Trigonometric Identities

1. Prove the following trigonometric identities:

(i) $(1-\sin^2\theta)\sec^2\theta = 1$ (ii) $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \tan\theta + \cot\theta$

Solution: (i) We have,

$$LHS = (1-\sin^2\theta)\sec^2\theta$$

$\Rightarrow \cos^2\theta\sec^2\theta$ $[\because 1-\sin^2\theta = \cos^2\theta]$

$\Rightarrow \cos^2\theta\left(\frac{1}{\cos^2\theta}\right) = 1 = RHS$ $\left[\because \sec\theta = \frac{1}{\cos\theta} \therefore \sec^2\theta = \frac{1}{\cos^2\theta}\right]$

(ii) We have,

$$LHS = \sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}$$
$$= \sqrt{(1+\tan^2\theta) + (1+\cot^2\theta)}$$
$$= \sqrt{\tan^2 + \cot^2\theta + 2)} = \sqrt{\tan^2\theta + \cot^2\theta + 2\tan\cot\theta} \quad [\because \tan\theta\cot\theta = 1]$$
$$= \sqrt{(\tan\theta + \cot\theta)^2} = \tan\theta + \cot\theta = RHS$$

2. Prove that following trigonometric identities:

(i) $(1+\tan^2\theta)(1+\sin\theta)(1-\sin\theta) = 1$

(ii) $\tan^2\theta - \frac{1}{\cos^2\theta} = -1$

Solution: (i) We have,

$$LHS = (1+\tan^2\theta)(1+\sin\theta)(1-\sin\theta)$$

$\Rightarrow (1+\tan^2\theta)\{(1+\sin\theta)(1-\sin\theta)\}$

$\Rightarrow (1+\tan^2\theta)(1-\sin^2\theta)$

$\Rightarrow \sec^2\theta\cos^2\theta$ $\left[\because 1+\tan^2\theta = \sec^2\theta \text{ and } 1-\sin^2\theta = \cos^2\theta\right]$

$\Rightarrow \frac{1}{\cos^2\theta}\times\cos^2\theta = 1 = RHS$ $\left[\because \sec\theta = \frac{1}{\cos\theta} \therefore \sec^2\theta = \frac{1}{\cos^2\theta}\right]$

(ii) We have,

$$LHS = \tan^2\theta - \frac{1}{\cos^2\theta}$$

$\Rightarrow \quad \tan^2\theta - \sec^2\theta \qquad \left[\because \frac{1}{\cos\theta} = \sec\theta \therefore \frac{1}{\cos^2\theta} = \sec^2\theta\right]$

$\Rightarrow \quad -(\sec^2\theta - \tan^2\theta) = -1 = RHS$

3. Prove the following trigonometric identities:

(i) $\cot\theta - \tan\theta = \dfrac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$ (ii) $\tan\theta - \cot\theta = \dfrac{2\sin^2\theta - 1}{\sin\theta\cos\theta}$

Solution: (i) We have,

$$LHS = \cot\theta - \tan\theta$$

$\Rightarrow \quad \dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta}$

$\Rightarrow \quad \dfrac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta} = \dfrac{\cos^2\theta - (1-\cos^2\theta)}{\sin\theta\cos\theta} \qquad [\because \sin^2\theta = 1 - \cos^2\theta]$

$\Rightarrow \quad \dfrac{\cos^2\theta - 1 + \cos^2\theta}{\sin\theta\cos\theta} = \dfrac{2\cos^2\theta - 1}{\sin\theta\cos\theta} = RHS$

(ii) We have,

$$LHS = \tan\theta - \cot\theta$$

$\Rightarrow \quad \dfrac{\sin\theta}{\cos\theta} - \dfrac{\cos\theta}{\sin\theta}$

$\Rightarrow \quad \dfrac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta} = \dfrac{\sin^2\theta - (1-\sin^2\theta)}{\sin\theta\cos\theta}$

$\Rightarrow \quad \dfrac{\sin^2\theta - 1 + \sin^2\theta}{\sin\theta\cos\theta} = \dfrac{2\sin^2\theta - 1}{\sin\theta\cos\theta} = RHS$

4. Prove the following trigonometric identities:

(i) $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$ (ii) $\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}} = \operatorname{cosec}\theta + \cot\theta$

