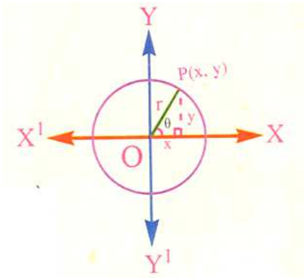
INTRODUCTION TO TRIGONOMETRY

➢ Trigonometric ratios of any angle :

Consider a circle with centre 'O' and radius 'r'. Let θ be any angle in the standard position such that its terminal ray intersects the circle in $P(x, y)$.

$\therefore$ We have $OP = r \text{ and } x^2 + y^2 = r^2$

The six trigonometric ratios are



i) $\sin\theta = \frac{y}{r}$

ii) $\cos\theta = \frac{x}{r}$

iii) $\tan\theta = \frac{y}{x} (x \neq 0)$

iv) $\operatorname{cosec}\theta = \frac{r}{y} (y \neq 0)$

v) $\sec\theta = \frac{r}{x} (x \neq 0)$

vi) $\cot\theta = \frac{x}{y} (y \neq 0)$

➢ Signs of the trigonometric ratios :

Signs of the trigonometric ratios depend on the quadrant in which the terminal side $\overline{OP}$ lies.

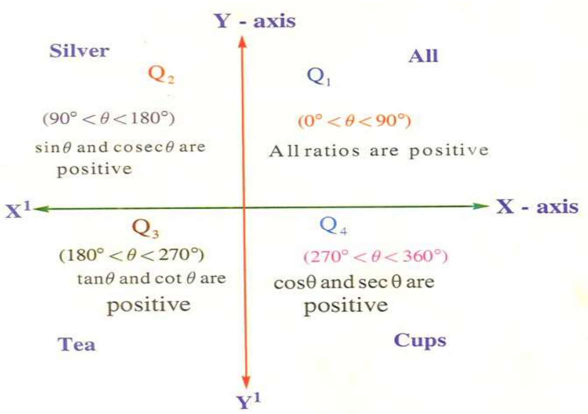
Case i): If $P(x, y)$ lies in Q_1, then $x > 0, y > 0 \text{ and } r > 0$. Hence all the six ratios are positive.

Case ii): If $P(x, y)$ lies in Q_2, then $x < 0, y > 0 \text{ and } r > 0$. $\therefore \sin\theta, \operatorname{cosec}\theta$ are positive and the rest are negative.

Case iii): If $P(x, y)$ lies in Q_3, then $x < 0, y < 0 \text{ and } r > 0$. $\therefore \tan\theta, \cot\theta$ are positive and the rest are negative.

Case iv): If $P(x, y)$ lies in Q_4, then $x > 0, y < 0 \text{ and } r > 0$. $\therefore \cos\theta, \sec\theta$ are positive and the rest are negative.

Note: If θ lies in Q_1, Q_2, Q_3, Q_4 then the signs of trigonometric ratios are as follows.



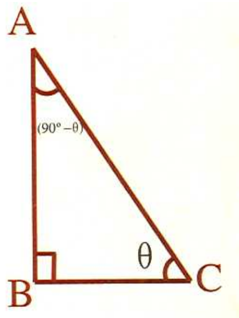
The signs of the trigonometric ratios can be remembered by using the phrase 'All Silver Tea Cups'.

- **Trigonometric ratios of complementary angles :**

Consider a right angled triangle ABC, right angled at B.

Let $\angle C = \theta$ *then* $\angle A = 90^0 - \theta$ where θ is measured in degrees

In ΔABC,



$$\sin(90^0 - \theta) = \frac{BC}{AC} = \cos\theta.$$

$$\cos(90^0 - \theta) = \frac{AB}{AC} = \sin\theta.$$

$$\tan(90^0 - \theta) = \frac{BC}{AB} = \cot\theta.$$

Similarly $\cos ec(90^0 - \theta) = \sec\theta$

$$\sec(90^0 - \theta) = \cos ec\theta$$

$$\cot(90^0 - \theta) = \tan\theta$$

- **The Values of trigonometric functions of certain angles in terms of angle in the first quadrant. :**

$x \rightarrow$	$90^0 - \theta$ Q_1	$90^0 - \theta$ Q_2	$180^0 - \theta$ Q_2	$180^0 - \theta$ Q_3	$270^0 - \theta$ Q_3	$270^0 - \theta$ Q_4	$360^0 - \theta$ Q_4	$360^0 - \theta$ Q_1
$\sin x$	$+\cos\theta$	$+\cos\theta$	$+\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$+\sin\theta$
$\cos x$	$+\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$+\sin\theta$	$+\cos\theta$	$+\cos\theta$
$\tan x$	$+\cot\theta$	$-\cot\theta$	$-\tan\theta$	$+\tan\theta$	$+\cot\theta$	$-\cot\theta$	$-\tan\theta$	$+\tan\theta$
$\csc x$	$+\sec\theta$	$+\sec\theta$	$+\csc\theta$	$-\csc\theta$	$-\sec\theta$	$-\sec\theta$	$-\csc\theta$	$+\csc\theta$
$\sec x$	$+\csc\theta$	$-\csc\theta$	$-\sec\theta$	$-\sec\theta$	$-\csc\theta$	$+\csc\theta$	$+\sec\theta$	$+\sec\theta$
$\cot x$	$+\tan\theta$	$-\tan\theta$	$-\cot\theta$	$+\cot\theta$	$+\tan\theta$	$-\tan\theta$	$-\cot\theta$	$+\cot\theta$

