Coordinate Geometry

1 Mark Questions

1. Where do these following points lie (0, –3), (0, –8), (0, 6), (0, 4)

A. Given points (0, –3), (0, –8), (0, 6), (0, 4)

The x– coordinates of each point is zero.

∴ Given points are on the y–axis.

2. What is the distance between the given points

(1) (–4, 0) and (6, 0)

(2) (0, –3), (0, – 8)

A. **1) Given points (– 4, 0), (6, 0)**

Given points lie on x–axis (∵ y–coordinates = 0)

∴The distance between two points = $|x_2 - x_1|$

$= |6 - (-4)| = 10$

2) Given points (0, –3), (0, –8)

Given points lie on y – axis (∵ x–coordinates = 0)

∴The distance between two points = $|y_2 - y_1|$

$= |-8 - (-3)| = 5$

3. Find the distance between the following pairs of points

(i) (–5, 7) and (–1, 3)

(ii) (a, b) and (–a, –b)

A. Distance between the points (x_1, y_1) and (x_2, y_2)

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 1$$

i) Distance between the points (–5, 7) and (–1, 3)

$$= \sqrt{(-1-(-5))^2 + (3-7)^2} = \sqrt{4^2 + (-4)^2}$$

$$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

ii) Distance between (a, b) and (–a, –b)

$$= \sqrt{(-a-a)^2 + (-b-b)^2} = \sqrt{(-2a)^2 + (-2b)^2} = \sqrt{4a^2 + 4b^2}$$

$$= \sqrt{4\left(a^2 + b^2\right)} = 2\sqrt{a^2 + b^2}$$

4. Find the radius of the circle whose centre is (3, 2) and passes through (–5, 6)

A.

Let the centre 'O' = (3, 2)

The point on the circle p = (–5, 6)

Radius of the circle = distance between the points O (3, 2) and p (–5, 6)

$$=\sqrt{(-5-3)^2+(6-2)^2}=\sqrt{64+16}=\sqrt{80}$$

$$=\sqrt{16\times 5}=4\sqrt{5}units$$

5. Find the values of y for which the distance between the points P (2, –3) and Q(10, y) is 10 units.

A. Given points P (2, –3) and Q (10, y)

Given that PQ = 10 units

i.e. $=\sqrt{(10-2)^2+(y-(-3))^2}=10$

$8^2 + (y + 3)^2 = 10^2 \Rightarrow 64 + (y + 3)^2 = 100$

$(y + 3)^2 = 100 - 64 \Rightarrow (y + 3)^2 = 36 \Rightarrow y + 3 = \sqrt{36} = \pm 6$

$y = \pm 6 - 3$; $y = 6 - 3$ or, $y = -6 - 3 \Rightarrow y = 3$, or -9

Hence, the required value of y is 3 or – 9.

6. Find the distance between the points (a sinα, – b cosα) and (–a cosα, b sinα)

A. Distance between the points (x_1, y_1) and (x_2, y_2) is

$$=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Similarly the distance between (a sinα, – b cosα) and (–a cosα, b sinα)

$$=\sqrt{(-a\cos\alpha-a\sin\alpha)^2+(b\sin\alpha-(-b\cos\alpha))^2}$$

$$=\sqrt{a^2(\cos\alpha+\sin\alpha)^2+b^2(\sin\alpha+\cos\alpha)^2}$$

$$=\sqrt{a^2\cos^2\alpha+\sin^2\alpha+2\sin\alpha\cos\alpha}$$

$$=\sqrt{a^2+b^2(\sin\alpha+\cos\alpha)^2}$$

$$=\left(\sqrt{a^2+b^2}\right)(\sin\alpha+\cos\alpha)$$

7. Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2:3

A. Given points (–1, 7) and (4, –3)

Given ratio 2 : 3 = $m_1 : m_2$

Let p (x, y) be the required point.

Using the section formula

$$p(x,y)=\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_2}{m_1+m_2}\right)$$

$$=\left(\frac{(2)(4)+(3)(-1)}{2+3},\frac{(2)(-3)+(3)(7)}{2+3}\right)$$

$$=\left(\frac{8-3}{5},\frac{-6+21}{5}\right)=\left(\frac{5}{5},\frac{15}{5}\right)=(1,3)$$

8. If A and B are (–2, –2) and (2, –4) respectively. Find the coordinates of p such that $AP=\frac{3}{7}$ AB and p lies on the segment AB

A. We have $AP=\frac{3}{7}AB\Rightarrow\frac{AP}{AB}=\frac{3}{7}$

$$\frac{AP}{AP+PB}=\frac{3}{7} \qquad (\because AB=AP+PB)$$

7AP = 3(AP + PB) $\Rightarrow$ 7AP – 3AP = 3PB $\Rightarrow$ 4AP = 3PB

$$\Rightarrow\frac{AP}{PB}=\frac{3}{4}$$

So p divides AB in the ratio = 3 : 4

A= (–2, –2); B = (2, –4)

$\therefore$Coordinates of P are $\left(\frac{(3)(2)+(4)(-2)}{3+4},\frac{(3)(-4)+(4)(-2)}{3+4}\right)$

$$P(x,y)=\left(\frac{6-8}{7},\frac{-12-8}{7}\right)=\left(\frac{-2}{7},\frac{-20}{7}\right)$$

