# AREAS RELATED TO CIRCLES

## Key Concepts:

### Review of perimeter and area of a circle

1. A circle is the locus of a point which moves in a plane in such a way that its distance from a fixed point always remains same.

2. The fixed point is called the centre and the4 given constant distance is known as the radius of the circle.

3. **Circumference:** The perimeter of a circle is generally known as its circumference.

   We know that circumference of a circle bears a constant ratio with its diameter. This constant ratio is denoted by the Greek letter $\pi$ ( read as 'pi').

   Thus, if C denotes the circumference of a circle of radius $r$. Then,

$$\pi = \frac{Circumference}{Diameter} \Rightarrow \pi = \frac{C}{2\,r} = C = 2\,\pi\,r$$

4. (i) Circumference = $2\pi\, r$

   Also, Circumference = $\pi$ d, where d = $2r$ is the diameter of the circle.

5. Area = $\pi\, r^2$, Also Area = $\pi\left(\frac{d}{2}\right)^2 = \frac{1}{4}\pi\, d^2$, Area of semi – circle = $\frac{1}{2}\pi\, r^2$, Area of a quadrant of a circle = $\frac{1}{4}\pi\, r^2$

6. If two circles touch internally, then the distance between their centres is equal to the difference of their radii.

7. If two circles touch externally, then the distance between their centres is equal to the sum of their radii.

8. Distance moved by a rotating wheel in one revolution is equal to the circumference of the wheel.

9. The number of revolutions completed by a rotating wheel in one minute

$$= \frac{Distance\ moved\ in\ one\ minute}{Circumference}$$

**1. Find the area of a circle whose circumference is 22 cm.**

**Solution:** Let $r$ be the radius of the circle. It is given that the circumference of the circle is 22 cm.

Now, Circumference = 22 cm.

$$\Rightarrow \quad 2\pi r = 22 \Rightarrow 2\times\frac{22}{7}\times r = 22 \Rightarrow r = \frac{7}{2}cm$$

$$\therefore \text{ Area of the circle} = \pi r^2 = \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}cm^2 = 38.5\ cm^2$$

**2. Find the area of a quadrant of a circle whose circumference is 22 cm.**

**Solution:** Let $r$ be the radius of the circle. It is given that the circumference of the circle is 22 cm.

Now, Circumference = 22 cm

$$\Rightarrow \quad 2\pi r = 22 \Rightarrow 2\times\frac{22}{7}\times r = 22 \Rightarrow r = \frac{7}{2}cm$$

$$\therefore \text{ Area of a quadrant } = \frac{1}{4}\pi r^2 = \left\{\frac{1}{4}\times\frac{22}{7}\left(\frac{7}{2}\right)^2\right\}cm^2$$

$$= \left\{\frac{1}{4}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\right\}cm^2 = \frac{77}{8}cm^2 = 9.625\ cm^2$$

3. **If the perimeter of a semi – circular protractor is 108 cm, find the diameter of the protractor (Take** $\pi = 22/7$).

**Solution:** Let the radius of the protractor be $r$ cm. It is given that its perimeter is 108 cm.

Now, Perimeter = 108 cm.

$$\Rightarrow \quad \frac{1}{2}(2\pi r) + 2r = 108 \qquad \left[\because \text{ Perimeter of a semi – circle} = \frac{1}{2}(2\pi r)\right]$$

$$\Rightarrow \quad \pi r + 2r = 108 \Rightarrow \frac{22}{7}\times r + 2r = 108 \Rightarrow 36r = 108\times 7 \Rightarrow r = 3\times 7 = 21$$

$\therefore$ Diameter of the protractor = $2r$ =(2 x 21) cm = 42 cm.