1. In a ΔABC, right angled at A, if AB =12, AC = 5 and BC =13, find all the six trigonometric ratios of angle B.

Solution:

Adjacent side = Base = AB = 12, opposite side = AC = 5 and, Hypotenuse = BC =13 Using the definitions of trigonometric ratios, we have

$$\sin B = \frac{AC}{BC} = \frac{5}{13}$$

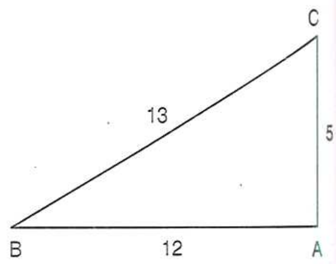
$$\cos B = \frac{AB}{BC} = \frac{12}{13}$$

$$\tan B = \frac{AC}{AB} = \frac{5}{12}$$

$$\operatorname{cosec} B = \frac{BC}{AC} = \frac{13}{5}$$

$$\sec B = \frac{BC}{AB} = \frac{13}{12}$$

$$and,\ \cot B = \frac{AB}{AC} = \frac{12}{5}$$



2. In a ΔABC, right angled at B, if AB=12 and BC=5, find:

(i) sin A and tan A (ii) cos C and cot C

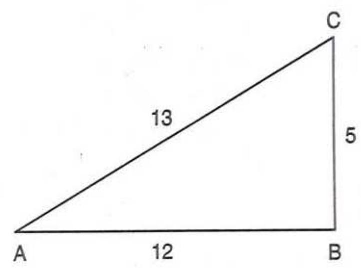
Solution: By Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = 12^2 + 5^2$$

$$\Rightarrow AC^2 = 169$$

$$\Rightarrow AC = 13$$



(i) When we consider t – ratios of $\angle A$, we have

Base = AB =12, opposite side = BC =5 and, Hypotenuse = AC = 13

$$\therefore \quad \sin A = \frac{\text{opposite side}}{Hypotenuse} = \frac{5}{13}\ and,\ \tan A = \frac{\text{opposite side}}{Base} = \frac{5}{12}$$

(ii) When we consider t- ratios of $\angle C$, we have

Base = BC =5, opposite side = AB = 12 and, Hypotenuse = AC = 13

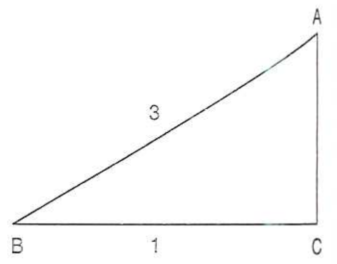
$$\therefore \quad \cos C = \frac{Base}{Hypotenuse} = \frac{5}{13}\ and,\ \cot C = \frac{Base}{\text{opposite side}} = \frac{5}{12}$$

3. If $\cos B = \frac{1}{3}$, find the other five trigonometric ratios.

Solution: We have,

$$\cos B = \frac{Base}{Hypotenuse} = \frac{1}{3}$$

By Pythagoras theorem, we have



$$AB^2 = BC^2 + AC^2$$

$\Rightarrow 3^2 = 1^2 + AC^2$

$\Rightarrow AC^2 = 9 - 1 = 8$

$\Rightarrow AC = \sqrt{8} = 2\sqrt{2}$

When we consider the t- ratios of $\angle B$, we have

Base = BC= 1, opposite side = AC = $2\sqrt{2}$ and, Hypotenuse = AB=3

4. In a right triangle ABC, right angled at C, if tan A =1, then verify that 2 sin A cos A= 1.

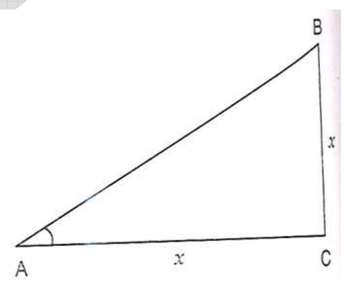
Solution: In ΔABC, we have

tan A= 1

$\Rightarrow \frac{BC}{AC} = 1$

$\Rightarrow BC = x \text{ and } AC = x$

By Pythagoras theorem, we have



$$AB^2 = AC^2 + BC^2$$

$\Rightarrow AB^2 = x^2 + x^2$

$\Rightarrow AB = \sqrt{2}x$

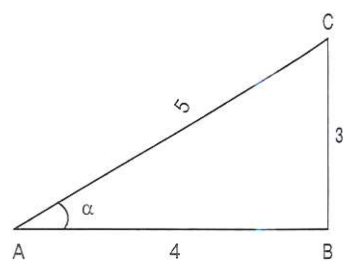
$\therefore \sin A = \frac{BC}{AB} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}} \text{ and } \cos A = \frac{AC}{AB} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}} = 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1$

5. If sec $\alpha = \frac{5}{4}$, evaluate $\frac{1 - \tan\alpha}{1 + \tan\alpha}$

Solution: We have,

$$\sec\alpha = \frac{Hypotenuse}{Base} = \frac{5}{4}$$

$$AC^2 = AB^2 + BC^2$$



$\Rightarrow 5^2 = 4^2 + BC^2$

$\Rightarrow BC^2 = 5^2 - 4^2 = 9$

$\Rightarrow BC = \sqrt{9} = 3$

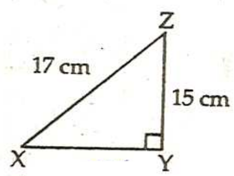
$\therefore \tan\alpha = \frac{BC}{AB} = \frac{3}{4}$

