CO- ORDINATE GEOMETRY

Key Concepts

→ A French mathematician **Rene – De- Cartes (1596 – 1650)** has developed the study of Co –ordinate Geometry.

→ The Cartesian plane is also called **Coordinate plane or XY plane.**

→ The x coordinate is called the **Abscissa** and the y coordinate is called the **ordinate.**

→ The intersection of X – axis and Y – axis is called the **origin.**

The coordinates of origin = (0, 0)

→ The distance between points lying on X – axis to the difference between the x coordinates. In general for the points $(x_1,0)$ $(x_2,0)$ on the X – axis.

The distance between these points = $|x_2 - x_1|$.

→ The distance between two points $(0, y_1), (0, y_2) = |y_2 - y_1|$.

→ The distance between two points (x_1, y_1) and (x_2, y_2) *is* $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

→ The distance of a points (x, y) from the origin is $\sqrt{x^2+y^2}$.

→ Points lie on the same line are called **Collinear Points.**

→ The distance between two points (x_1, y_1) and (x_2, y_2) on a line parallel to Y- axis is $|y_2 - y_1|$.

→ The distance between two points (x_1, y_1) and (x_2, y_2) on a line parallel to X – axis is $|x_2 - x_1|$.

→ The coordinates of the point $P(x, y)$ which divides the line segment joining the points $A(x_1, y_1)$ *and* $B(x_2, y_2)$ internally in the ratio $m_1 : m_2$ are $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$ This is known as "**Section formula**".

→ The midpoint of the linesegment joining the points $P(x_1, y_1)$ *and* $Q(x_2, y_2)$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

→ The point that divides each median in the ratio 2 : 1 is the **centroid** of a triangle.

→ The **centroid** of a triangle is the point of intersection of its medians.

$A(x_1, y_1), B(x_2, y_2)$ *and* $C(x_{3,} y_3)$ are the vertices of the triangle ABC coordinates of the centroid = $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

→ The area of the triangle formed by the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is the numerical value of the expression

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

→ Area of Rhombus $= \frac{1}{2} \times$ product of its diagonals.

→ The diagonals of a parallelogram bisect each other.

→ The area of a triangle is zero then the three points said to be **Collinear points.**

→ Area of a triangle $= \frac{1}{2} \times$ **base** $\times$ **height.**

→ Area of a triangle formula **' Heron's formula'**

$$A = \sqrt{S(S-a)\,(S-b)\,(S-c)}$$

Problems

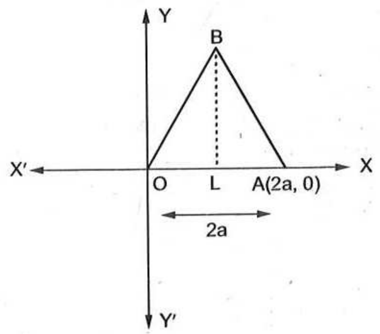
1 Find the coordinates of the vertices of an equilateral triangle of side 2a as shown in

Solution: Since OAB is an equilateral triangle of side 2a therefore,

OA = AB= OB = 2a

Let BL perpendicular from B on OA. Then,

OL = LA = a



In ΔOLB, we have

$$OB^2 = OL^2 + LB^2$$

$$\Rightarrow (2a)^2 = a^2 + LB^2$$
$$\Rightarrow LB^2 = 3a^2$$
$$\Rightarrow LB = \sqrt{3}\,a$$

Clearly, coordinates of O are (0,0) and that of A are (2a,0). Since OL = a and LB = $\sqrt{3}\,a$. So, the coordinates of B are $(a, \sqrt{3}\,a)$.