9. If the mid–point of the line segment joining $A\left[\frac{x}{2},\frac{y+1}{2}\right]$ and B (x + 1, y – 3) is C (5, –2), find x, y

A. Midpoint of the line segment joining A (x_1, y_1), B (x_2, y_2) is $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

Given that midpoint of $A\left[\frac{x}{2},\frac{y+1}{2}\right]$and B (x + 1, y – 3) is C (5, –2)

$$\therefore (5,-2)=\left(\frac{\frac{x}{2}+x+1}{2},\frac{\frac{y+1}{2}+y-3}{2}\right)$$

$$\frac{\frac{x}{2}+x+1}{2}=5\Rightarrow\frac{x+2x+2}{4}=5$$

$\Rightarrow$ 3x + 2 = 20 $\Rightarrow$ x = 6

$$\frac{\frac{y+1}{2}+(y-3)}{2}=-2\Rightarrow\frac{y+1+2y-6}{4}=-2$$

$\Rightarrow$ 3y – 5 = –8 y = –1

$\therefore$ x = 6, y = –1

10. The points (2, 3), (x, y), (3, –2) are vertices of a triangle. If the centroid of this triangle is origin, find (x, y)

A. Centroid of (x_1, y_1), (x_2, y_2) and (x_3, y_3) $=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$

Given that centroid of (2, 3), (x, y), (3, –2) is (0, 0)

i.e $(0,0)=\left(\frac{2+x+3}{3},\frac{3+y-2}{3}\right)$

$$(0,0)=\left(\frac{5+x}{3},\frac{y+1}{3}\right)$$

$$\frac{5+x}{3}=o\Rightarrow x=-5$$

$$\frac{y+1}{3}=0\Rightarrow y=-1$$

11. If the points A (6, 1), B (8, 2), C (9, 4) and D (P, 3) are the vertices of a parallelogram, taken in order find the value of p.

A. We know that diagonals of parallelogram bisect each other. Given A (6, 1), B (8, 2), C (9, 4), D (P, 3)

So, the coordinates of the midpoint of AC =

Coordinates of the midpoint of BD

i.e. $\left(\frac{6+9}{2},\frac{1+4}{2}\right)=\left(\frac{8+p}{2},\frac{2+3}{2}\right)$

$$\Rightarrow\left(\frac{15}{2},\frac{5}{2}\right)=\left(\frac{8+p}{2},\frac{5}{2}\right)\Rightarrow\frac{8+p}{2}=\frac{15}{2}$$

$\Rightarrow$ p = 15 – 8 = 7.

12. Find the area of the triangle whose vertices are (0, 0), (3, 0) and (0, 2)

A. Area of triangle $\Delta = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$$\Delta = \frac{1}{2}|0(0-2)+3(2-0)+0(0-0)| = \frac{1}{2}|6| = 3 sq\,units$$

Note: Area of the triangle whose vertices are (0,0), (x_1, y_1), (x_2, y_2) is

$\frac{1}{2}|x_1y_2 - x_2y_1|$

13. Find the slope of the line joining the two points A (–1.4, –3.7) and B (–2.4, 1.3)

A. Given points A (–1.4, –3.7), B (–2.4, 1.3)

Slope of $AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(1.3)-(-3.7)}{-2.4-(-1.4)} = \frac{1.3+3.7}{-2.4+1.4} = \frac{5}{-1} = -5$

14. Justify that the line $\overline{AB}$ line segment formed by (–2, 8), (–2,–2) is parallel to y–axis. What can you say about their slope? Why?

A. Slope of $AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-8}{(-2)-(-2)} = \frac{-10}{0} = undefined$

The slope of AB cannot defined, because the line segment $\overline{AB}$ is parallel to y–axis.

15. If x–2y +k = 0 is a median of the triangle whose vertices are at points A (–1,3), B (o,4) and C (–5, 2), find the value of K.

A. The coordinates of the centroid G of ΔABC

$$= \left(\frac{(-1)+0+(-5)}{3}, \frac{3+4+2}{3}\right) = (-2,3)$$

Since G lies on the median x – 2y + k = 0,

⇒ Coordinates of G satisfy its equation

∴ –2 –2(3) + K = 0 ⇒K=8.

16. Determine x so that 2 is the slope of the line through P (2, 5) and Q (x, 3)

A. Given points P (2, 5) and Q (x, 3)

Slope of $\overline{PQ}$ is 2

$$\therefore slope = 2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-5}{x-2} = \frac{-2}{x-2}$$

$$\Rightarrow \frac{-2}{x-2} = 2 \Rightarrow -2 = 2(x-2)$$

⇒ –2 = 2x – 4 ⇒ 2x = 2 ⇒ x = 1.

17. The coordinates of one end point of a diameter of a circle are (4, –1) and the coordinates of the centre of the circle are (1, –3). Find the coordinates of the other end of the diameter.

A. Let AB be a diameter of the circle having its centre at C (1, –3) such that the coordinates of one end A are (4, –1)

Let the coordinates of other end be B (x, y) since C is the mid–point of AB.