Solution: (i) We have,

$$LHS = \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} \Rightarrow \sqrt{\frac{1-\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}} \quad \left[\textit{Multiplying and dividing by } (1-\sin\theta)\right]$$

$$\Rightarrow \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$

$$\Rightarrow \sqrt{\left(\frac{1-\sin\theta}{\cos\theta}\right)^2} = \frac{1-\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = \sec\theta - \tan\theta = RHS$$

(ii) We have

$$LHS = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$$

$$\Rightarrow \sqrt{\frac{1+\cos\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}} \quad \left[\begin{array}{l}\textit{Multiplying and dividing within the}\\ \textit{square root sign by } (1+\cos\theta)\end{array}\right]$$

$$\Rightarrow \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} = \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}}$$

$$\Rightarrow \sqrt{\left(\frac{1+\cos\theta}{\sin\theta}\right)^2} = \frac{1+\cos\theta}{\sin\theta} = \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \operatorname{cosec}\theta + \cot\theta = RHS$$

5. Prove the following identities:

(i) $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

(ii) $(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

(iii) $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$

Solution: (i) We have,

$$LHS = (\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$$

$\Rightarrow \left(\sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\operatorname{cosec}\theta\right) + \left(\cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta\right)$

$\Rightarrow \left(\sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\frac{1}{\sin\theta}\right) + \left(\cos^2\theta + \sec^2\theta + 2\cos\theta\frac{1}{\cos\theta}\right)$

$\Rightarrow (\sin^2\theta + \operatorname{cosec}^2\theta + 2) + (\cos^2\theta + \sec^2\theta + 2)$

$\Rightarrow \sin^2\theta + \cos^2\theta + \operatorname{cosec}^2\theta + \sec^2\theta + 4$

$\Rightarrow 1 + (1+\cot^2\theta) + (1+\tan^2\theta) + 4$ $\left[\because \operatorname{cosec}^2\theta = 1+\cot^2\theta,\ \sec^2\theta = 1+\tan^2\theta\right]$

$\Rightarrow 7 + \tan^2\theta + \cot^2\theta = RHS$

(ii) We have

$LHS = (\operatorname{cosec}\theta - \cot\theta)^2$

$\Rightarrow \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$

$\Rightarrow \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$

$\Rightarrow \frac{(1-\cos\theta)^2}{\sin^2\theta} = \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$ $[\therefore \sin^2\theta = 1-\cos^2\theta]$

$\Rightarrow \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta} = RHS$

(iii) We have,

$LHS = \sec^4\theta - \sec^2\theta$

$\Rightarrow \sec^2\theta(\sec^2\theta - 1)$

$\Rightarrow (1+\tan^2\theta)(1+\tan^2\theta - 1)$ $[\because \sec^2\theta = 1+\tan^2\theta]$

$\Rightarrow (1+\tan^2\theta)\tan^2\theta = \tan^2\theta + \tan^4\theta = RHS$

6. Prove the following identities:

(i) $\frac{\sin\theta}{1-\cos\theta} + \frac{\tan\theta}{1+\cos\theta} = \sec\theta\operatorname{cosec}\theta + \cot\theta$

Solution: (i) We have

$$LHS = \frac{\sin\theta}{1-\cos\theta} + \frac{\tan\theta}{1+\cos\theta}$$

$$\frac{\sin\theta(1+\cos\theta)}{(1+\cos\theta)\,(1-\cos\theta)} + \frac{\tan\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta} + \frac{\tan\theta(1-\cos\theta)}{1-\cos^2\theta}$$

$$= \frac{\sin\theta(1+\cos\theta)}{\sin^2\theta} + \frac{\tan\theta(1-\cos\theta)}{\sin^2\theta} = \frac{\sin\theta(1+\cos\theta)}{\sin^2\theta} + \frac{\sin\theta(1-\cos\theta)}{\cos\theta\sin^2\theta}$$

$$= \frac{1+\cos\theta}{\sin\theta} + \frac{1-\cos\theta}{\cos\theta\sin\theta} = \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} + \frac{1}{\cos\theta\sin\theta} - \frac{\cos\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} + \frac{1}{\cos\theta\sin\theta} - \frac{1}{\sin\theta} = \cot\theta + \sec\theta\,\cos ec\theta = RHS$$