2 If the point (x, y) **is equidistant from the points** $(a+b, b-a)$ **and** $(a-b, a+b)$, **prove that** $bx = ay$.

Solution: Let $P(x, y), Q(a+b, b-a)$ and $R(a-b, a+b)$ be the given points. Then, $PQ = PR$.

$$\Rightarrow \sqrt{\{x-(a+b)\}^2 + \{y-(b-a)\}^2} = \sqrt{\{x-(a-b)\}^2 + \{y-(a+b)\}^2}$$
$$\Rightarrow \{x-(a+b)\}^2 + \{y-(b-a)\}^2 = \{x-(a-b)\}^2 + \{y-(a+b)\}^2$$

$$\Rightarrow x^2 - 2x(a+b) + (a+b)^2 + y^2 - 2y(b-a) + (b-a)^2$$
$$= x^2 + (a-b)^2 - 2x(a-b) + y^2 - 2y(a+b) + (a+b)^2$$
$$\Rightarrow -2x\,(a+b) - 2y(b-a) = -2x\,(a-b) - 2y\,(a+b)$$
$$\Rightarrow ax + bx + by - ay = ax - bx + ay + by$$
$$\Rightarrow 2bx = 2ay \Rightarrow bx = ay$$

<u>Remark :</u> We know that a point which is equidistant from points P and Q lies on the perpendicular bisector of PQ. Therefore, bx = ay is the equation of the perpendicular bisector of PQ.

3 If the points A (4,3) and B $(x,5)$ are on the circle with centre O (2,3) find the value of x.

Solution: Since A and B lie on the circle having centre O.

$\therefore \quad OA = OB$

$$\Rightarrow \sqrt{(4-2)^2+(3-3)^2} = \sqrt{(x-2)^2+(5-3)^2}$$
$$\Rightarrow 2 = \sqrt{(x-2)^2+4}$$
$$\Rightarrow 4 = (x-2)^2+4 \Rightarrow (x-2)^2 = 0 \Rightarrow x-2=0 \Rightarrow x=2.$$

4 Find a point on x- axis which is equidistant from A (2,-5) and B (-2,9).

Solution: We know that a point on x- axis is of the form $(x,0)$. So, let P $(x,0)$ be the point equidistant from A (2, -5) and B (-2,9). Then,

PA = PB

$$\Rightarrow \sqrt{(x-2)^2+(0+5)^2} = \sqrt{(x+2)^2+(0-9)^2}$$
$$\Rightarrow (x-2)^2+25 = (x+2)^2+81$$
$$\Rightarrow x^2-4x+4+25 = x^2+4x+4+81 \Rightarrow -8x = 56 \Rightarrow x = -7$$

Hence, the required point is (-7, 0).

5 The x- coordinate of a point P is twice its y- coordinate. If P is equidistant from Q (2, -5) and R (-3, 6), then find the coordinates of P.

Solution: Let the coordinates of P be (x, y). It given that $x = 2y$. It is also given that

PQ = PR

$$\Rightarrow \sqrt{(x-2)^2+(y+5)^2} = \sqrt{(x+3)^2+(y-6)^2}$$
$$\Rightarrow \sqrt{(2y-2)^2+(y+5)^2} = \sqrt{(2y+3)^2+(y-6)^2}$$
$$\Rightarrow \sqrt{5y^2+2y+29} = \sqrt{5y^2+45}$$
$$\Rightarrow 5y^2+2y+29 = 5y^2+45 \Rightarrow 2y = 16 \Rightarrow y = 8$$

Hence, the coordinates of P are (16,8)

6 Show that the points (1, -1), (5, 2) and (9, 5) are collinear.

Solution: Let A (1,-1), B (5, 2) and (9,5) be the given points. Then, we have

$$AB = \sqrt{(5-1)^2 + (2+1)^2} = \sqrt{16+9} = 5$$
$$BC = \sqrt{(5-9)^2 + (2-5)^2} = \sqrt{16+9} = 5$$

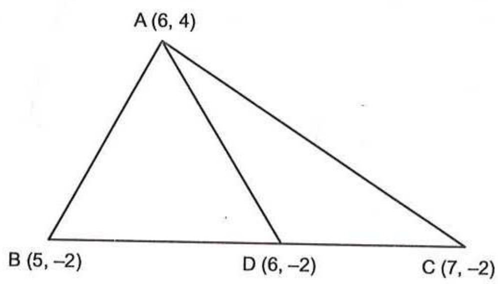
and, $AC = \sqrt{(1-9)^2 + (-1-5)^2} = \sqrt{64+36} = 10$

Clearly, AC = AB + BC. Hence, A, B, C are collinear points.

7 Show that A (6,4), B (5, -2) and C (7, -2) are the vertices of an isosceles triangle. Also find the length of the median through A.

