume of 10 cones with hemispherical tops = Volume of the cylindrical container

$\Rightarrow\ 20\pi r^3 = 540\pi$

$\Rightarrow\ r^3 = 27$

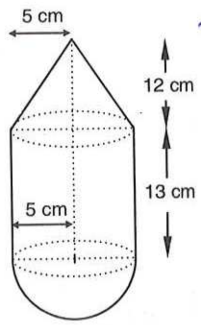
$\Rightarrow\ r = 3\ cm$

Hence, radius of the ice – cream cone is 3 cm.

19.A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The height and radius of the cylindrical part are 13 cm and 5 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Calculate the surface area of the toy if height of the conical part is 12 cm.

Solution: Let r cm be the radius and h cm the height of the cylindrical part. It is given that $r = 5\ cm$ and h= 13 cm. Clearly, radii of the spherical part and base of the conical part are also r cm. Let h_1 cm be the height, $l\ cm$ be the slant height of the conical part. Then,

$$l^2 = r^2 + h_1^{\,2}$$



$$\Rightarrow \quad l = \sqrt{r^2 + h_1^{\,2}}$$

$$\Rightarrow \quad l = \sqrt{5^2 + 12^2} = \sqrt{169} = 13\ cm \qquad [\because h_1 = 12\ cm,\ r = 5cm]$$

Let S be the surface area of the toy. Then,

S = Curved surface area of the cylindrical part

+ Curved surface area of hemispherical part

+ Curved surface area of conical part

$$\Rightarrow \quad S = (2\pi\ rh + 2\ \pi r^2 + \pi\ r\ l)\ cm^2$$

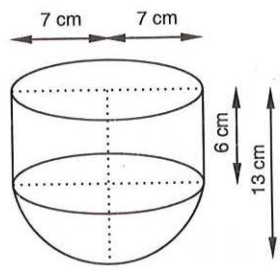
$$\Rightarrow \quad S = \pi\ r\ (2h = 2r + l)\ cm^2$$

$$\Rightarrow \quad S = \left\{\frac{22}{7}\times 5\times(2\times 13 + 2\times 5 + 13)\right\} cm^2 = \left(\frac{22}{7}\times 5\times 49\right) cm^2 = 770\ cm^2$$

20.A vessel is in the form of a hemispherical bowl mounted by a hollow cylinder. The diameter of the sphere is 14 cm and the total height of the vessel is 13 cm. Find its capacity. (Take $\pi = 22/7$).

Solution: Let r be the radius of the hemispherical bowl and h be the height of the cylinder. It is given that r = 7 cm and h = 6 cm. Let V be the total capacity of the bowl. Then,

V = Volume of the cylinder + Volume of the hemisphere



$$\Rightarrow \quad V = \left(\pi \; r^2 h + \frac{2}{3} \pi \, r^3 \right) cm^3$$

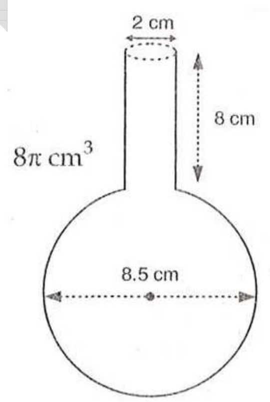
$$\Rightarrow \quad V = \pi \, r^2 \left(h + \frac{2}{3} r \right) cm^3$$

$$\Rightarrow \quad V = \frac{22}{7} \times 7^2 \times \left(6 + \frac{2}{3} \times 7 \right) cm^3$$

$$\Rightarrow \quad V = 22 \times 7 \times \frac{32}{3} cm^3 = \frac{4928}{3} \, cm^3 = 1642.66 \, cm^3$$

21.A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter : the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm³. Check whether she is correct, taking the above as the inside measurements and $\pi = 3.16$

Solution: We have,



h = Length of the cylindrical neck = 8 cm

r = Radius of the cylindrical neck = 1 cm

∴ Volume of the cylindrical neck = $\pi \, r^2 h = \pi \times 1^2 \times 8 \, cm^3 = 8\pi \; cm^3$

Volume of the spherical part = $\frac{4}{3} \pi \times \left(\frac{8.5}{2} \right)^3 \; cm^3$

$$= \frac{4\pi}{3} \times (4.25)^3 \; cm^3$$

∴ Amount of water in the vessel = $\left\{ 8\pi + \frac{4\pi}{3} \times (4.25)^3 \right\} cm^3$

$$= \pi \left\{ 8 + \frac{4}{3} \times (4.25)^3 \right\} cm^3$$

$$= 3.14 \times \left\{ 8 + \frac{4}{3} \times 4.25 \times 4.25 \times 4.25 \right\} cm^3$$

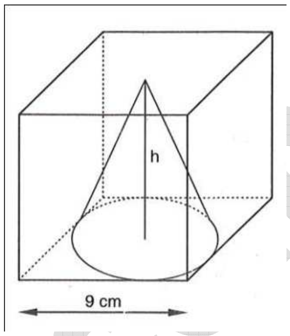
$$= 3.14 \times (8 + 102.354) \; cm^3$$

$$= 346.511 \; cm^3 \cong 346.5 \; cm^3$$

Hence, the volume of found by the child is not correct.

22.Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 9 cm.

Solution: The base of the largest right circular cone will be the circle inscribed in a face of the cube and its height will be equal to an edge of the cube.



$\therefore\ r =$ Radius of the base of the cone $\frac{9}{2} cm$ $\quad [\because edge = 9\ cm]$

and, h = Height of cone = 9 cm

$\therefore$ Volume of the cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \frac{9}{2} \times \frac{9}{2} \times 9\ cm^3 = \frac{2673}{14}\ cm^3 = 190.93\ cm^3$

23.The radii of the circular ends of a frustum of height 6 cm are 14 cm and 6 cm respectively. Find the lateral surface area and total surface area of the frustum.

Solution: We have, = $r_1 = 14 cm, r_2 = 6\ cm\ and\ h = 6 cm.$ Let l be the slant height of the frustum. Then,

$$l = \sqrt{h^2 + (r_1 - r_2)^2} \Rightarrow l = \sqrt{36 + (14-6)^2} = \sqrt{36+64} = 10\ cm$$

Let LSA and TSA respectively be the lateral surface area and total surface area of the frustum. Then,

$$\therefore\ LSA = \pi(r_1 + r_2)l = \frac{22}{7} \times (14+6) \times 10\ cm^2 = \frac{22}{7} \times 200\ cm^2 = 628.57\ cm^2$$

$$TSA = \pi\left\{ r_1^2 + r_2^2 + (r_1 + r_2)\ l \right\} = \frac{22}{7} \times (196 + 36 + 20 \times 10)\ cm^2 = \frac{22}{7} \times 432\ cm^2 = 1357.71\ cm^2$$