**4. Find the diameter of the circle whose area is equal to the sum of the areas of two circles of diameters 20 cm and 48 cm.**

**Solution:** Let d be the diameter of the circle whose area is equal to the sum of the areas of tow circles of diameters $d_1 = 20cm \ and \ d_2 = 48$ cm. Then,

$$\pi\left(\frac{d}{2}\right)^2 = \pi\left(\frac{20}{2}\right)^2 + \pi\left(\frac{48}{2}\right)^2$$

$$\Rightarrow \frac{d^2}{4} = 10^2 + 24^2$$

$$\Rightarrow \frac{d^2}{4} = 100 + 576 = 676$$

$$\Rightarrow d^2 = 676 \times 4 = 26^2 \times 2^2$$

$$\Rightarrow d = 26 \times 2 = 52 \ cm.$$

**5.A wire is looped in the form of a circle of radius 28 cm. It is re- bent into a square form. Determine the length of the side of the square.**

**Solution:** Let the side of the square be $x$ cm. The wire is in the form of a circle of radius 28 cm.

$\therefore$ Length of the wire = Circumference of the circle

$$= \left\{2 \times \frac{22}{7} \times 28\right\} cm \qquad [\text{ Using } C = 2\pi r]$$

= 176 cm

The wire is re- bent in the form of a square of side $x$ cm. ... (i)

$\therefore$ Perimeter of the square = Length of the wire

$\Rightarrow \quad 4x = 176$ [using (i) ]

$\Rightarrow \quad x = 44cm$

Hence, the length of the side of the square is 44 cm.

**6.A bicycle wheel makes 5000 revolutions in moving 11 km. find the diameter of the wheel.**

**Solution:** Let the radius of the wheel be $r$ cm. We observe that the distance covered by the wheel in one revolution is equal to the circumference of the wheel.

$\therefore$ Distance covered by the wheel in one revolution = $2\pi\ r\ cm$

$\Rightarrow$ Distance covered by the wheel in 5000 revolutions = 5000 x $2\pi\ r\ cm$

$$= 10000\times\frac{22}{7}\times r\ cm$$

$$= \frac{10000\times\frac{22}{7}\times 4}{100}$$

$$= \frac{10000\times\frac{22}{7}\times r}{100\times 1000} km = \frac{11}{35} r\ km$$

It is given that the bicycle wheel covers 11 km distance in 5000 revolutions.

$\therefore \frac{11}{35} r = 11 \Rightarrow r = 35$

$\therefore$ Diameter = $2r\ cm$ = ( 2 x 35)cm = 70 cm.

Hence, the diameter of the wheel is 70 cm.

**7. A wheel has diameter 84 cm. Find how many complete revolutions must it take to cover 792 meters.**

**Solution:** Suppose the wheel makes n complete revolutions in covering 792 meters.

Let $r$ be the radius of the wheel. It is given that the diameter of the wheel is 84 cm.

$\therefore\ 2r = 84 \Rightarrow r = 42cm$

$\therefore$ Circumference of the wheel = $2\pi\ r\ cm = 2\times\frac{22}{7}\times 42\ cm = 264cm = 2.64m$

Distance covered by the wheel in one revolution = 2.64 cm.

Distance covered by wheel in n revolutions = (2.64) n metres

It is given that the wheel covers 792 metres in n revolutions.

$$\therefore \quad (2.64)\ n = 792 \Rightarrow n = \frac{792}{2.64} = 300$$

Hence, the wheel takes 300 revolutions in covering 792 meters.

**8. A car has wheels which are 80 cm in diameter. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?**

**Solution:** Suppose each wheel of the car makes $n$ complete revolutions in 10 minutes. This means that the distance covered by each wheel in $n$ revolutions is same as the distance travelled by the car in 10 minutes.

It is given that :

Speed of the car = 66 km / hr

$\therefore$ Distance travelled by the car in 1 hour = 66 km.

$\Rightarrow$ Distance travelled by the car in 10 min $= \left(\frac{66}{60}\times 10\right) km = 11 km = 11\times 1000 \quad \times 100 cm ...(i)$

It is given that:

Radius of car wheels = 40 cm

$\therefore$ Circumference of the wheels = $2\times\frac{22}{7}\times 40cm$

In a revolution each wheel covers the distance equal to its circumference.

$\therefore$ Distance covered by each wheel in one complete revolution = $2\times\frac{22}{7} = 40m$

$\Rightarrow$ Distance covered by each wheel in n revolutions = $\left(2\times\frac{22}{7}\times 40\times n\right) cm$ ... (i)

But, distance covered by each wheel in completing n revolutions is equal to the distance travelled by the car in 10 minutes.

$$\therefore \quad 2\times\frac{22}{7}\times 40\times n = 11\times 1000\times 100 \Rightarrow n = \frac{11\times 1000\times 100\times 7}{2\times 22\times 40} = 4375$$

Hence, each wheel makes 4375 revolutions in 10 minutes.