Now, $\frac{1-\tan\alpha}{1+\tan\alpha} = \frac{1-\frac{3}{4}}{1+\frac{3}{4}} = \frac{\frac{1}{4}}{\frac{7}{4}} = \frac{1}{7}$

6. In a triangle XYZ, $\angle$Y is right angle. XZ = 17 cm and YZ = 15 cm, then find (i) sin X (ii) cos Z (iii) tan X

Sol. In ΔXYZ, $\angle$Y = 90^0

$\therefore$ By Pythagoras theorem



XY = $\sqrt{XZ^2 - YZ^2} = \sqrt{17^2 - 15^2}$

$= \sqrt{64} = 8cm$

(i) $\sin x = \frac{yz}{xz} = \frac{15}{17}$

(ii) $\cos Z = \frac{ZY}{XZ} = \frac{8}{17}$

(iii) $\tan X = \frac{YZ}{XY} = \frac{15}{8}$

(iv)

7. Is it right to say cos (60^0+30^0) = cos 60^0. cos 30^0 – sin 60^0. sin 30^0 ?

Sol.

$LHS = \cos(60^0 + 30^0) = \cos 90^0 = 0$

$RHS = \cos 60^0 . \cos 30^0 - \sin 60^0 . \sin 30^0$

$= \frac{1}{2}.\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}.\frac{1}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

$\therefore LHS = RHS$

Thus, the given equation is right.

8. For which value of acute angle

$(i) \frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4 \text{ is true?}$

For which value of $0^0 \le \theta \le 90^0$, *above equation is not definded?*

Sol. $\frac{\cos\theta}{1-\sin\theta}+\frac{\cos\theta}{1+\sin\theta}=4$

$$\cos\theta\left[\frac{1}{1-\sin\theta}+\frac{1}{1+\sin\theta}\right]=4$$

$$\cos\theta\left[\frac{1+\sin\theta+1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)}\right]=4$$

$$\cos\theta\left[\frac{2}{(1-\sin^2\theta)}\right]=4$$

$$\cos\theta\, x\frac{2}{\cos^2\theta}=4$$

$$\frac{2}{\cos\theta}=4$$

$$\cos\theta=\frac{2}{4}=\frac{1}{2}=\cos 60^0$$

$$\theta=60^0$$

For above equation θ = 90^0 is not defined.

9. If sin A = cos B, then prove that A + B = 90^0

Sol. Suppose sin A = cos B

⇒ sin A = sin (90^0 – B)

⇒ A = 90^0 – B

[If both are acute angles]

⇒ A + B = 90^0

10. Given ΔABC right angled at C in which AB=29 units, BC=21 unit and $\angle ABC=\theta$. Determine the values of

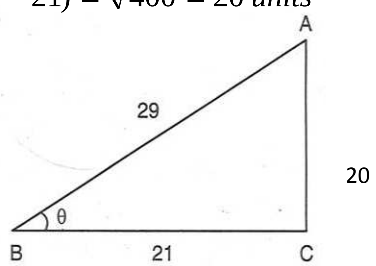
(i) $\cos^2\theta+\sin^2\theta$ **(ii)** $\cos^2\theta-\sin^2\theta$

Solution: In ΔABC, we have

$$AB^2=AC^2+BC^2$$

$$\Rightarrow AC=\sqrt{AB^2-BC^2}=\sqrt{29^2-21^2}=\sqrt{(29+21)(29-21)}=\sqrt{400}=20\ units$$

$$\therefore\quad \sin\theta=\frac{AC}{AB}=\frac{20}{29}\ and\ \cos\theta=\frac{BC}{AB}=\frac{21}{29}$$



(i) Using the values of $\sin\theta$ *and* $\cos\theta$, we obtain

$$\cos^2\theta + \sin^2\theta = \left(\frac{21}{29}\right)^2 + \left(\frac{20}{29}\right)^2$$

$$= \frac{441+400}{841} = 1$$

(ii) Using the values of $\sin\theta$ *and* $\cos\theta$, we obtain

$$\cos^2\theta - \sin^2\theta = \left(\frac{21}{29}\right)^2 - \left(\frac{20}{29}\right)^2 = \frac{21^1 - 20^2}{29^2} = \frac{(21+20)(21-20)}{841} = \frac{41}{841}$$

11. In ΔABC, right angled at B, if tan A = $\frac{1}{\sqrt{3}}$, find the value of

(i) sin A cos C + cos A sin C **(ii) cos A cos C – sin A sin C**

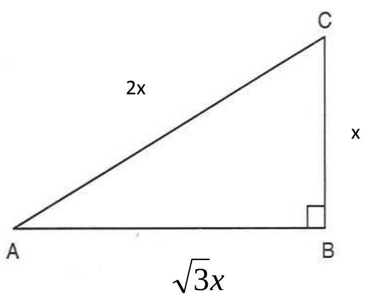
Solution: We have,

$$\tan A = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow BC = x \text{ and } AB = \sqrt{3}x$$