Solution: We have

$$AB = \sqrt{(6-5)^2 + (4+2)^2} = \sqrt{37}, \quad AC = \sqrt{(6-7)^2 + (4+2)^2} = \sqrt{37}$$



$\therefore$ AB= AC

So, ΔABC is isosceles.

Let D be the mid – point of BC. Then, coordinates of D are $\left(\frac{5+7}{2}, \frac{-2-2}{2}\right)$ i.e. (6, -2).

$\therefore \ AD = \sqrt{(6-6)^2 + (4+2)^2} = \sqrt{36} = 6$

8 If (-4, 0) and (4, 0) are two vertices of an equilateral triangle, find the coordinates of its third vertex.

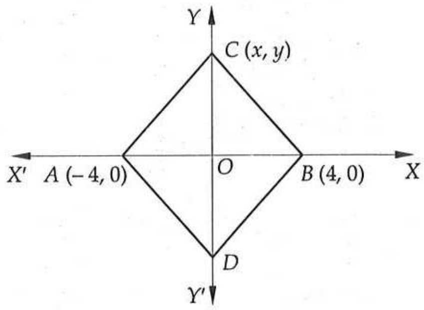
Solution: Let C (x, y) be the third vertex of triangle ABC having two vertices at A (-4,0) and B (4, 0). Since ΔABC is equilateral. Therefore, AC= BC = AB

Now, AC = BC

$$\Rightarrow \sqrt{(x+4)^2(y-0)^2} = \sqrt{(x-4)^2+(y-0)^2}$$
$$\Rightarrow (x+4)^2+y^2 = (x-4)^2+y^2$$
$$\Rightarrow 16x = 0$$
$$\Rightarrow x = 0$$

Again,

$$AC = BC = AB$$
$$\Rightarrow AC = AB$$
$$\Rightarrow \sqrt{(x+4)^2+(y-0)^2} = \sqrt{(4+4)^2+0^2}$$
$$\Rightarrow (0+4)^2+y^2 = 64$$
$$\Rightarrow y^2 = 48$$
$$\Rightarrow y = \pm 4\sqrt{3}$$

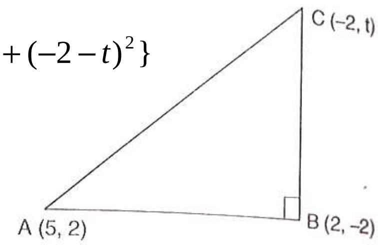


Hence, the coordinates of the third vertex are C $(0, 4\sqrt{3})$ *and* $D(0, -4\sqrt{3})$.

9 If A (5, 2), B (2, -2) and C (2, t) are the vertices of right angled triangle with $\angle B = 90^0$, then find the value of t.

Solution: Using Pythagoras theorem in right triangle ABC, we obtain

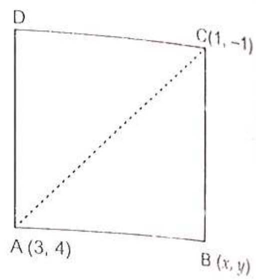
$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow (5+2)^2+(2-t)^2 = \{(5-2)^2+(2+2)^2\}+\{(2+2)^2+(-2-t)^2\}$$
$$\Rightarrow 49+(4-4t+t^2) = (9+16)+(16+4+4t+t^2)$$
$$\Rightarrow t^2-4t+53 = t^2+4t \times 45$$
$$\Rightarrow -8t = -8$$
$$\Rightarrow t = 1$$



10 Let the opposite angular points of a square be (3,4) and (1, -1). Find the coordinates of the remaining angular points.

Solution: Let ABCD be a square and let A (3,4) and C (1, -1) be the given angular points. Let B (x, y) be the unknown vertex.