**9. Find the number of revolutions made by a circular wheel of area 1.54 m² in rolling a distance of 176 m.**

**Solution:** Let $r$ be the radius of the circular wheel. It is given that its area is 1.54 $m^2$

$$\therefore \quad \pi r^2 = 1.54 \Rightarrow \frac{22}{7} r^2 = 1.54 \Rightarrow r^2 = 7 \times 0.07 = 0.49 \Rightarrow r = 0.7$$

Suppose the wheel makes n revolutions in rolling a distance of 176 m.

$\therefore \quad n \times$ Distance rolled in one revolution = 176

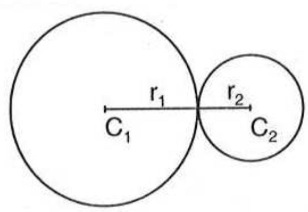
$\Rightarrow \quad n \times 2\pi r = 176$ [$\therefore$ Distance rolled in one revolution = Circumference]

$$\Rightarrow \quad n \times 2 \times \frac{22}{7} \times 0.7 = 176 \Rightarrow n = \frac{176 \times 7}{2 \times 22 \times 0.7} = 40$$

Hence, the circular wheel makes 40 revolutions.

**10. Two circles touch externally. The sum of their areas is 130 $\pi$ sq. cm. And the distance between their centres is 14 cm. Find the radii of the circles.**

**Solution:** If two circles touch externally, then the distance between their centres is equal to the sum of their radii. Let the radii of the two circles be $r_1$ cm and $r_2$ cm respectively. Let $C_1$ and $C_2$ be the centres of the given circles. Then,




$$C_1C_2 = r_1 + r_2$$

$\Rightarrow 14 = r_1 + r_2$ [$\therefore$ $C_1C_2 = 14cm$ (given) ] ...(i)

$\Rightarrow r_1 + r_2 = 14$

It is given that the sum of the areas of two circles is equal to 130 $\pi\ cm^2$.

$\therefore \quad \pi r_1^2 + \pi r_2^2 = 130\pi$

$\Rightarrow r_1^2 + r_2^2 = 130$

Now, $(r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1r_2$

$\Rightarrow 14^2 = 130 + 2r_1r_2$ [Using (i) and (ii)]

$\Rightarrow 196 - 130 = 2r_1r_2$

$\Rightarrow r_1r_2 = 33$ ...(ii)

Now,

$$(r_1 - r_2)^2 = r_1^2 + r_2^2 - 2r_1r_2$$

$\Rightarrow (r_1 - r_2)^2 = 130 - 2 \times 33$ [Using (ii) and (iii)]

$\Rightarrow (r_1 - r_2)^2 = 64$

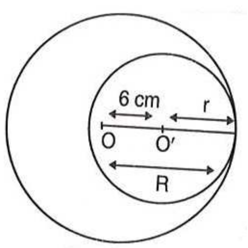
$\Rightarrow r_1 - r_2 = 8$ ...(iv)

Solving (i) and (iv), we get $r_1 = 11\,cm$ and $r_2 = 3cm.$

Hence, the radii of the two circles are 11 cm and 3 cm.

**11. Two circles touch internally. The sum of their areas is 116 $\pi$ cm² and distance between their centres is 6 cm. Find the radii of the circles.**

**Solution:** Let R and $r$ be the radii of the circles having centres at O and O′ respectively. It is given that the sum of the areas is 116 $\pi\ cm^2$ and the distance between the centres is 6 cm.




Now, sum of areas = 116 $\pi\ cm^2$

$\Rightarrow \pi R^2 + \pi r^2 = 116\pi$ ..(i)

$\Rightarrow R^2 + r^2 = 116$

Distance between the centres = 6 cm

$\Rightarrow OO' = 6cm$

$\Rightarrow R - r = 6$ ...(ii)

Now, $(R+r)^2 + (R-r)^2 = 2(R^2 + r^2)$

$\Rightarrow (R+r)^2 + 36 = 2 \times 116$ [Using (i) and (ii) ]

$\Rightarrow (R+r)^2 = (2 \times 116 - 36) = 196$

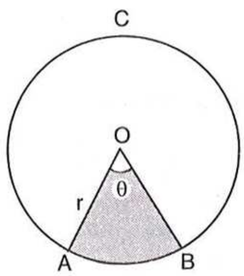
$\Rightarrow R = r = 14$ ....(iii)

Solving (ii) and (iii), we get : R = 10 and $r$ = 4.