$\therefore$ The coordinates of C are $\left(\frac{x+4}{2}, \frac{y-1}{2}\right)$

But, the coordinates of C are given to be (1, –3)

$\therefore \left(\frac{x+4}{2}, \frac{y-1}{2}\right) = (1, -3) \Rightarrow \frac{x+4}{2} = 1 \Rightarrow x = -2$

$\frac{y-1}{2} = -3 \Rightarrow y = -5$

The other end point is (–2, –5).

2 Marks

1. Find a relation between x and y such that the point (x, y) is equidistant from the points (–2, 8) and (–3, –5)

A. Let P(x, y) be equidistant from the points A (–2, 8) and B (–3, –5)

Given that AP = BP $\Rightarrow AP^2 = BP^2$

i.e. $(x-(-2))^2 + (y-8)^2 = (x-(-3))^2 + (y-(-5))^2$

i.e. $(x+2)^2 + (y-8)^2 = (x+3)^2 + (y+5)^2$

$x^2 + 4x + 4 + y^2 - 16y + 64 = x^2 + 6x + 9 + y^2 + 10y + 25$

$-2x - 26y + 68 - 34 = 0$

$-2x - 26y = -34$

Model Problem: Find x + 13y =17, Which is the required relation. A relation between x and y such that the point (x, y) is equidistant from the point (7, 1) and (3, 5)

2. Find the point on the x – axis which is equidistant from (2, –5) and (–2, 9)

A. We know that a point on the x–axis is of the form (–x, 0). So, let the point P(x, 0) be equidistant from A (2, –5), and B (–2, 9)

Given that PA = PB

$PA^2 = PB^2$

$(x-2)^2 + (0-(-5))^2 = (x-(-2))^2 + (0-9)^2$

$x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 36$

$-4x - 4x + 29 - 40 = 0$

$-8x - 11 = 0$

$x = -\frac{11}{8}$

So, the required point is $\left(-\frac{11}{8}, 0\right)$.

Model problem:

Find a point on the y –axis which is equidistant from both the points A (6, 5) and B (–4, 3)

3. Verify that the points (1, 5), (2, 3) and (–2, –1) are collinear or not

A. Given points let A (1, 5), B (2, 3) and C (–2, –1)

$\overline{AB} = \sqrt{(2-1)^2 + (3-5)^2} = \sqrt{1+4} = \sqrt{5}$

$\overline{BC} = \sqrt{(-2-2)^2 + (-1-3)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

$$\overline{CA} = \sqrt{(5-(-1))^2 + (1-(-2))^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$$

We observe that AB + BC ≠ CA

∴ Given points are not collinear.

Model Problem: Show that the points A(4, 2), B(7, 5) and C(9, 7) are three points lie on a same line.

Note: we get AB + BC = AC, so given points are collinear.

Model problem: Are the points (3, 2), (–2, –3) and (2, 3) form a triangle.

Note: We get AB + BC ≠ AC, so given points form a triangle.

4. Check whether (5, –2), (6, 4) and (7, –2) are the vertices of an isosceles triangle.

A. Let the points are A (5, –2), B (6, 4) and C (7, –2)

$$AB = \sqrt{(6-5)^2 + (4-(-2))^2} = \sqrt{1+36} = \sqrt{37}$$

$$BC = \sqrt{(7-6)^2 + (-2-4)^2} = \sqrt{1+36} = \sqrt{37}$$

$$CA = \sqrt{(7-5)^2 + (-2+2)^2} = \sqrt{1+0} = 1$$

Since AB = BC, Given vertices form an isosceles triangle.

5. In what ratio does the point (–4, 6) divide the line segment joining the points A (–6, 10) and B (3, –8)

A. Let (–4, 6) divide AB internally in the ratio $m_1 : m_2$

using the section formula, we get

$$(-4, 6) = \left(\frac{3m_1 - 6m_2}{m_1 + m_2}, \frac{-8m_1 + 10m_2}{m_1 + m_2}\right)$$

$$\Rightarrow \frac{3m_1 - 6m_2}{m_1 + m_2} = -4 \qquad \frac{-8m_1 + 10m_2}{m_1 + m_2} = 6$$

$\Rightarrow 3m_1 - 6m_2 = -4m_1 - 4m_2 \Rightarrow 7m_1 - 2m_2 = 0$

$7m_1 - 2m_2$

$$\Rightarrow \frac{m_1}{m_2} = \frac{2}{7}$$

$\therefore m_1 : m_2 = 2 : 7$

Model Problem: Find the ratio in which the line segment joining The points (–3, 10) and (6, –8) is divided by (–1, 6).

6.Find the ratio in which the y–axis divides the line segment joining the points (5, –6) and (–1, –4). Also find the point of intersection.