Using Pythagoras theorem in ΔABC, we have



$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = (\sqrt{3}x)^2 + x^2$$

$$\Rightarrow AC^2 = 4x^2$$

$$\Rightarrow AC = 2x$$

Now ,

$$\sin A = \frac{BC}{AC} = \frac{x}{2x} = \frac{1}{2}, \cos A = \frac{AB}{AC} = \frac{\sqrt{3}x}{2x} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{AB}{AC} = \frac{\sqrt{3}x}{2x} = \frac{\sqrt{3}}{2} \text{ and}, \cos C = \frac{BC}{AC} = \frac{x}{2x} = \frac{1}{2}$$

(*i*) $\sin A\cos C + \cos A\sin C = \frac{1}{2}\times\frac{1}{2} + \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = 1$

(ii) $\cos A\cos C - \sin A\sin C = \frac{\sqrt{3}}{2}\times\frac{1}{2} - \frac{1}{2}\times\frac{\sqrt{3}}{2} = 0$

12. Give that 16 cot A = 12; find the value of $\frac{\sin A+\cos A}{\sin A-\cos A}$

Solution: We have, 16 cot A = 12 $\Rightarrow \cot A=\frac{12}{16} \Rightarrow \cot A=\frac{3}{4}$.

Now, $\frac{\sin A+\cos A}{\sin A-\cos A}=\frac{\frac{\sin A+\cos A}{\sin A}}{\frac{\sin A-\cos A}{\sin A}}$ (Dividing Numerator and Denominator by sin A)

$$=\frac{1+\cot A}{1-\cot A}=\frac{1+\frac{12}{16}}{1-\frac{12}{16}} \qquad \left[\because 16\cot A=12 \Rightarrow \cot A=\frac{12}{16}\right]$$

$$=\frac{1+\frac{3}{4}}{1-\frac{3}{4}}=\frac{\frac{7}{4}}{\frac{1}{4}}=7$$

13. If tan $\theta=\frac{12}{13}$, *evaluate* $\frac{2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}$.

Solution: We have, $\tan\theta=\frac{12}{13}$

Now, $\frac{2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}=\frac{\frac{2\sin\theta\cos\theta}{\cos^2\theta}}{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}$ $[Dividing\ N^r and\ D^r by\ \cos^2\theta]$

$$=\frac{2\tan\theta}{1-\tan^2\theta}=\frac{2\times\frac{12}{13}}{1-\left(\frac{12}{13}\right)^2}=\frac{\frac{24}{13}}{1-\frac{144}{169}}=\frac{\frac{24}{13}}{\frac{25}{169}}=\frac{24}{13}\times\frac{169}{25}=\frac{312}{25}$$

14. If tan $\theta+\frac{1}{\tan\theta}=2$, find the value of $\tan^2\theta+\frac{1}{\tan^2\theta}$

Solution: We have,

$$\tan\theta+\frac{1}{\tan\theta}=2$$

$\Rightarrow \tan^2\theta + 1 = 2\tan\theta$

$\Rightarrow \tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow (\tan\theta - 1)^2 = 0$

$\Rightarrow \tan\theta = 1$

$\therefore\ \tan^2\theta + \dfrac{1}{\tan^2\theta} = 1 + 1 = 2$

15. Show that

(i) $\tan 48^0 . \tan 16^0 . \tan 42^0 . \tan 74^0 = 1$

(ii) $\cos 36^0 . \cos 54^0 - \sin 36^0 . \sin 54^0 = 0$

Sol. (i) LHS = $\tan 48^0 . \tan 16^0 . \tan 42^0 . \tan 74^0$

$= \tan 48^0 . \tan 42^0 . \tan 16^0 . \tan 74^0$

$= \tan(90^0 - 42^0) . \tan 42^0 .$

$\tan(90^0 - 74^0) . \tan 42^0 .$

$= \cot 42^0 . \tan 42^0 . \cot 74^0 . \tan 74^0$

$= \dfrac{1}{\tan 42^0} . \tan 42^0 . \dfrac{1}{\tan 74^0} . \tan 74^0$

$= 1 =$ RHS

(ii) $\cos 36^0 . \cos 54^0 = \cos(90^0 - 54^0).$

$\cos(90^0 - 36^0)$

$= \sin 54^0 . \sin 36^0$

$[\because \cos(90^0 - \theta) = \sin\theta]$

$\therefore \cos 36^0 . \cos 54^0 - \sin 36^0 . \sin 54^0 = 0$

16. Express $\sin 75^0 + \cos 65^0$ in terms of trigonometric ratios of angles between 0^0 and 45^0.

Sol. $\sin 75^0 = \sin(90^0 - 15^0)$

$= \cos 15^0\ [\sin(90^0 - \theta) = \cos\theta]$

$\cos 65^0 = \cos(90^0 - 25^0)$

$= \sin 25^0\ [\because \cos(90^0 - \theta) = \sin\theta].$

$\therefore \sin 75^0 + \cos 65^0 = \cos 15^0 + \sin 25^0$

17. Show that cot θ + tan θ = sec θ . cosec θ

Sol. LHS = cot θ + tan θ

$$=\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}$$

$$=\frac{\cos^2\theta+\sin^2\theta}{\sin\theta.\cos\theta}$$

$$=\frac{1}{\sin\theta.\,\cos\theta}$$

$[\because \cos^2\theta+\sin^2\theta=1]$

$$=\frac{1}{\sin\theta}.\frac{1}{\cos\theta}=\cos ec\theta.\sec\theta$$

$$=\sec\theta.\cos ec\theta=RHS$$

Following table gives the values of various trigonometric ratios of $0^0, 30^0, 45^0, 60^0$ and 90^0 for ready reference.