Hence, radii of the given circles are 10 cm and 4 cm respectively.

## Sector of A Circle and Its Area

Consider a circle of radius $r$ having its centre at the point O. Let A, B, and C be three points on the circle as shown in. The area enclosed by the circle is divided into two regions, namely, OBA and OBCA. These regions are called sectors of the circle. Each of these two sectors has an arc of the circle as a part of its boundary. The sector OBA has arc AB as a part of its boundary whereas the sector OBCA has arc ACB as a part of its boundary. These sectors are known as minor and major sectors of the circle as defined below.




**Minor Sector:** A sector of a circle is called a minor sector if the minor arc of the circle is a part of its boundary

In sector OAB is the minor sector.

**Major Sector:** A sector of a circle is called a major sector if the major arc of the circle is a part of its boundary.

In sector OACB is the major sector.

Following are some important points to remember:

(i) A minor sector has an angle $\theta$, (say), subtended at the centre of the circle, whereas a major sector has no angle.

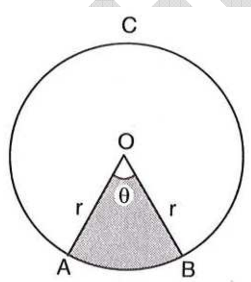
(ii) The sum of the arcs of major and minor sectors of a circle is equal to the circumference of the circle.

(iii) The sum of the areas of major and minor sectors of a circle is equal to the area of the circle.

(iv) The boundary of a sector consists of an arc of the circle and the two radii.

## Area of a Sector

Consider a circle of radius $r$ having its centre at O. Let AOB be a sector of the circle such that $\angle AOB = \theta$. If $\theta < 180^0$, then the arc AB is a minor arc of the circle. Now, if $\theta$ increases the length of the arc AB also increases and if $\theta$ becomes $180^0$ then arc AB becomes the circumference of a semi – circle. Thus, if an arc subtends an angle of $180^0$ at the centre, then its arc length is $\pi r$




$\therefore$ If the arc subtends an angle of $\theta$ at the centre, then its arc length $= \frac{\theta}{180} \times \pi r$

Hence, the arc length $l$ of a sector of angle $\theta$ in a circle of radius $r$ is given by

$$l = \frac{\theta}{180} \times \pi r \qquad \text{...(i)}$$

$$\Rightarrow l = \frac{\theta}{360} \times 2\pi r = \frac{\theta}{360} \times \text{(Circumference of the circle)}$$

As discussed above, if the arc subtends an angle of $180^0$ then the area of the corresponding sector is equal to the area of a semi – circle i.e. $\frac{1}{2}\pi r^2$.

$\therefore$ If the arc subtends an angle $\theta$, then area of the corresponding sector is $\frac{\theta}{180}\times\frac{1}{2}\pi r^2 = \frac{\pi r^2\theta}{360}$

Thus, the area A of a sector of angle $\theta$ in a circle of radius $r$ is given by

$$A = \frac{\theta}{360}\times\pi r^2 = \frac{\theta}{360}\times \text{(Area of the circle)} \qquad \text{...(ii)}$$

Area of a sector

Also $A=\frac{\theta}{360}\times\pi r^2 \Rightarrow A=\frac{1}{2}\left(\frac{\theta}{180}\times\pi r\right)r \Rightarrow A=\frac{1}{2}lr$ [Using (i)]

**12. Find the area of a sector of a circle of radius 28 cm and central angle $45^0$.**

**Solution:** We know that the area A of a sector of a circle of radius $r$ and central angle $\theta$ (in degrees) is given by

$$A=\frac{\theta}{360}\times\pi r^2$$

Here, $r$ = 28 cm and $\theta$ = 45.

$$\therefore \quad A=\frac{45}{360}\times\pi\times(28)^2 = \frac{1}{8}\times\frac{22}{7}\times 28\times 28\, cm^2 = 308\, cm^2$$

**13. Find the difference of the areas of a sector of angle $120^0$ and its corresponding major sector of a circle of radius 21 cm.**