A.Let the ratio be K : 1. Then by the section formula, the coordinates of the point which divides AB in the ratio K : 1 are K : 1 (5, –6) (–1, –4)

$$\left(\frac{k(-1)+1(5)}{k+1}, \frac{k(-4)+1(-6)}{K+1}\right)$$

$$i.e.\left(\frac{-k+5}{k+1}, \frac{-4k-6}{K+1}\right)$$

This point lies on the y–axis, and we know that on the y–axis the x coordinate is o

$$\therefore \frac{-k+5}{K+1}=0 \quad \Rightarrow -k+5=0 \Rightarrow k=5$$

So the ratio is K : 1 = 5 :1

Patting the value of k = 5, we get the point of intersection as

$$\left(\frac{-5+5}{5+1}, \frac{-4(5)-6}{5+1}\right)=\left(0, \frac{-26}{6}\right)=\left(0, \frac{-13}{3}\right)$$

7. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

A. Let the Given points A (1, 2), B (4, y), C (x, 6) and D (3, 5) are the vertices of a parallelogram.

We know that diagonally of parallelogram bisect each other

$\therefore$ Midpoint of AC = Midpoint of BD.

$$\left(\frac{1+x}{2}, \frac{2+6}{2}\right)=\left(\frac{4+3}{2}, \frac{y+5}{2}\right)$$

$$\Rightarrow \frac{1+x}{2}=\frac{4+3}{2} \Rightarrow \frac{1+x}{2}=\frac{7}{2} \Rightarrow 1+x=7 \Rightarrow x=6$$

$$\frac{y+5}{2}=\frac{2+6}{2} \Rightarrow y+5=8 \Rightarrow y=3$$

$\therefore$ x = 6, y = 3

8. Find the area of a triangle whose vertices are (1, –1), (–4, 6) and (–3, –5)

A. Let the points are A (1, –1), B (–4, 6) and C (–3, –5)

$$\text{Area of } \Delta ABC=\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$$

$$=\frac{1}{2}|1(6-(-5))+(-4)(-5-(-1))+(-3)(-1-6)|$$

$$=\frac{1}{2}|11+16+21|=\frac{1}{2}\times 48=24 squareunits$$

Model Problem

Find the area of a triangle formed by the points A (3, 1), B (5, 0), C (1, 2)

9. Find the area of the square formed by (0, –1), (2, 1), (0, 3) and (–2, 1) taken in order are as vertices

A. Area of the square

$= 2 \times \text{area of } \Delta ABC \rightarrow (1)$

Area of $\Delta ABC = \frac{1}{2}|0(1-3)+2(3+1)+0(-1-1)|$

$= 4$ sq.units

$\therefore$ From eqn (1), we get

Area of the given square = $2 \times 4 = 8$ sq.units.

10. The points (3, –2), (–2, 8) and (0, 4) are three points in a plane. Show that these points are collinear.

A. By using area of the triangle formula

$$\Delta ABC = \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$$

Given points A (3, –2), B(–2, 8), C(0, 4)

$$\Delta = \frac{1}{2}|3(8-4)+(-2)(4-(-2))+0(-2-8)|$$

$$= \frac{1}{2}|12-12+0| = 0$$

The area of the triangle is o. Hence the three points are collinear or they lie on the same line.

4 Marks Questions

1.Show that following points form a equilateral triangle A (A, 0), B(–a, 0), C (0, a√3)

A. Given points A (a, 0), B (–a, 0), C (0, a√3)

Distance between two points

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$AB=\sqrt{(-a-a)^2+(0-0)^2}=\sqrt{(2a)^2}=2a$$

$$BC=\sqrt{(0-(-a))^2+\left(a\sqrt{3}-0\right)^2}=\sqrt{a^2+3a^2}=\sqrt{4a^2}=2a$$

$$CA=\sqrt{(0-a)^2+\left(a\sqrt{3}-0\right)^2}=\sqrt{a^2+3a^2}=\sqrt{4a^2}=2a$$

Since AB = BC = CA, Given points form a equilateral triangle.

2.Name the type of quadrilateral formed. If any, by the following points, and give reasons for your answer the points are (–3, 5), (3, 1), (0, 3),(–1, –4).

A. Let the Given points A (–3, 5), B(3, 1), C (0, 3), D (–1, –4).

$$AB=\sqrt{(3-(-3))^2+(1-5)^2}=\sqrt{36+16}=\sqrt{52}$$

$$BC=\sqrt{(0-3)^2+(3-1)^2}=\sqrt{9+4}=\sqrt{13}$$

$$CD=\sqrt{(-1-0)^2+(-4-3)^2}=\sqrt{1+49}=\sqrt{50}$$

$$DA=\sqrt{(-3+1)^2+(5+4)^2}=\sqrt{4+81}=\sqrt{85}$$

AB ≠ BC ≠ CD ≠ DA

∴The points does not form a quadrilateral

Note: A, B, C and D are four vertices of a quadrilateral

i) If AB = BC = CD = DA and AC = BD, then it is square

ii) If AB = BC = CD = DA and AC ≠ BD, then it is Rhombus

iii) If AB = CD, BC = DA and AC = BD, then it is Rectangular

iv) If AB = CD, BC = DA and AC ≠ BD, then it is parallelogram

v) Any two sides are not equal then it is quadrilateral

3.Prove that the points (–7, –3), (5, 10), (15, 8) and (3, –5) taken in order are the corners of a parallelogram.