Then, AB = BC



$\Rightarrow\ AB^2 = BC^2$

$\Rightarrow\ (x-3)^2 + (y-4)^2 = (x-1)^2 + (y+1)^2$

$\Rightarrow\ 4x + 10y - 23 = 0$

$\Rightarrow\ x = \dfrac{23-10y}{4}$ ...(i)

In right - angled triangle ABC, we have

$AB^2 + BC^2 = AC^2$

$\Rightarrow\ (x-3)^2 + (y-4)^2 + (x-1)^2 + (y+1)^2 = (3-1)^2 + (4+1)^2$

$\Rightarrow\ x^2 + y^2 - 4x - 3y - 1 = 0$

Substituting the value of x from (i) into (ii), we get

$$\left(\frac{23-10y}{4}\right)^2 + y^2 - (23-10y) - 3y - 1 = 0$$

$\Rightarrow\ 4y^2 - 12y + 5 = 0 \Rightarrow (2y-1)(2y-5) = 0 \Rightarrow y = \dfrac{1}{2} or, \dfrac{5}{2}$

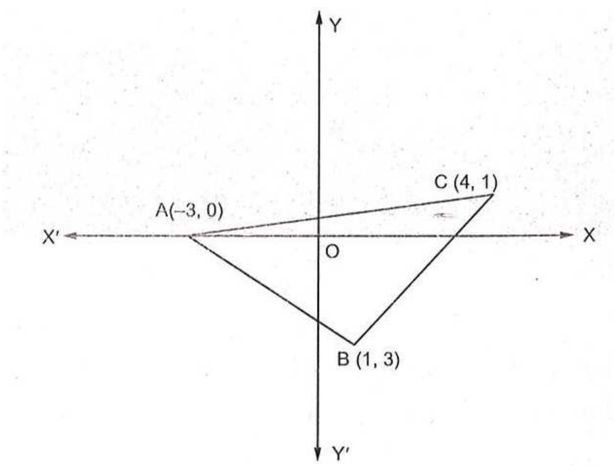
Putting $y = \dfrac{1}{2}$ and $y = \dfrac{5}{2}$ respectively in (i), we get $x = \dfrac{9}{2}$ and $x = \dfrac{-1}{2}$ respectively.

Hence, the required vertices of the square are (9/2, ½) and (-1/2, 5/2).

11 Prove that the points (-3, 0), (1, -3) and (4, 1) are the vertices of an isosceles right angled triangle. Find the area of this triangle.

Solution: Let A (-3, 0), B (1, -3) and C (4, 1) be the given points. Then,

$AB = \sqrt{\{1-(-3)\}^2 + (-3-0)^2} = \sqrt{4^2 + (-3)^2} = \sqrt{16+9} = 5\ units.$



$BC = \sqrt{(4-1)^2 + (1+3)^2} = \sqrt{9+16} = 5\ units$

and, $CA = \sqrt{(4+3)^2 + (1-0)^2} = \sqrt{49+1} = 5\sqrt{2}\ units$

clearly, AB = BC. Therefore, ΔABC is isosceles.

Also, $AB^2 + BC^2 = 25 + 25 = (5\sqrt{2})^2 = CA^2$

$\therefore$ $\Delta\ ABC$ is right – angled at B.

Thus, $\Delta\ ABC$ is a right - angled isosceles triangle.

Now, Area of $\Delta ABC = \frac{1}{2}(Base \times Height) = \frac{1}{2}(AB \times BC) = \left(\frac{1}{2} \times 5 \times 5\right) sq.\ units = \frac{25}{2}$ sq. units

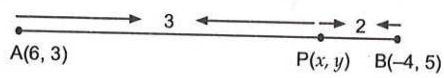
Type I On finding the section point when the section ratio is given

12 Find the coordinates of the point which divides the line segment joining the points (6,3) and (-4, 5) in the ratio 3 : 2 internally.

Solution: Let P (x, y) be the required point. Then,

$$x = \frac{3 \times -4 + 2 \times 6}{3+2}\ and\ y = \frac{3 \times 5 + 2 \times 3}{3+2}$$

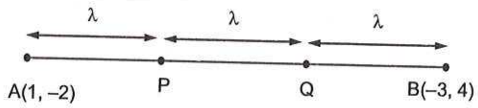
$$\Rightarrow\ x = 0\ and\ y = \frac{21}{5}$$



So, the coordinates of P are (0, 21/5).