**Solution:** Let $A_1$ *and* $A_2$ be the areas of the given sector and the corresponding major sector respectively. We have, $\theta$ = 120 and $r$ = 21 cm.

$$\therefore A_1 = \frac{\theta}{360}\times\pi r^2 = \frac{120}{360}\times\pi\times(21)^2 = 147\,\pi\, cm^2$$

and, $A_2$ = Area of the circle - $A_1$

$$\Rightarrow \quad A_2 = \{\pi\times(21)^2 - 147\pi\}\, cm^2 = \pi(441-147)\, cm^2 = 294\,\pi\, cm^2$$

Required differences = $A_2 - A_1 = (294\pi - 147\pi)\, cm^2 = 147\pi\, cm^2 = \left(147 \times \frac{22}{7}\right)\, cm^2 = 462\, cm^2$

**14. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115⁰. Find the total area cleaned at each sweep of the blades.**

**Solution:** Clearly, each wiper sweeps a sector of a circle of radius 25 cm and sector angle 115⁰. Therefore, total area A cleaned at each sweep is given by

$$\therefore \quad A = 2 \times \frac{\theta}{360} \times \pi r^2$$

$$\Rightarrow \quad A = 2 \times \frac{115}{360} \times \frac{22}{7} \times 25 \times 25\, cm^2 = 1254.96\, cm^2$$

**15. A pendulum swings through an angle of 30⁰ and describes an arc 8.8 cm in length. Find the length of the pendulum.**

**Solution:** Here, $\theta = 30^0, l = arc = 8.8\, cm$

$$\therefore \quad l = \frac{\theta}{360} \times 2\pi r \Rightarrow 8.8 = \frac{30}{360} \times 2 \times \frac{22}{7} \times r \Rightarrow r = \frac{8.8 \times 6 \times 7}{22} cm = 16.8\, cm$$

**16. Area of a sector of a circle of radius 36 cm is 54 $\pi\, cm^2$. Find the length of the corresponding arc of the sector.**

**Solution:** Let A be the area of the sector of a circle of radius $r = 36\, cm$ and $l$ be the length of the corresponding arc. Then,

$$A = \frac{1}{2} l r$$

$$\Rightarrow \quad 54\pi = \frac{1}{2} \times l \times 36 \qquad [\therefore \ A = 54\pi\, cm^2 \text{ (given) and } r = 36\, cm\ ]$$

$$\Rightarrow \quad l = 3\pi\, cm$$

Alter Let the central angle (in degrees) be $\theta$. It is given that $r = 36\, cm$ and area of the sector is 54 $\pi\, cm^2$

$$\therefore \quad \frac{\theta}{360} \times \pi \times (36)^2 = 54\pi \qquad [\text{Using : Area } = \frac{\theta}{360} \times \pi r^2]$$

$$\Rightarrow \ \theta = \frac{54\pi \times 360}{\pi(36)^2} = 15$$

Let $l$ be the length of the corresponding arc. Then,

$$l = \frac{\theta}{360} \times 2\pi\, r \Rightarrow l = \frac{15}{360} \times 2\pi \times 36\ cm = 3\pi\ cm$$

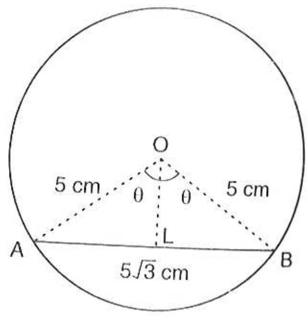
**17. In a circle with centre O and radius 5 cm, AB is a chord of length $5\sqrt{3}$ cm. Find the area of sector AOB.**

**Solution:** It is given that AB = $5\sqrt{3}$ cm.

$$\Rightarrow \ AL = BL = \frac{5\sqrt{3}}{2} cm$$

Let $\angle AOB = 2\theta$. Then, $\angle AOL = \angle BOL = \theta$.