7. Prove the following identities:

(i) $\cos^4 A - \cos^2 A = \sin^4 A - \sin^2 A$

(ii) $\sin^4 A + \cos^4 A = 1 - 2\sin^2 A\cos^2 A$

(iii) $\sin^6 A + \cos^6 A = 1 - 3\sin^2 A\cos^2 A$

Solution: (i) We have,

$$LHS = \cos^4 A - \cos^2 A$$

$\Rightarrow \cos^2 A\,(\cos^2 A - 1)$

$\Rightarrow -\cos^2 A\,(1-\cos^2 A)$

$\Rightarrow -\cos^2 A\sin^2 A = -(1-\sin^2 A)\sin^2 A = -\sin^2 A + \sin^4 A$

$\Rightarrow \sin^4 A - \sin^2 A = RHS$

(ii) We have,

$$LHS = \sin^4 A + \cos^4 A$$

$\Rightarrow (\sin^2 A)^2 + (\cos^2 A)^2 + 2\sin^2 A\cos^2 A - 2\sin^2 A\cos^2 A$

[*Adding and subtracting* $2\sin^2 A\cos^2 A$]

$\Rightarrow (\sin^2 A + \cos^2 A)^2 - 2\sin^2 A\cos^2 A = 1 - 2\sin^2 A\cos^2 A = RHS$

(iii) We have,

$LHS = (\sin^2 A)^3 + (\cos^2 A)^3$

$\Rightarrow \quad (\sin^2 A + \cos^2 A)^3 - 3\sin^2 A\cos^2 A(\sin^2 A + \cos^2 A)$

$[\because a^3 + b^3 = (a+b)^3 - 3ab(a+b)]$

$\Rightarrow \quad 1 - 3\sin^2 A\cos^2 A = RHS$

8. Prove that: $(1-\sin\theta+\cos\theta)^2 = 2(1+\cos\theta)(1-\sin\theta)$

Solution: We know that $(a-b+c)^2 = a^2+b^2+c^2-2ab-2bc+2ac$

$LHS = (1-\sin\theta+\cos\theta)^2$

$\Rightarrow 1+\sin^2\theta+\cos^2\theta-2\sin\theta+2\cos\theta-2\sin\theta\cos\theta$

$\Rightarrow 2-2\sin\theta+2\cos\theta-2\sin\theta\cos\theta$

$\Rightarrow 2(1-\sin\theta)+2cso\theta(1-\sin\theta) = 2(1-\sin\theta)(1+\cos\theta) = RHS$

9.If $\sin\theta+\sin^2\theta=1$, **prove that** $\cos^2\theta+\cos^4\theta=1$

Solution: We have,

$\sin\theta+\sin^2\theta=1 \Rightarrow \sin\theta = 1-\sin^2\theta \Rightarrow \sin\theta = \cos^2\theta$

Now $\cos^2\theta+\cos^4\theta = \cos^2\theta+(\cos^2\theta)^2 = \cos^2+\sin^2\theta = 1$ $\quad [\because \cos^2\theta = \sin\theta]$

10. If $\tan\theta+\sin\theta = m$ *and* $\tan\theta-\sin\theta = n$, *show that* $m^2-n^2 = 4\sqrt{mn}$

Solution: We have, $m = \tan\theta+\sin\theta$ *and*, $n = \tan\theta-\sin\theta$.

$\therefore \quad LHS = m^2-n^2 = (m+n)(m-n)$

$= (\tan\theta+\sin\theta+\tan\theta-\sin\theta)(\tan\theta+\sin\theta-\tan\theta+\sin\theta)$

$= (2\tan\theta)(2\sin\theta) = 4\tan\theta\sin\theta = 4\sqrt{\tan^2\theta\sin^2\theta}$

$= 4\sqrt{\tan^2\theta(1-\cos^2\theta)} = 4\sqrt{(\tan^2\theta-\tan^2\cos^2\theta}$

$= 4\sqrt{\tan^2\theta-\sin^2\theta} = 4\sqrt{(\tan\theta+\sin\theta)(\tan\theta-\sin\theta)} = 4\sqrt{mn} = RHS$

11. **If** $\cos\theta+\sin\theta=\sqrt{2}\cos\theta$, **show that** $\cos\theta-\sin\theta=\sqrt{2}\sin\theta$