θ / T. ratios	0^0	30^0	45^0	60^0	90^0
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
Cosec θ	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot\theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

18. Prove that: $\frac{\cos 30^0+\sin 60^0}{1+\cos 60^0+\sin 30^0}=\frac{\sqrt{3}}{2}$

Solution: We have,

$$\frac{\cos 30^0+\sin 60^0}{1+\cos 60^0+\sin 30^0}=\frac{\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}}{1+\frac{1}{2}+\frac{1}{2}}=\frac{2\left(\frac{\sqrt{3}}{2}\right)}{2}=\frac{\sqrt{3}}{2}$$

19. Show that:

(i) $2(\cos^2 45^0 + \tan^2 60^0) - 6(\sin^2 45^0 - \tan^2 30^0) = 6$

Solution: (i) We have,

$$2(\cos^2 45^0 + \tan^2 60^0) - 6(\sin^2 45^0 - \tan^2 30^0)$$

$$= 2\left\{\left(\frac{1}{\sqrt{2}}\right)^2 + (\sqrt{3})^2\right\} - 6\left\{\left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2\right\}$$

$$= 2\left(\frac{1}{2} + 3\right) - 6\left(\frac{1}{2} - \frac{1}{3}\right) = 2\left(\frac{1+6}{2}\right) - 6\left(\frac{3-2}{6}\right) = 7 - 1 = 6$$

20. Find the value of θ in each of the following:

(i) $2\sin 2\theta = \sqrt{3}$ **(ii)** $2\cos 3\theta = 1$ **(iii)** $\sqrt{3}\tan 2\theta - 3 = 0$

Solution: (i) We have,

$$2\sin 2\theta = \sqrt{3}$$

$$\Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2} \Rightarrow \sin 2\theta = \sin 60^0 \Rightarrow 2\theta = 60^0 \Rightarrow \theta = 30^0$$

(ii) We have,

$$2\cos 3\theta = 1$$

$$\Rightarrow \cos 3\theta = \frac{1}{2} \Rightarrow \cos 3\theta = \cos 60^0 \Rightarrow 3\theta = 60^0 \Rightarrow \theta = 20^0$$

(iii) We have,

$$\sqrt{3}\tan 2\theta - 3 = 0$$

$$\Rightarrow \sqrt{3}\tan 2\theta = 3$$

$$\Rightarrow \tan 2\theta = \frac{3}{\sqrt{3}} = \sqrt{3} \Rightarrow \tan 2\theta = \tan 60^0 \Rightarrow 2\theta = 60^0 \Rightarrow \theta = 30^0$$

21. Find the value of x in each of the following:

(i) $\tan 3x = \sin 45^0 \cos 45^0 + \sin 30^0$

Solution: (i) We have,

$$\tan 3x = \sin 45^0 \cos 45^0 + \sin 30^0$$

$$\Rightarrow \tan 3x = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2}$$

$\Rightarrow \tan 3x = \frac{1}{2} + \frac{1}{2}$

$\Rightarrow \tan 3x = 1 \Rightarrow \tan 3x = \tan 45^0 \Rightarrow 3x = 45^0 \Rightarrow x = 15^0$

22. If θ is an acute angle and tan θ + cot θ=2, find the value of $\tan^7\theta + \cot^7\theta$h

Solution: We have,

tan θ + cot θ=2

$\Rightarrow \tan\theta + \frac{1}{\tan\theta} = 2$

$\Rightarrow \frac{\tan^2\theta + 1}{\tan\theta} = 2$

$\Rightarrow \tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow (\tan\theta - 1)^2 = 0$

$\Rightarrow \tan\theta - 1 = 0 \Rightarrow \tan\theta = 1 \Rightarrow \tan\theta = \tan 45^0 \Rightarrow \theta = 45$

$\therefore \tan^7\theta + \cot^7\theta = \tan^7 45^0 = \cot^7 45^0 = (\tan 45^0)^7 + (\cot 45^0)^7 = (1)^7 + (1)^7 = 2$

23. Show that $\tan^2\theta + \tan^4\theta = \sec^4\theta - \sec^2\theta$

Sol. LHS = $\tan^2\theta + \tan^4\theta = \tan^2\theta\,(1+\tan^2\theta)$

$= \tan^2\theta.\sec^2\theta [\because \sec^2\theta - \tan^2\theta = 1]$

$= (\sec^2\theta - 1).\sec^2\theta$

$= \sec^4\theta - \sec^2\theta = RHS$

24. Prove that $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \operatorname{cosec}\theta + \cot\theta$

Sol. $LHS = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$

$= \sqrt{\frac{1+\cos\theta}{1-\cos\theta} x \frac{1+\cos\theta}{1+\cos\theta}}$

$= \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} = \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}}$

$= \frac{1+\cos\theta}{\sin\theta} = \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$

$= \operatorname{cosec}\theta + \cot\theta = RHS$

25. $Show that\ (\cos ec\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

Sol. LHS = $(\cos ec\theta - \cot\theta)^2$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2 = \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} \quad \because \sin^2\theta = 1-\cos^2\theta$$

$$= \frac{(1-co\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$

[provided cosθ ≠ 1 or θ ≠ 0]