In $\Delta OLA$, we have




$$\sin\theta = \frac{AL}{OA} = \frac{\frac{5\sqrt{3}}{2}}{5} = \frac{\sqrt{3}}{2}$$

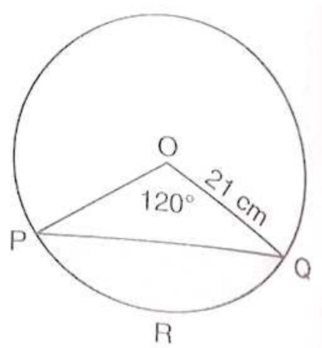
$$\Rightarrow \ \theta = 60^0$$
$$\Rightarrow \ \angle AOB = 120^0$$

$$\therefore \ Area\ of\ sector\ AOB = \frac{120}{360} \times \pi \times 5^2\ cm^2 = \frac{25\pi}{3}\ cm^2$$

**18. Find the area of the segment of a circle, given that the angle of the sector is $120^0$ and the radius of the circle is 21 cm. (Take $\pi = 22/7$)**

**Solution:** The area A of a minor segment of a circle of radius $r$ and the corresponding sector angle $\theta$ (in degrees) is given by

$$A = \left\{\frac{\pi}{360} \times \theta - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2$$




Here, $r = 21\,cm\ and\ \theta = 120^0$.

$$\therefore \quad \text{Area of the segment} = \left\{\frac{\pi}{360} \times \theta - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2$$

$$= \left\{\frac{22}{7} \times \frac{120}{360} - \sin 60^0 \cos 60^0\right\}(21)^2 cm^2$$

$$\Rightarrow \left\{\frac{22}{21} - \frac{1}{2} \times \frac{\sqrt{3}}{2}\right\}(21)^2\ cm^2$$

$$= \left\{\frac{22}{21} \times (21)^2 - (21)^2 \times \frac{\sqrt{3}}{4}\right\} cm^2$$

$$= \left(462 - \frac{441}{4}\sqrt{3}\right) cm^2 = \frac{21}{4}(88 - 21\sqrt{3}) cm^2$$

**19. Find the difference of the areas of two segments of circle formed by a chord of length 5 cm subtending an angle of $90^0$ at the centre.**

**Solution:** Let $r$ be the radius of the circle. Using Pythagoras theorem in $\Delta AOB$, we obtain

$$AB^2 = OA^2 + OB^2$$

$$\Rightarrow\ 5^2 = r^2 + r^2$$

$$\Rightarrow\ 2r^2 = 25 \Rightarrow r^2 = \frac{25}{2} \Rightarrow r = \frac{5}{\sqrt{2}}$$

Let $A_1$ *and* $A_2$ be the areas of minor segment ACB and major segment ADB respectively. Then,

$$A_1 = \left(\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) r^2$$

$$\Rightarrow\ A_1 = \left(\frac{\pi}{360} \times 90 - \sin 45^0 \cos 45^0\right) \times \left(\frac{5}{\sqrt{2}}\right) \qquad \left[\because \theta = 90^0 \text{ and } r = \frac{5}{\sqrt{2}}\right]$$

$$\Rightarrow\ A_1 = \left(\frac{\pi}{4} - \frac{1}{2}\right) \times \frac{25}{2} cm^2 = \left(\frac{25\pi}{8} - \frac{25}{4}\right) cm^2$$

and,

$A_2$ = Area of the circle - $A_1$

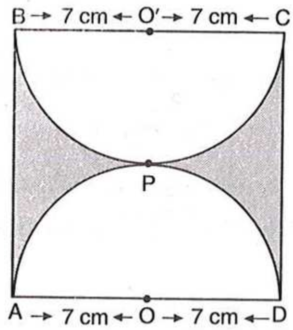
$$\Rightarrow\ A_2 = \left\{\pi \times \left(\frac{5}{\sqrt{2}}\right)^2 - \left(\frac{25\pi}{8} - \frac{25}{4}\right)\right\} cm^2 = \left(\frac{25\pi}{2} - \frac{25\pi}{8} + \frac{25}{4}\right) cm^2 = \left(\frac{75\pi}{8} + \frac{25}{4}\right) cm^2$$

$$\therefore \text{ Required difference} = A_2 - A_1 = \left\{\left(\frac{75\pi}{8} + \frac{25}{4}\right) - \left(\frac{25\pi}{8} - \frac{25}{4}\right)\right\} cm^2$$

$$= \left(\frac{25\pi}{4} + \frac{25}{2}\right)cm^2 = \frac{25}{4}(\pi + 2)cm^2$$