13 Find the coordinates of points which trisect the line segment joining (1, -2) and (-3, 4).

Solution: Let A (1, -2) and B (-3, 4) be the given points. Let the points of trisection be P and Q. Then, AP = QB = λ (say).



$$\therefore\ PB = PQ + QB = 2\lambda\ and\ AQ = AP + PQ = 2\lambda$$
$$\Rightarrow\ AP : PB = \lambda : 2\lambda = 1 : 2\ and\ AQ : QB = 2\lambda : \lambda = 2 : 1$$

So, P divides AB internally in the ratio 1 : 2 while Q divides internally in the ratio 2 : 1. Thus, the coordinates of P and Q are

$$P\left(\frac{1\times -3+2\times 1}{1+2},\frac{1\times 4+2\times -2}{1+2}\right)=P\left(\frac{-1}{3},0\right)$$

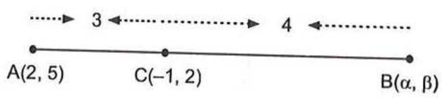
$$Q\left(\frac{2\times -3+1\times 1}{2+1},\frac{2\times 4+1\times (-2)}{2+1}\right)=Q\left(\frac{-5}{3},2\right)\ \textit{respectively}$$

Hence, the two points of trisection are (-1/3, 0) and (-5/3, 2).

Remark : As Q is the mid – point of BP. So, the coordinates of Q can also be obtained by using mid – point formula.

14 If the pint C (-1, 2) divides internally the line segment joining A (2,5) and B in ratio 3:4, find the coordinates of B.

Solution: Let the coordinates of B be (α,β). It is given that AC : BC = 3:4 So, the co-ordinates of Care



$$\left(\frac{3\alpha+4\times 2}{3+4},\frac{3\beta+4\times 5}{3+4}\right)=\left(\frac{3\alpha+8}{7},\frac{3\beta+20}{7}\right)$$

But, the coordinates of C are (-1, 2).

$$\therefore\ \frac{3\alpha+8}{7}=-1\ \textit{and}\ \frac{3\beta+20}{7}=2$$

$$\Rightarrow\ \alpha=-5\ \textit{and}\ \beta=-2$$

Thus, the coordinates of B are (-5, -2).

15 Find the ratio in which the point (-3, p) divides the line segment joining the points (-5, -4) and (-2,3). Hence, find the value of p.

Solution: Suppose the point P (-3, p) divides the line segment joining points A (-5, -4) and B (-2, 3) in the ratio k : 1.

Then, the coordinates of p are $\left(\frac{-2k-5}{k+1}, \frac{3k-4}{k+1}\right)$

But, the coordinates of P are given as (-3, p).

$$\therefore \quad \frac{-2k-5}{k+1} = -3 \; and \; \frac{3k-4}{k+1} = p$$

$$\Rightarrow \; -2k-5 = -3k-3 \; and \; \frac{3k-4}{k+1} = p$$

$$\Rightarrow \; k = 2 \; andp = \frac{3k-4}{k+1}$$

$$\Rightarrow \; k = 2 \; and \; p = 2/3$$

Hence, the ratio is 2 : 1 and p = 2/3.

Type II On determination of the type of a given quadrilateral

16 Prove that the points (-2, -1), (1, 0), (4, 3) and (1, 2) are the vertices of a parallelogram. Is it a rectangle ?

Solution: Let the given point be A, B, C and D respectively. Then,

Coordinates of the mid – point of Ac are $\left(\frac{-2+4}{2}, \frac{-1+3}{2}\right) = (1, 1)$

Coordinates of the mid – point of BD are $\left(\frac{1+1}{2}, \frac{0+2}{2}\right) = (1, 1)$

Thus, AC and BD have the same mid – point. Hence, ABCD is a parallelogram.

Now, we shall see whether ABCD is a rectangle or not.

We have,

AC = $\sqrt{(4-(-2))^2 + (3-(-1))^2} = 2\sqrt{13}$

and, $BD = \sqrt{(1-1)^2 + (0-2)^2} = 2$

Clearly, AC ≠ BD. So, ABCD is not a rectangle.

Type III On finding the unknown vertex from given points

17 If the points A (6, 1), B (8, 2), C (9,4) and D (p,3) are the vertices of a parallelogram, taken in order, find the value of p.