A. Given corners of a parallelogram

A(–7, –3), B (5, 10), C(15, 8) D (3, –5)

$$AB=\sqrt{(5-(-7))^2+(10-(-3))^2}=\sqrt{144+169}=\sqrt{313}$$

$$BC = \sqrt{(15-5)^2 + (8-10)^2} = \sqrt{100+4} = \sqrt{104}$$

$$CD = \sqrt{(3-15)^2 + (-5-8)^2} = \sqrt{144+169} = \sqrt{313}$$

$$DA = \sqrt{(3+7)^2 + (-5+3)^2} = \sqrt{100+4} = \sqrt{104}$$

$$AC = \sqrt{(15+7)^2 + (8+3)^2} = \sqrt{484+121} = \sqrt{605}$$

$$BD = \sqrt{(3-5)^2 + (-5-10)^2} = \sqrt{4+225} = \sqrt{229}$$

Since AB = CD, BC = DA and AC ≠ BD

$\therefore$ ABCD is a parallelogram

4. Given vertices of a rhombus A (–4, –7), B (–1, 2), C (8, 5), D (5, –4)

A.

$$AB = \sqrt{(-1-(-4))^2 + (2-(-7))^2} = \sqrt{3^2+9^2} = \sqrt{9+81} = \sqrt{90}$$

$$BC = \sqrt{(8-(-1))^2 + (5-2)^2} = \sqrt{9^2+3^2} = \sqrt{90}$$

$$CD = \sqrt{(5-8)^2 + (-4-5)^2} = \sqrt{9+81} = \sqrt{90}$$

$$DA = \sqrt{(-4-5)^2 + (-7+4)^2} = \sqrt{81+9} = \sqrt{90}$$

$$AC = \sqrt{(8-(-4))^2 + (5-(-7))^2} = \sqrt{144+144} = \sqrt{288}$$

$$BD = \sqrt{(5-(-1))^2 + (-4-2)^2} = \sqrt{36+36} = \sqrt{72}$$

Since AB = BC = CD = DA and AC ≠ BD

$\therefore$ ABCD is a rhombus

Area of rhombus

$$= \frac{1}{2} \times product\ of\ diagonals$$

$$= \frac{1}{2} \times \sqrt{288} \times \sqrt{72} = \frac{1}{2}\sqrt{288 \times 72}$$

$$= \frac{1}{2}\sqrt{72 \times 4 \times 72} = \frac{1}{2} \times 72 \times 2 = 72\ sq.units$$

Model Problem: Show that the points A (2, –2), B (14, 10), C (11, 13) and D (–1, 1) are the vertices of a rectangle.

Model Problem: Show the points A (3, 9), B (6, 4), C (1, 1) and D (–2, 6) are the vertices of a square ABCD.

5.Find the coordinates of the points of trisection (The points which divide a line segment into 3 equal part are said to be the trisection points) of the line segment joining the points A (2, –2) and (–7, 4)

(A) Trisection points: The points which divide a line segment into 3 equal parts are said to be the trisection points.

(or)

The points which divide the given line segment in the ratio 1:2 and 2:1 are called points of trisection.

A.

A (2, –2) B (–7, 4)

Let P and Q be the points of trisection of AB i.e. AP = PQ= QB.

Therefore, P divides AB internally in the ratio 1:2

By applying the section formula $m_1:m_2 = 1:2$

$$p(x,y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{(1)(-7)+(2)(2)}{1+2}, \frac{(1)(4)+(2)(-2)}{1+2}\right) = (-1, 0)$$

Q divides AB internally in the ratio 2:1

$$Q(x,y) = \left(\frac{(2)(-7)+(1)(2)}{2+1}, \frac{(2)(4)+(1)(-2)}{2+1}\right) = \left(\frac{-12}{3}, \frac{6}{3}\right) = (-4, 2)$$

∴ The coordinates of the points of trisection of the line segment are p (–1. 0) and Q (–4, 2)

Model problem: Find the trisection points of line joining (2, 6) and (–4, 3)

6.Find the coordinates of the points which divides the line segment joining A (–2, 2) and B (2, 8) into four equal parts.

A. Given points A (–2, 2) and B (2, 8)

Let P, Q, R divides $\overline{AB}$ into four equal parts

A (–2, 2) B (2, 8)

P divides $\overline{AB}$ into four equal parts

$$p(x,y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{(1)(2)+(3)(2)}{1+3}, \frac{(1)(8)+(3)(2)}{1+3}\right) = \left(-1, \frac{7}{2}\right)$$

Q divides $\overline{AB}$ in the ratio 2:2= 1:1

i.e Q is the midpoint of AB

$$Q(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-2+2}{2},\frac{2+8}{2}\right)=(0,5)$$

R divides $\overline{AB}$ in the ratio 3:1

$$R(x,y)=\left(\frac{(3)(2)+(1)(-2)}{3+1},\frac{(3)(8)=(1)(2)}{3+1}\right)=\left(\frac{4}{4},\frac{26}{4}\right)=\left(1,\frac{13}{2}\right)$$

$\therefore$ The points divide $\overline{AB}$ into four equal parts are $P\left(-1,\frac{7}{2}\right),Q(0,5),R\left(1,\frac{13}{2}\right)$

Model Problem: Find the coordinates of points which divide the line segment joining A (–4, 0) and B (0, 6) into four equal parts.