**20. Find the area of the shaded region in if ABCD is a square of side 14 cm and APD and BPC are semi – circles.**

**Solution:** Let A be the area of the shaded region. Then,

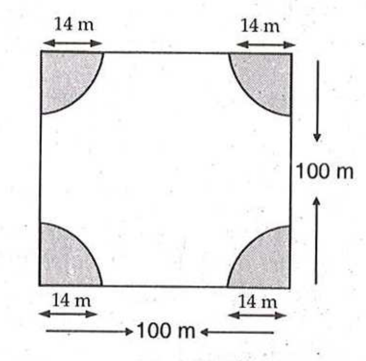



$A =$ Area of square ABCD – Area of two semi – circles

$$\Rightarrow A = 14 \times 14\ cm^2 - 2\left(\frac{1}{2} \times \frac{22}{7} \times 7^2\right)cm^2 = 196\ cm^2 - 154\ cm^2 = 42\ cm^2$$

**21. A square park has each side of 100 m. At each corner of the park, there is a flower bed in the form of a quadrant of radius 14 m as shown in. find the area of the remaining part of the park (Use** $\pi = 22/7$).

**Solution:** Let A be the area of each quadrant of a circle of radius 14 m. Then,




$$A = \frac{1}{4}(\pi r^2) = \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 = 154\ m^2$$

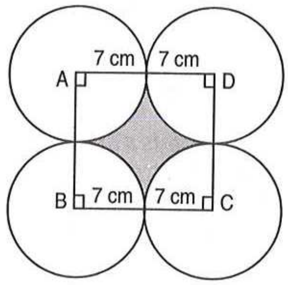
$\therefore$ Area of 4 quadrants = 4A = $(4 \times 154)\ m^2 = 616\ m^2$

Area of square park having side 100 m long = $(100 \times 100)\ m^2 = 10{,}000\ m^2$

Hence,

Area of the remaining part of the park = 10,000 – 616 = 9384 $m^2$

**22. Four equal circles are described about the four corners of a square so that each touches two of the others as shown in. Find the area of the shaded region, each side of the square measuring 14 cm.**




**Solution:** Let ABCD be the given square each side of which is 14 cm long. Clearly, the radius of each circle is 7 cm.

Area of the square f side 14 cm long = $(14\times14)\ cm^2 = 196\ cm^2$

Area of each quadrant of a circle of radius 7 cm = $\frac{1}{4}(\pi r^2)$

$$= \left\{\frac{1}{4}\times\frac{22}{7}\times(7)^2\right\}cm^2 = 38.5\ cm^2$$

$\therefore$ Area of 4 quadrants = $4\times 38.5\ cm^2 = 154\ cm^2$

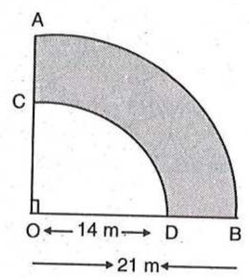
Hence,

Area of the shaded region = Area of the square ABCD – Area of 4 quadrants

$$= (196-154)\ cm^2 = 42\ cm^2$$

**23. ABCD is a flower bed. If OA = 21 m and OC = 14 m, find the area of the bed. (Take** $\pi = 22/7$).

**Solution:** We have, OA = R = 21 m and OC = $r$ = 14 m



$\therefore$ Area of the flower bed = Area of a quadrant of a circle of radius R

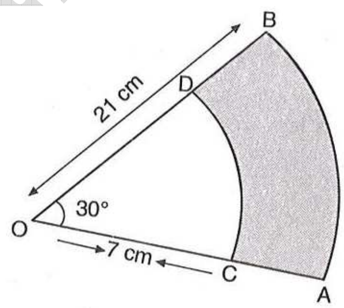
- Area of a quadrant of a circle of radius $r$

$$= \frac{1}{4}\pi R^2 - \frac{1}{4}\pi r^2$$
$$= \frac{\pi}{4}(R^2 - r^2)$$
$$= \frac{1}{4}\times\frac{2}{7}(21^2 - 14^2)\ cm^2 \quad [\because R = 21m \ and \ r = 14m]$$
$$= \left\{\frac{1}{4}\times\frac{22}{7}\times(21+14)(21-14)\right\}m^2 = \left\{\frac{1}{4}\times\frac{22}{7}\times 35\times 7\right\}m^2$$
$$= 192.5\ m^2$$

**24. AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O. If $\angle AOB = 30^0$ find the area of the shaded region.**