Solution: We have,

$$\cos\theta+\sin\theta=\sqrt{2}\cos\theta$$

$$\Rightarrow (\cos\theta+\sin\theta)^2=(\sqrt{2}\cos\theta)^2$$
$$\Rightarrow \cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta=2\cos^2\theta$$
$$\Rightarrow \cos^2\theta-\sin^2\theta=2\sin\theta\cos\theta$$
$$\Rightarrow (\cos\theta+\sin\theta)(\cos\theta-\sin\theta)=2\sin\theta\cos\theta$$
$$\Rightarrow \cos\theta-\sin\theta=\frac{2\sin\theta\cos\theta}{\cos\theta+\sin\theta}$$
$$\Rightarrow \cos\theta-\sin\theta=\frac{2\sin\theta\cos\theta}{\sqrt{2}\cos\theta} \qquad [\because \cos\theta+\sin\theta=\sqrt{2}\cos\theta]$$
$$\Rightarrow \cos\theta-\sin\theta=\sqrt{2}\sin\theta$$

12. **If** $x=a\sin\theta$ *and* $y=b\tan\theta$, **then prove that** $\frac{a^2}{x^2}-\frac{b^2}{y^2}=1$

Solution: We have $x=a\sin\theta$ *and* $y=b\tan\theta$

$$\therefore \quad LHS=\frac{a^2}{x^2}-\frac{b^2}{y^2}$$
$$\Rightarrow \frac{a^2}{a^2\sin^2\theta}-\frac{b^2}{b^2\tan^2\theta} \qquad [\because x=a\sin\theta,\ y=b\tan\theta]$$
$$\Rightarrow \frac{1}{\sin^2\theta}-\frac{1}{\tan^2\theta}$$
$$\Rightarrow \operatorname{cosec}^2\theta-\cot^2\theta=1=RHS \quad [\because 1+\cot^2\theta=\operatorname{cosec}^2\theta \therefore \operatorname{cosec}^2\theta-\cot^2=1]$$

13. **If** $\tan\theta+\cot\theta=2$, *find the value of* $\tan^2\theta+\cot^2\theta$.

Solution: We have

$$\tan\theta+\cot\theta=2$$

$$\Rightarrow (\tan\theta+\cot\theta)^2=4 \qquad [\textit{On squaring both sides}]$$
$$\Rightarrow \tan^2\theta+\cot^2\theta+2\tan\theta\cot\theta=4$$
$$\Rightarrow \tan^2\theta+\cot^2\theta+2=4 \qquad [\because \tan\theta\cot\theta=1]$$
$$\Rightarrow \tan^2\theta+\cot^2\theta=2$$

14. Prove the following identities:

(i) $2(\sin^6\theta+\cos^6\theta)-3(\sin^4\theta+\cos^4\theta)+1=0$

(ii) $\sin^6\theta+\cos^6\theta+3\sin^2\theta\cos^2\theta=1$

Solution: (i) We have

$$LHS=2(\sin^6\theta+\cos^6\theta)-3(\sin^4\theta+\cos^4\theta)+1$$

$$\Rightarrow\ 2\{(\sin^2\theta)^3+(\cos^2\theta)^3\}-3(\sin^4\theta+\cos^4\theta)+1$$

Using $a^3+b^3=(a+b)^3-3ab(a+b)$ *and* $a^2+b^2=(a+b)^2-2ab$, *we obtain*

$$2\{(\sin^2\theta+\cos^2\theta)^3-3\sin^2\theta\cos^2\theta(\sin^2\theta+\cos^2\theta)\}$$
$$-3\{(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta\cos^2\theta+1\}$$
$$\Rightarrow 2(1-3\sin^2\theta\cos^2\theta)-3(1-2\sin^2\theta\cos^2\theta)+1$$
$$\Rightarrow 2-6\sin^2\theta\cos^2\theta-3+6\sin^2\theta\cos^2\theta+1=0=RHS$$

(ii) We have

$$LHS=\sin^6\theta+\cos^6\theta+3\sin^2\theta\cos^2\theta$$

$$\Rightarrow(\sin^2\theta)^3+(\cos^2\theta)^3+3\sin^2\theta\cos^2\theta$$
$$\Rightarrow(\sin^2\theta+\cos^2\theta)^3-3\sin^2\theta\cos^2\theta(\sin^2\theta+\cos^2\theta)+3\sin^2\theta\cos^2\theta$$
$$[\because a^3+b^3=(a+b)^3-3ab(a+b)]$$
$$\Rightarrow 1-3\sin^2\theta\cos^2\theta+3\sin^2\theta\cos^2\theta=1=RHS$$