7. Find the area of the quadrilateral whose vertices taken in order, are (–4, –2), (–3, –5), (3, –2) and (2, 3)

A. Let the given vertices of a quadrilateral are A(–4, –2), B (–3, –5), C (3, –2) D(2, 3)

Area of quadrilateral ABCD = Area of ΔABC + Area of ΔACD

<u>Area of ΔABC</u>

A(–4, 2), B(–3, –5), C(3, –2)

Area of ΔABC $=\frac{1}{2}|-4(-5-(-2))+(-3)((-2)-(-2))+3(-2-(-5))|$

$$=\frac{1}{2}|(-4)(-3)+(-3)(0)+(3)(3)|=\frac{1}{2}|12+9|=\frac{21}{2}=10.5\,sq.units$$

<u>Area of ΔACD</u>

A(–4, 2), B(3, –2), C(2, 3)

Area of ΔABC $=\frac{1}{2}|-4(-2-3)+(3)(3-(-2))+2(-2-(-2))|$

$$=\frac{1}{2}|20+15+0|=\frac{35}{2}=17.5\,sq.units$$

Area of quadrilateral ABCD = Ar (ΔABC) + Ar (ΔACD)

= 10.5 + 17.5 = 28 sq. units

Model Problem: If A (–5, 7), B (–4, –5), C(–1, –6) and D (4, 5)

Are the vertices of a quadrilateral. Then find the area of the quadrilateral ABCD.

8. Find the value of 'K' for which the points (k, k) (2,3) and (4, –1) are collinear

A. Let the given points A (k, k), B (2, 3), C (4, –1)

If the points are collinear then the area of $\Delta ABC = 0$.

$$\therefore \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|=0$$

$$\therefore \frac{1}{2}|k(3-(-1))+2(-1-k)+4(k-3)|=0$$

$$\therefore \frac{1}{2}|4k-2-2k+4k-12|=0$$

$|6k - 14| = 0 \Rightarrow 6k - 14 = 0 \Rightarrow 6k = 14$

$$k=\frac{14}{6}=\frac{7}{3}$$

Model problem: Find the value of 'k' for which the points (7, –2), (5, 1), (3, k) are collinear

Model Problem: Find the value of 'b' for which the points

A (1, 2), B (–1, b), C (–3, –4)

9. Find the area of the triangle formed by the points (0,0), (4, 0), (4, 3) by using Heron's formula.

A. Let the given points be A (0, 0), B (4, 0), C (4, 3)

Let the lengths of the sides of ΔABC are a, b, c

$$a=\overline{BC}=\sqrt{(4-4)^2+(3-0)^2}=\sqrt{0+9}=3$$

$$b=\overline{CA}=\sqrt{(4-0)^2+(3-0)^2}=\sqrt{16+9}=5$$

$$c=\overline{AB}=\sqrt{(4-0)^2+(0-0)^2}=4$$

$$S=\frac{a+b+c}{2}=\frac{3+5+4}{2}=6$$

Heron's formula

$$Area\ of\ \Delta ABC=\sqrt{s(s-a)(s-b)(s-a)}=\sqrt{6(6-3)(6-5)(6-4)}$$

$$=\sqrt{6(3)(1)(2)}=6\ sq.units.$$

10. Find the area of the triangle formed by joining the mid–points of the sides of the triangle whose vertices are (0, –1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

A. Let the given points of the triangle of the triangle A (0, –1), B (2, 1) and C (0, 3).

Let the mid–points of AB, BC, CA are D, E, F

$$D=\left(\frac{0+2}{2},\frac{-1+1}{2}\right)=(1,0)$$

$$E=\left(\frac{2+0}{2},\frac{1+3}{2}\right)=(1,2)$$

$$F=\left(\frac{0+0}{2},\frac{3-1}{2}\right)=(0,1)$$

$$Area\ of\ \Delta ABC=\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$$

A (0, –1), B (2, 1), C (0, 3).

$$=\frac{1}{2}|0(1-3)+2(3-(-1))+0(-1-1)|=\frac{1}{2}|8|=4 sq.units$$

$$Area\ of\ \Delta DEF=\frac{1}{2}|1(2-1)+1(1-0)+0(0-2)|$$

D (1, 0), E (1, 2), F(0, 1)

$$=\frac{1}{2}|2+0|=\frac{1}{2}\times 2=1 sq.units$$

Ratio of the ΔABC and ΔDEF = 4 : 1

11. Find the area of the square formed by (0, –1), (2, 1), (0, 3) and (–2, 1)

A. In a square four sides are equal

Length of a side of the square

Area of the square = side × side

= √8 × √8

= 8 sq. units.

12. Find the coordinates of the point equidistant from. Three given points A (5, 1), B (–3, –7) and C (7, –1)

A. Let p(x, y) be equidistant from the three given points A(5, 1), B (–3, –7) and C(7, –1)

Then PA = PB = PC ⇒ $PA^2 = PB^2 = PC^2$

$PA^2 = PB^2 \Rightarrow (x-5)^2 + (y-1)^2 = (x+3)^2 + (y+7)^2$

$\Rightarrow x^2 - 10x + 25 + y^2 - 2y + 1 = x^2 + 6x + 9 + y^2 + 14y + 49$

$\Rightarrow -16x - 16y + 26 - 58 = 0$

$\Rightarrow -16x - 16y - 32 = 0$

$\Rightarrow x + y + 2 = 0 \quad \to (1)$

$PB^2 = PC^2 \Rightarrow (x + 3)^2 + (y + 7)^2 = (x - 7)^2 + (y + 1)^2$

$\Rightarrow x^2 + 6x + 9 + y^2 + 14y + 49 = x^2 - 14x + 49 + y^2 + 2y + 1$

$\Rightarrow 6x + 14x + 14y - 2y + 58 - 50 = 0$

$20x + 12y + 8 = 0$

$5x + 3y + 2 = 0 \to (2)$

Solving eqns (1) & (2)