= RHS

26. $Show that\ \frac{1-\tan^2 A}{\cot^2 A - 1} = \tan^2 A$

Sol. LHS = $\frac{1-\tan^2 A}{\cot^2 A - 1}$

$$= \frac{1-\tan^2 A}{\frac{1}{\tan^2 A} - 1} \quad \left[\because \cot A = \frac{1}{\tan A}\right]$$

$$= \frac{1-\tan^2 A}{\frac{1-\tan^2 A}{\tan^2 A}} = \tan^2 A = RHS$$

27. Find an acute angle θ, when $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$

Solution: We have,

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

$$\Rightarrow \frac{\frac{\cos\theta - \sin\theta}{\cos\theta}}{\frac{\cos\theta + \sin\theta}{\cos\theta}} = \frac{1-\sqrt{3}}{1+\sqrt{3}} \quad \left[\begin{array}{l} Dividing\ Numerator\ and \\ Deno\min ator\ by\ \cos\theta \end{array}\right]$$

$$\Rightarrow \frac{1-\tan\theta}{1+\tan\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

$$\Rightarrow \tan\theta = \sqrt{3}$$

$$\Rightarrow \tan\theta = \tan 60^0 \qquad [\textit{On comparing two sides}]$$

$$\Rightarrow \theta = 60^0$$

28. If $\sin(A+B) = 1$ **and** $\cos(A-B) = \frac{\sqrt{3}}{2}, 0^0 < A+B \le 90^0, A > B$ **then find A and B.**

Solution: We have,

$$\sin(A+B) = 1$$

$$\Rightarrow \sin(A+B) = \sin 90^0$$

$$\Rightarrow A+B = 90^0 \qquad ..(i)$$

$$and, \quad \cos(A-B) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos(A-B) = \cos 30^0$$

$$\Rightarrow A-B = 30^0$$

Adding (i) and (ii), we get ..(ii)

$$(A+B)+(A-B) = 90^0 + 30^0 \Rightarrow 2A = 120^0 \Rightarrow A = 60^0$$

Putting $A = 60^0$ *in* (i), *we get*

$$60^0 + B = 90^0 \Rightarrow B = 30^0$$

Hence, $A = 60^0$ *and* $B = 30^0$

29. If θ **is an acute angle and** $\sin\theta = \cos\theta$, **find the value of** $2\tan^2\theta + \sin^2\theta - 1$.

Solution: We have,

$$\sin\theta = \cos\theta,$$

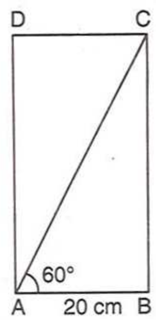
$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{\cos\theta}{\cos\theta} \qquad [\textit{Dividing both sides by } \cos\theta]$$

$$\Rightarrow \tan\theta = 1 \Rightarrow \tan\theta = \tan 45^0 \Rightarrow \theta = 45^0$$

$$\therefore 2\tan^2\theta + \sin^2\theta - 1$$

$$= 2\tan^2 45^0 + \sin^2 45^0 - 1 = 2(1)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - 1 = 2 + \frac{1}{2} - 1 = \frac{5}{2} - 1 = \frac{3}{2}$$

30. In a rectangle ABCD, AB=20cm, $\angle BAC = 60^0$ **Calculate side BC.**



Solution: In ΔABC, we have

$$AB = 20, \angle BAC = 60^0$$

$\therefore \quad \tan \angle BAC = \frac{BC}{AB}$

$\Rightarrow \tan 60^0 = \frac{BC}{20}$

$\Rightarrow \sqrt{3} = \frac{BC}{20}$

$\Rightarrow BC = 20\sqrt{3}\ cm$

31. *Showtaht* $\frac{1}{\cos\theta} - \cos\theta = \tan\theta . \sin\theta$

Sol. $LHS = \frac{1}{\cos\theta} - \cos\theta \quad = \frac{1-\cos^2\theta}{\cos\theta}$

$= \frac{\sin^2\theta}{\cos\theta} \quad [\because \sin^2\theta + \cos^2\theta = 1]$

$= \frac{\sin\theta}{\cos\theta} . \sin\theta = \tan\theta . \sin\theta = RHS$

32. Simplify (1-cosθ) (1+cosθ) (1+cot²θ)

Sol. $(1-\cos\theta)(1+\cos\theta)(1+\cot^2\theta)$

$= (1-\cos^2\theta)(1+\cot^2\theta)$

$= \sin^2\theta \ (1+\cot^2\theta)$

$= \sin^2\theta . \mathrm{cosec}^2\theta \ [1+\cot^2\theta = \mathrm{cosec}^2\theta]$

$= \sin^2\theta . \frac{1}{\sin^2\theta} . = 1$

33. If secθ + tanθ = p then what is the value of secθ - tanθ ?

Sol. Suppose $\sec\theta + \tan\theta = p$

We have, $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow p(\sec\theta - \tan\theta) = 1.$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{P}$

34. Prove sec²θ + cosec²θ = sec²θ . cosec²θ

Sol. $LHS = \sec^2\theta + \mathrm{cosec}^2\theta$

$= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \sin^2\theta}$

$= \frac{1}{\cos^2\theta \sin^2\theta} = \frac{1}{\cos^2\theta} . \frac{1}{\sin^2\theta}$

$= \sec^2\theta . \cos ec^2\theta = RHS$

35. Evaluate the following:

(i) $\sin 39^0 - \cos 51^0$ **(ii)** $\cos ec25^0 - \sec 65^0$ **(iii)** $\cot 34^0 - \tan 56^0$

(iv) $\dfrac{\sin 36^0}{\cos 54^0} - \dfrac{\sin 54^0}{\cos 36^0}$ **(v)** $\cos^2 13^0 - \sin^2 77^0$