**Solution:** Let A be the area oif the shaded region. Then,

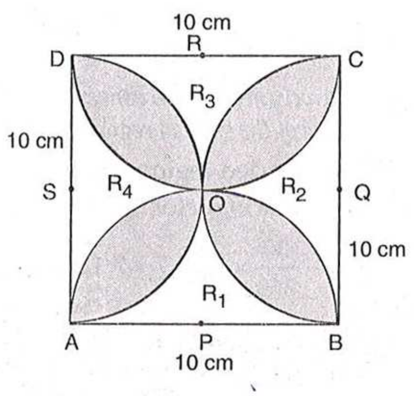
A = Area of sector OAB – Area of sector OCD




$$\Rightarrow A = \left(\frac{30}{360}\times\frac{22}{7}\times 21\times 21 - \frac{30}{360}\times\frac{22}{7}\times 7\times 7\right)cm^2$$
$$\Rightarrow A = \frac{30}{360}\times\frac{22}{7}\times(21\times 21 - 7\times 7)\ cm^2$$
$$\Rightarrow A = \frac{11}{42}\times(21+7)\times(21-7)cm^2 = \frac{11}{42}\times 28\times 14\ cm^2 = 102.67\ cm^2$$

**25. ABCD is a square of side 10 cm. Semi – circles are drawn with each side of square as diameter. Find the area of (i) the un shaded region (ii) the shaded region**

**Solution:** Let us mark the four unshaded regions as $R_1, R_2, R_3$ and $R_4$




Clearly,

Area of $R_1$ + Area of $R_3$

= Area of square ABCD – Area of two semi – circles having centres at Q and S

$$= \left(10\times10 - 2\times\frac{1}{2}\times3.14\times5^2\right)cm^2 \qquad [\because\ Radius = AP = 5cm]$$

$$= (100 - 3.14\times25)\ cm^2 = (100-78.5)cm^2 = 21.5cm^2$$

Similarly, we have

$$\text{Area of } R_2 + Area\ of\ R_4 = 21.5\ cm^2$$

(i) Area of the unshaded region = Area $R_1 + Area\ R_2 + Area\ R_3 + Area\ R_4$

$$= (Area\ R_1 + Area\ R_3) + (Area\ R_2 + Area\ R_4)$$

$$= 2\ (21.5)cm^2 = 43\ cm^2$$

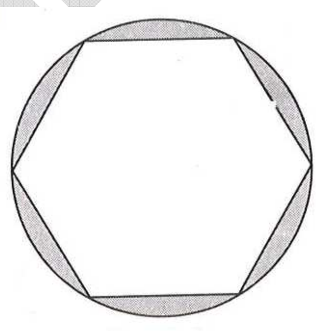
(ii) Area of the shaded region

= Area of square ABCD – (Area of $R_1 + Area\ of\ R_2 + Area\ of\ R_3 + Area\ of\ R_4$)

$$= (100 - 2\times21.5)\ cm^2 = 57\ cm^2$$

**26. A round table cover has six equal designs as shown in. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs. 3.50 per** $cm^2$. $(Use\ \sqrt{3} = 1.7)$

**Solution:** We observe that the designs form six segments of a circle of radius $r$ = 28 cm and each of angle $\theta = 60^0$



Let A be the area of the six designs. Then,

$$A = 6\left\{\frac{\theta}{360}\times\pi\ r^2 - \sin\frac{\theta}{2}\cos\frac{\theta}{2}r^2\right\}cm^2$$

$$\Rightarrow A = 6\left\{\frac{60}{360}\times\frac{22}{7}\times(28)^2 - \sin 30^0 \cos 30^0 \times (28)^2\right\}cm^2 \; [\because r = 28\; cm\; and\; \theta = 60^0)$$

$$\Rightarrow A = 6\left\{\frac{1}{6}\times\frac{22}{7}\times 28\times 28 - \frac{1}{2}\times\frac{\sqrt{3}}{2}\times 28\times 28\right\}cm^2$$

$$\Rightarrow A = (88\times 28 - 6\times\sqrt{3}\times 7\times 28)cm^2 = (2464 - 1999.2)cm^2 = 464.8\; cm^2$$

Hence, Cost of making the designs at the rate of Rs. 3.50 per $cm^2 = Rs.464.8\times 3.50 = Rs.1626.80$