From (1) $x + y + 2 = 0 \Rightarrow 2 + y + 2 = 0$

$y = -4$

$(1) \times 3 \qquad 3x + 3y + 6 = 0$

$(2) \times 1 \qquad 5x + 3y + 2 = 0$

$\quad - \quad - \quad -$

$-2x \qquad + 4 = 0$

$$x = \frac{-4}{-2} = 2$$

Hence, The required point is (2, –4)

13. Prove that the points (a, b + c), (b, c + a) and (c, a + b) are collinear.

A. Let the given points A (a, b + c), B (b, c + a), C (c, a + b)

$$Area\ of\ \Delta ABC = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$= \frac{1}{2}|a((c+a)-(a+b)) + b((a+b)-(b+c)) + c((b+c)-(c+a))|$$

$$= \frac{1}{2}|a(c-b) + b(a-c) + c(b-a)|$$

$$= \frac{1}{2}|ac - ab + ba - bc + cb - ca|$$

$$= \frac{1}{2}|0| = 0$$

Since area of ΔABC = 0, the given points are collinear.

14. A (3, 2) and B (–2, 1) are two vertices of a triangle ABC, Whose centroid G has a coordinates $\left(\frac{5}{3},-\frac{1}{3}\right)$. Find the coordinates of the third vertex c of the triangle.

A. Given points are A (3, 2) and B (–2, 1)

Let the coordinates of the third vertex be C(x, y)

Centroid of ABC, $\left(\frac{5}{3},-\frac{1}{3}\right)$

$$G=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

$$\left(\frac{5}{3},-\frac{1}{3}\right)=\left(\frac{3+(-2)x}{3},\frac{2+1+y}{3}\right)$$

$$\left(\frac{5}{3},-\frac{1}{3}\right)=\left(\frac{x+1}{3},\frac{y+3}{3}\right)$$

$$\frac{x+1}{3}=\frac{5}{3}\Rightarrow x+1=5\Rightarrow x=5-1=4$$

$$\frac{y+3}{3}=-\frac{1}{3}\Rightarrow y+3=-1\Rightarrow y=-1-3=-4$$

$\therefore$ The third vertex is (4, –4)

15. The two opposite vertices of a square are (–1, 2) and (3, 2). Find the coordinates of the other two vertices.

A. Let the opposite vertices of a square A (–1, 2), C (3, 2)

Let B (x, y) be the unknown vertex

AB = BC (∵ In a square sides are equal)

$\Rightarrow AB^2 = BC^2$

$(x-(-1))^2 + y(y-2)^2 = (3-x)^2 + (2-y)^2 \Rightarrow x^2 + 2x + 1 + y^2 - 4y + 4$

$\Rightarrow 8x = 13 - 5 \Rightarrow x = 1 \rightarrow (1)$

Also By pythagoras theorem

$AC^2 = AB^2 + BC^2$

$(3+1)^2 + (2-2)^2 = (x+1)^2 + (y-2)^2 + (x-3)^2 + (y-2)^2$

$16 = x^2 + 2x + 1 + y^2 - 4y + 4 + x^2 - 6x + 9 + y^2 - 4y + 4$

$2x^2 + 2y^2 - 4x - 8y + 18 = 16$

$x^2 + y^2 - 2x - 4y + 1 = 0$

From (1) x = 1

i.e. $1^2 + y^2 - 2(1) - 4y + 1 = 0$

$y^2 - 4y = 0$

$y(y-4) = 0 \Rightarrow y = 0$ or $y - 4 = 0$

$\Rightarrow y = 0$ or $y = 4$

Hence the other vertices are (1, 0) and (1, 4).

Multiple Choice Questions

1. **For each point on x–axis, y–coordinate is equal to** [ ]

   a) 1 b) 2 c) 3 4) 0

2. **The distance of the point (3, 4) from x – axis is** [ ]

   a) 3 b) 4 c) 1 d) 7

3. **The distance of the point (5, –2) from origin is** [ ]

   a) $\sqrt{29}$ b) $\sqrt{21}$ c) $\sqrt{30}$ d) $\sqrt{28}$

4. **The point equidistant from the points (0, 0), (2, 0), and (0, 2) is** [ ]

   a) (1, 2) b) (2, 1) c) (2, 2) d) (1, 1)

5. **If the distance between the points (3, a) and (4, 1) is $\sqrt{10}$, then, find the values of a** [ ]

   a) 3, –1 b) 2, –2 c) 4, –2 d) 5, –3

6. **If the point (x, y) is equidistant from the points (2, 1) and (1, –2), then** [ ]

   a) $x + 3y = 0$ b) $3x + y = 0$ c) $x + 2y = 0$ d) $2y + 3x = 0$

7. **The closed figure with vertices (–2, 0), (2, 0), (2,2) (0, 4) and (2, –2) is a** [ ]

   a) Triangle b) quadrilateral c) pentagon d) hexagon

8. **If the coordinates of p and Q are (a cosθ, b sinθ) and (–a sinθ, b cosθ). Then $OP^2 + OQ^2$ =** [ ]

   a) $a^2 + b^2$ b) a + b c) ab d) 2ab

9. In which quadrant does the point (–3, –3) lie? [ ]

a) I b) II c) III d) IV

10. Find the value of K if the distance between (k, 3) and (2, 3) is 5. [ ]

a) 5 b) 6 c) 7 d) 8

11. What is the condition that A, B, C are the successive points of a line? [ ]

a) AB + BC = AC b) BC + CA = AB

c) CA + AB = BC d) AB + BC = 2AC

12. The coordinates of the point, dividing the join of the point (0, 5) and (0, 4) in the ratio 2 : 3 internally, are [ ]

a) $\left(3, \frac{8}{5}\right)$ b) $\left(1, \frac{4}{5}\right)$ c) $\left(\frac{5}{2}, \frac{3}{4}\right)$ d) $\left(2, \frac{12}{5}\right)$