Solution: (i) We have,

$$\sin 39^0 - \cos 51^0 = \sin(90^0 - 51^0) - \cos 51^0$$
$$= \cos 51^0 - \cos 51^0 = 0 \qquad [\because \sin(90^0 - \theta) = \cos\theta]$$

(ii) We have,

$$\cos ec25^0 - \sec 65^0 = \cos ec(90^0 - 65^0) - \sec 65^0$$
$$= \sec 65^0 - \sec 65^0 = 0 \qquad [\because \cos ec(90^0 - \theta) = \sec\theta]$$

(iii) We have,

$$\cot 34^0 - \tan 56^0 = \cot(90^0 - 56^0) - \tan 56^0$$
$$= \tan 56^0 - \tan 56^0 = 0 \qquad [\because \cot(90^0 - \theta) = \tan\theta]$$

(iv) We have,

$$\frac{\sin 36^0}{\cos 54^0} - \frac{\sin 54^0}{\cos 36^0} = \frac{\sin(90^0 - 54^0)}{\cos 54^0} - \frac{\sin(90^0 - 36^0)}{\cos 36^0}$$
$$= \frac{\cos 54^0}{\cos 54^0} - \frac{\cos 36^0}{\cos 36^0} = 1 - 1 = 0 \qquad \begin{bmatrix} \because \sin(90^0 - \theta) = \cos\theta, \\ \cos(90^0 - \theta) = \sin\theta \end{bmatrix}$$

(v) We have,

$$\cos^2 13^0 - \sin^2 77^0 = \cos^2(90^0 - 77^0) - \sin^2 77^0$$
$$= \sin^2 77^0 - \sin^2 77^0 = 0 \qquad [\because \cos(90^0 - \theta) = \sin\theta]$$

36. Express each of the following in terms of trigonometric ratios of angles between θ^0 *and* 45^0 :

(i) $\sin 85^0 + \cos ec85^0$ (ii) $\tan 68^0 + \sec 68^0$ (iii) $\cos ec69^0 + \cot 69^0$

(iv) $\sin 81^0 + \tan 81^0$ (v) $\sin 72^0 + \cot 72^0$

Solution: (i) We have

$$\sin 85^0 + \cos ec85^0$$

$$= \sin(90^0 - 5^0) + \cos ec(90^0 - 5^0)$$
$$= \cos 5^0 + \sec 5^0 \qquad [\because \sin(90^0 - \theta) = \cos\theta \text{ and } \cos ec\,(90^0 - \theta) = \sec\theta]$$

(ii) We have

$$\tan 68^0 + \sec 68^0$$

$$= \tan(90^0 - 22^0) + \sec(90^0 - 22^0)$$
$$= \cot 22^0 + \cos ec 22^0 \qquad [\because \tan(90-\theta) = \cot\theta, \sec(90-\theta) = \cos ec\theta]$$

(iii) We have,

$$\cos ec 69^0 + \cot 69^0$$

$$= \cos ec(90^0 - 21^0) + \cot(90^0 - 21^0)$$
$$= \sec 21^0 + \tan 21^0 \qquad [\because \cos ec(90^0 - \theta) = \sec\theta \text{ and } \cot(90^0 - \theta) = \tan\theta]$$

(iv) We have,

$$\sin 81^0 + \tan 81^0$$
$$= \sin(90^0 - 9) + \tan(90^0 - 9^0)$$
$$= \cos 9^0 + \cot 9^0 \qquad [\because \sin(90^0 - \theta^0) = \cos\theta \text{ and}, \tan(90^0 - 18^0) = \cot 90^0]$$

(v) We have,

$$\sin 72^0 + \cot 72^0$$
$$= \sin(90^0 - 18^0) + \cot(90^0 - 18^0)$$
$$= \cos 18^0 + \tan 18^0 \qquad [\because \sin(90^0 - 18^0) = \cos 18^0 \text{ and}, \tan(90^0 - 18^0) = \cot 18^0]$$

37. If A,B,C are the interior angles of a triangle ABC, prove that $\tan\frac{B-C}{2} = \cot\frac{A}{2}$

Solution: In ΔABC, we have

$$A + B + C = 180^0$$

$$\Rightarrow B + C = 180^0 - A$$
$$\Rightarrow \frac{B+C}{2} = 90^0 - \frac{A}{2}$$

$$\Rightarrow \tan\left(\frac{B+C}{2}\right) = \tan\left(90^0 - \frac{A}{2}\right) \Rightarrow \tan\left(\frac{B+C}{2}\right) = \cot\frac{A}{2}$$

38. If $\sin 5\theta = \cos 4\theta$, **where** 5θ *and* 4θ **are acute angles, find the value of** θ.

Solution: We have,

$$\sin 5\theta = \cos 4\theta,$$

$$\Rightarrow \sin 5\theta = \sin(90^0 - 4\theta)$$
$$\Rightarrow 5\theta = 90^0 - 4\theta$$
$$\Rightarrow 9\theta = 90^0$$
$$\Rightarrow \theta = 10^0$$