15. If $\sin\theta+\cos\theta=p$ *and* $\sec\theta+\operatorname{cosec}\theta=q$, *show that* $q\,(p^2-1)=2p$.

Solution: We have, $p=\sin\theta+\cos\theta$ *and* $q=\sec\theta+\operatorname{cosec}\theta$

$$\therefore\quad LHS=q\,(p^2-1)$$
$$=(\sec\theta+\operatorname{cosec}\theta)\,\{(\sin\theta+\cos\theta)^2-1\}$$
$$=\left(\frac{1}{\cos\theta}+\frac{1}{\sin\theta}\right)\{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1\}$$
$$=\left(\frac{\sin\theta+\cos\theta}{\cos\theta\sin\theta}\right)(1+2\sin\theta\cos\theta-1)$$
$$=\left(\frac{\sin\theta+\cos\theta}{\cos\theta\sin\theta}\right)(2\sin\theta\cos\theta)=2(\sin\theta+\cos\theta)=2p=RHS$$

16. **If** $\operatorname{cosec}\theta + \cot\theta = p$, *then prove that* $\cos\theta = \dfrac{p^2-1}{p^2+1}$

Solution: We know that $\operatorname{cosec} - \cot\theta = \dfrac{1}{\operatorname{cosec}\theta + \cot\theta}$

$$\therefore \quad \operatorname{cosec}\theta + \cot\theta = p$$

$$\Rightarrow \quad \operatorname{cosec}\theta - \cot\theta = \frac{1}{p}$$

Adding (i) and (ii), we obtain

$$2\operatorname{cosec}\theta = p + \frac{1}{p} \Rightarrow \operatorname{cosec}\theta = \frac{p^2+1}{2p} \Rightarrow \sin\theta = \frac{2p}{p^2+1}$$

Subtracting (ii) from (i), we obtain

$$2\cot\theta = p - \frac{1}{p} \Rightarrow \cot\theta = \frac{p^2-1}{2p}.$$

Now, $\cos\theta = \cot\theta \times \sin\theta$

$$\Rightarrow \quad \cos\theta = \frac{p^2-1}{2p^2} \times \frac{2p}{p^2+1} = \frac{p^2-1}{p^2+1}.$$

17. **If** $\operatorname{cosec}\theta - \sin\theta = m$ *and* $\sec\theta - \cos\theta = n$, *prove that* $(m^2n)^{2/3} + (mn^2)^{2/3} = 1$

Solution: We have,

$$\operatorname{cosec}\theta - \sin\theta = m \text{ and } \sec\theta - \cos\theta = n$$

$$\Rightarrow \quad \frac{1}{\sin\theta} - \sin\theta = m \text{ and } \frac{1}{\cos\theta} - \cos\theta = n$$

$$\Rightarrow \quad \frac{1-\sin^2\theta}{\sin\theta} = m \text{ and } \frac{1-\cos^2\theta}{\cos\theta} = n$$

$$\Rightarrow \quad \frac{\cos^2\theta}{\sin\theta} = m \text{ and } \frac{\sin^2\theta}{\cos\theta} = n$$

$$\therefore \quad (m^2n)^{2/3} + (mn^2)^{2/3} = \left(\frac{\cos^4\theta}{\sin^2\theta} \times \frac{\sin^2\theta}{\cos\theta}\right)^{2/3} + \left(\frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^4\theta}{\cos^2\theta}\right)^{2/3}$$

$$= (\cos^3\theta)^{2/3} + (\sin^3\theta)^{2/3} = \cos^2\theta + \sin^2\theta = 1$$

Hence, $(m^2n)^{2/3} + (mn^2)^{2/3} = 1$

18. **If** $\sin\theta+\cos\theta=\sqrt{3}$, *then prove that* $\tan\theta+\cot\theta=1.$

Solution We have

$$\sin\theta+\cos\theta=\sqrt{3}$$

$\Rightarrow (\sin\theta+\cos\theta)^2=(\sqrt{3})^2$

$\Rightarrow \sin^2+\cos^2\theta+2\sin\theta\cos\theta=3$

$\Rightarrow 1+2\sin\theta\cos\theta=3$

$\Rightarrow 2\sin\theta\cos t\theta=2$

$\Rightarrow \sin\theta\cos\theta=1$

$\Rightarrow \sin\theta\cos\theta=\sin^2\theta+\cos^2\theta$ $\quad [\because 1=\sin^2\theta+\cos^2\theta]$

$\Rightarrow \dfrac{\sin\theta\cos\theta}{\sin\theta\cos\theta}=\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$