13. If the point (0. 0), (a, 0) and (0, b) are collinear, then [ ]

a) a = b b) $a + b \neq 0$ c) ab = 0 d) $a \neq b$

14. The coordinates of the centroid of the triangle whose vertices are (8, –5), (–4, 7) and (11, 13) [ ]

a) (2, 2) b) (3, 3) c) (4, 4) d) (5, 5)

15. The coordinates of vertices A, B and C of the triangle ABC are (0, –1), (2,1) and (0, 3). Find the length of the median through B. [ ]

a) 1 b) 2 c) 3 d) 4

16. The vertices of a triangle are (4, y), (6, 9) and (x, y). The coordinates of it centroid are (3, 6). Find the value of x and y. [ ]

a) –1, –5 b) 1, –5 c) 1, 5 d) –1, 5

17. If a vertex of a parallelogram is (2, 3) and the diagonals cut at (3, –2). Find the opposite vertex. [ ]

a) (4, –7) b) (4, 7) c) (–4, 7) d) (–4, –7)

18. Three consecutive vertices of a parallelogram are (–2, 1), (1, 0) and (4, 3). Find the fourth vertex. [ ]

a) (1, 4) b) (1, –2) c) (–1, 2) d) (–1, –2)

19. If the points (1, 2), (–1, x) and (2, 3) are collinear then the value of x is [ ]

a) 2 b) 0 c) –1 d) 1

20. If the points (a, 0), (o, b) and (1, 1) are collinear then $\frac{1}{a}+\frac{1}{b}=$ [ ]

a) 0 b) 1 c) 2 d) –1

Key:

1) d; 2) b; 3) a; 4) d; 5) c; 6) a; 7) c; 8) a; 9) c; 10) c;

11) a; 12) a; 13) c;14) d; 15) b; 16) a; 17) a; 18) a; 19) b; 20) b;

Fill in the Blanks:

1. The coordinates of the point of intersection of x – axis and y – axis are ___________.
2. For each point on y–axis, x– coordinate is equal to __________.
3. The distance of the point (3, 4) from y –axis is _________.
4. The distance between the points (0, 3) and (–2, 0) is ______.
5. The opposite vertices of a square are (5, 4) and (–3, 2). The length of its diagonal is _____________.
6. The distance between the points (a cosθ+ b sinθ, 0) and (0, a sinθ – b cosθ) is ____________.
7. The coordinates of the centroid of the triangle with vertices (0, 0) (3a, 0) and (0, 3b) are ______.
8. If OPQR is a rectangle where O is the origin and p (3, 0) and R (0, 4), Then the Coordinates of Q are __________.
9. If the centroid of the triangle (a, b), (b, c) and (c, a) is O (0, 0), then the value of $a^3 + b^3 + c^3$ is ___________.

10. If (–2, –1), (a, 0), (4, b) and (1, 2) are the vertices of a parallelogram, then the values of a and b are _______.
11. The area of the triangle whose vertices are (0, 0), (a, 0) and (o, b) is _________.
12. One end of a line is (4, 0) and its middle point is (4, 1), them the coordinates of the other end ______.
13. The distance of the mid–point of the line segment joining the points (6, 8) and (2, 4) from the point (1, 2) is _______.
14. The area of the triangle formed by the points (0, 0), (3, 0) and (0, 4) is ________.
15. The co–ordinates of the mid–point of the line segment joining the points (x_1, y_1) and (x_2, y_2) are ___________.
16. The distance between the points (a $\cos 25^0$, 0) and (0, a $\cos 65^0$) is _______.
17. The line segment joining points (–3, –4) and (1, –2) is divided by y –axis in the ratio _________.
18. If A (5, 3), B (11, –5) and p(12, y) are the vertices of a right triangle right angled at p, Then y = _________.
19. The perimeter of the triangle formed by the points (0, 0), (1, 0) and (0, 1) is ___________.
20. The coordinates of the circumventer of the triangle formed by the points O (0,0), A (a, 0) and B (o, b) is __________.

Key:

1) (0, 0); 2) 0; 3) 3; 4) √13; 5) 10; 6) $\sqrt{a^2+b^2}$; 7) (a, b);

8) (3, 4); 9) 3abc; 10) a = 1, b = 2; 11) $\frac{1}{2}ab$; 12) (4, 2)

13) 5; 14) 6; 15) $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$; 16) a; 17) 3 : 1; 18) 2 or -4;

19) 2 + √2; 20) $\left(\frac{a}{2}, \frac{b}{2}\right)$;