39. If $\tan 2A = \cot(A - 18^0)$, where 2A is an acute angle, find the value of A.

Solution: We have,

$$\tan 2A = \cot(A - 18^0),$$

$$\Rightarrow \tan 2A = \tan\{90^0 - (A - 18^0)\}$$

$$\Rightarrow \tan 2A = \tan(108^0 - A)$$

$$\Rightarrow 2A = 108^0 - A \Rightarrow 3A = 108^0 \Rightarrow A = 36^0$$

40. If $\tan A = \cot B$, prove that A+B=90⁰

Solution: We have

$$\tan A = \cot B$$

$$\Rightarrow \tan A = \tan(90^0 - B) \Rightarrow A = 90^0 - B \Rightarrow A + B = 90^0$$

41. Prove that:

(i) $\tan 10^0 \tan 15^0 \tan 75^0 \tan 80^0 = 1$ (ii) $\tan 1^0 \tan 2^0 \tan 3^0 \ldots \tan 89^0 = 1$

(iii) $\cos 1^0 \cos 2^0 \cos 3^0 \ldots \cos 180^0 = 0$

Solution: (i) We have,

$$LHS = \tan 10^0 \tan 15^0 \tan 75^0 \tan 80^0$$

$$= \tan(90^0 - 80^0)\tan(90^0 - 75^0)\tan 75^0 \tan 80^0$$

$$= \cot 80^0 \cot 75^0 \tan 75^0 \tan 80^0 \qquad [\because \tan(90^0 - \theta) = \cot\theta]$$

$$= (\cot 80^0 \tan 80^0)(\cot 75^0 \tan 75^0) = 1 \times 1 = 1 = RHS \qquad [\because \cot\theta.\tan\theta = 1]$$

(ii) We have,

$$LHS = \tan 1^0 \tan 2^0 \tan 3^0 \ldots \tan 89^0$$

$$= \tan(90^0 - 89^0)\tan(90^0 - 88^0)\tan(90^0 - 87^0)\ldots\tan 87^0 \tan 88^0 \tan 89^0$$

$$= \cot 89^0 \cot 88^0 \cot 87^0 \ldots \tan 87^0 \tan 88^0 \tan 89^0$$

$$= (\cot 89^0 \tan 89^0)(\cot 88^0 \tan 88^0)(\cot 87^0 \tan 87^0)\ldots(\cot 44^0 \tan 44^0).\tan 45^0$$

$$= 1 \times 1 \times 1 \ldots \times 1 = 1 = RHS \qquad [\because \cot\theta \tan\theta = 1 \; and \; \tan 45^0 = 11]$$

(iii) We have,

$$LHS = \cos 1^0 \cos 2^0 \cos 3^0 \ldots \cos 180^0$$

$$= \cos 1^0 \cos 2^0 \cos 3^0 \ldots \cos 89^0 \cos 90^0 \cos 91^0 \ldots \cos 180^0$$

$$= \cos 1^0 \cos 2^0 \cos 3^0 \ldots . \times \cos 89^0 \times 0 \times \cos 91^0 \times \ldots \cos 180^0 = 0 = RHS \qquad [\because \cos 90^0 = 0]$$

42. If secθ + tanθ = $\frac{4}{3}$ then find secθ - tanθ.

Sol. Given secθ + tanθ = $\frac{4}{3}$

We have sec²θ - tan²θ = 1

(secθ + tanθ) (secθ - tanθ) = 1

$\frac{4}{3}$ X (secθ - tanθ) = 1

$\Rightarrow$ secθ - tanθ = 1 X $\frac{3}{4}$ = $\frac{3}{4}$

$\therefore$ secθ - tanθ = $\frac{3}{4}$.

43. Prove that sin²A + cos²A = 1.

Sol. Suppose ΔABC is a right angled triangle.

By Pythagoras theorem

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow \frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2}$$

$$\Rightarrow \left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = 1$$

$$\Rightarrow \cos^2 A + \sin^2 A = 1 \qquad (or)$$

$$\sin^2 A + \cos^2 A = 1$$

44. Prove Sec $A\left(\sqrt{1-\sin^2 A}\right) = 1$

Sol. LHS $= \text{Sec } A.\sqrt{1-\sin^2 A}$

$$= \sec A.\sqrt{\cos^2 A}$$

$$[\because \sin^2 A + \cos^2 A = 1]$$

$$= SecA.CosA$$

$$= \frac{1}{\cos A}.\cos A = 1 = RHS$$

45. Prove that $\frac{\cot\theta - \cos\theta}{\cot\theta + \cos\theta} = \frac{\cos ec\theta - 1}{\cos ec\theta + 1}$.

Sol. $$LHS = \frac{\cot\theta - \cos\theta}{\cot\theta + \cos\theta} = \frac{\frac{\cos\theta}{\sin\theta} - \cos\theta}{\frac{\cos\theta}{\sin\theta} + \cos\theta}$$

$$=\frac{\cos\theta\left[\frac{1}{\sin\theta}-1\right]}{\cos\theta\left[\frac{1}{\sin\theta}+1\right]}=\frac{\cos ec\theta-1}{\cos ec\theta+1}$$

[provided cosθ ≠ 0 or θ ≠ 90^0]

= RHS

46. $\frac{1+\sec A}{\sec A}=\frac{\sin^2 A}{1-\cos A}$ *prove it.*

Sol. $LHS=\frac{1+\sec A}{\sec A}=\frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$

$$=\frac{\frac{(1+\cos A)}{\cos A}}{\frac{1}{\cos A}}$$

$=1+\cos A$

$$=\frac{(1+\cos A)(1-\cos A)}{1-\cos A}$$

$$=\frac{1-\cos^2 A}{1-\cos A}=\frac{\sin^2 A}{1-\cos A}=RHS$$