$\Rightarrow 1=\dfrac{\sin^2\theta}{\sin\theta\cos\theta}+\dfrac{\cos^2\theta}{\sin\theta\cos\theta}\Rightarrow 1=\tan\theta+\cot\theta$

19. **If** $\cos\theta=\dfrac{3}{5}$, *find the value of* $\cot\theta+\cos ec\theta.$

Solution: We have, $\cos\theta=\dfrac{3}{5}$

$\therefore \quad \sin\theta=\sqrt{1-\cos^2\theta}\Rightarrow\sin\theta=\sqrt{1-\left(\dfrac{3}{5}\right)^2}=\sqrt{1-\dfrac{9}{25}}=\dfrac{4}{5}$

and, $\cot\theta=\dfrac{\cos\theta}{\sin\theta}\Rightarrow\cot\theta=\dfrac{3/5}{4/5}=\dfrac{3}{4}, \cos ec\theta=\dfrac{1}{\sin\theta}\Rightarrow\cos ec\theta=\dfrac{5}{4}$

$\therefore \quad \cot\theta+\cos ec\theta=\dfrac{3}{4}+\dfrac{5}{4}=\dfrac{8}{4}=2$

20. **If** $\cos ecA=\sqrt{2}$, *find the value of* $\dfrac{2\sin^2 A+3\cot^2 A.}{4\tan^2 A-\cos^2 A}.$

Solution: We have, $\cos ecA=\sqrt{2}$

$\therefore \quad \sin A=\dfrac{1}{\cos ecA}\Rightarrow\sin A=\dfrac{1}{\sqrt{2}}$

$$\cos A=\sqrt{1-\sin^2 A}\Rightarrow\cos A=\sqrt{1-\left(\dfrac{1}{\sqrt{2}}\right)^2}=\dfrac{1}{\sqrt{2}}$$

$$\tan A = \frac{\sin A}{\cos A} \Rightarrow \tan A = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1\ and,\ \cot A = \frac{1}{\tan A} \Rightarrow \cot A = \frac{1}{1} = 1$$

Hence, $$\frac{2\sin^2 A + 3\cot^2 A}{4\tan^2 A - \cos^2 A} = \frac{2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 3(1)^2}{4(1)^2 - \left(\frac{1}{\sqrt{2}}\right)^2} = \frac{2 \times \frac{1}{2} + 3}{4 - \frac{1}{2}} = \frac{1+3}{7/2} = \frac{8}{7}$$

21. **If** $\sin\theta = \frac{a}{\sqrt{a^2+b^2}}, 0 < \theta < 90^0$, *find the values of* $\cos\theta$ *and* $\tan\theta$.

Solution: We have, $\sin\theta = \frac{a}{\sqrt{a^2+b^2}}$

$$\therefore \quad \cos\theta = \sqrt{1-\sin^2\theta} \Rightarrow \cos\theta = \sqrt{1 - \frac{a^2}{a^2+b^2}} = \sqrt{\frac{b^2}{a^2+b^2}} = \frac{b}{\sqrt{a^2+b^2}}$$

and $$\tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \tan\theta = \frac{a/\sqrt{a^2+b^2}}{b/\sqrt{a^2+b^2}} = \frac{a}{b}$$

22. **If** $\sin\theta + \cos\theta = \sqrt{2}\sin(90^0 - \theta)$, determine $\cot\theta$

Solution: We have,

$$\sin\theta + \cos\theta = \sqrt{2}\sin(90^0 - \theta)$$

$$\Rightarrow \quad \sin\theta + \cos\theta = \sqrt{2}\cos\theta$$

$$\Rightarrow \quad \sin\theta = \sqrt{2}\cos\theta - \cos\theta$$

$$\Rightarrow \quad \sin\theta = (\sqrt{2}-1)\cos\theta$$

$$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = \frac{(\sqrt{2}-1)\cos\theta}{\cos\theta} \qquad [\textit{Dividing throughout by } \cos\theta]$$

$$\Rightarrow \quad \tan\theta = (\sqrt{2}-1)$$

$$\Rightarrow \quad \cot\theta = \frac{1}{\sqrt{2}-1} \qquad \left[\because \cot\theta = \frac{1}{\tan\theta}\right]$$

$$\Rightarrow \quad \cot\theta = \frac{(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} = \frac{\sqrt{2}+1}{2-1} = \sqrt{2}+1$$