Solution: We know that the diagonals of a parallelogram bisect each other. So, coordinates of the mid – point of diagonal AC are same as the coordinates of the mid – point of diagonal BD.

$$\therefore \left(\frac{6+9}{2},\frac{1+4}{2}\right)=\left(\frac{8+p}{2},\frac{2+3}{2}\right)$$

$$\Rightarrow \left(\frac{15}{2},\frac{5}{2}\right)=\left(\frac{8+p}{2},\frac{5}{2}\right)$$

$$\Rightarrow \frac{15}{2}=\frac{8+p}{2}\Rightarrow 15=8+p\Rightarrow p=7$$

18 If A (-2, -1), B (a, 0), C (4, b) and D (1, 2) are the vertices of a parallelogram, find the values of a and b.

Solution: We know that the diagonals of a parallelogram bisect each other. Therefore, the coordinates of the mid – point of AC are same as the coordinates of the mid – point of BD i.e.,

$$\left(\frac{-2+4}{2},\frac{-1+b}{2}\right)=\left(\frac{a+1}{2},\frac{0+2}{2}\right)$$

$$\Rightarrow \left(1,\frac{b-1}{2}\right)=\left(\frac{a+1}{2},1\right)$$

$$\Rightarrow \frac{a+1}{2}=1 \text{ and } \frac{b-1}{2}=1$$

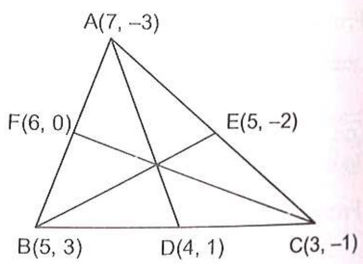
$$\Rightarrow a+1=2 \text{ and } b-1=2$$

$$\Rightarrow a=1 \text{ and } b=3$$

Hence, a = 1 and b = 3

19 Find the lengths of the medians of a ΔABC whose vertices are A (7,-3), B (5,3) and C (3, -1).

Solution: Let D, E, F be the mid – points of the sides BC, CA and AB respectively. Then, the coordinates of D, E and F are



$$D\left(\frac{5+3}{2},\frac{3-1}{2}\right)=D\,(4,1),\ E\left(\frac{3+7}{2},\frac{-1-3}{2}\right)=E(5,-2)$$

and $F\left(\frac{7+5}{2},\frac{-3+3}{2}\right)=F\ (6,0)$

$\therefore\ AD=\sqrt{(7-4)^2+(-3-1)^2}=\sqrt{9+16}=5\ units$

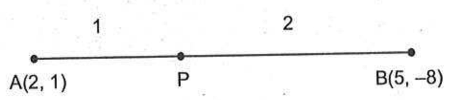
$BE=\sqrt{(5-5)^2+(-2-3)^2}=\sqrt{0+25}=5\ units$

and $CF=\sqrt{(6-3)^2+(0+1)^2}=\sqrt{9+1}=\sqrt{10}\ units.$

20 Point P divides the line segment joining the pints A (2, 1) and B (5, -8) such that $\frac{AP}{AB}=\frac{1}{3}$. If P lies on the line $2x-y+k=0$. find the value of k.

Solution: We have,

$$\frac{AP}{AB}=\frac{1}{3}$$



$$\Rightarrow \frac{AP}{AP=PB}=\frac{1}{3}$$

$$\Rightarrow 3AP=AP+BP$$

$$\Rightarrow 2AP=BP$$

$$\Rightarrow \frac{AP}{BP}=\frac{1}{2}$$

So, P divides AB in the ratio 1 : 2.

$\therefore$ Coordinates of P are $\left(\frac{1\times5+2\times2}{1+2},\frac{1\times-8+2\times1}{1+2}\right)=(3,2)$

Since, P (3, 2) lies on the line $2x-y+k=0$

$\therefore\ 2\times3-2+k=0\Rightarrow k=-4$

21 Find the coordinates of the centroid of a triangle whose vertices are (0, 6), (8, 12) and (8, 0)

Solution: We know that the coordinates of the centroid of a triangle whose angular points are $(x_1,y_1),(x_2,y_2),(x_3,y_3)$ *are*

$$\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

So, the coordinates of the centroid of a triangle whose vertices are (0, 6),(8, 12) and (8, 0) are

$$\left(\frac{0+8+8}{3},\frac{6+12+0}{3}\right) or, \left(\frac{16}{3},6\right)$$

22 If $x-2y+k=0$ is a median of the triangle whose vertices are at points A(-1, 3), B (0, 4) and C (-5, 2) find the value of k.

Solution: The coordinates of the centroid G of ΔABC are

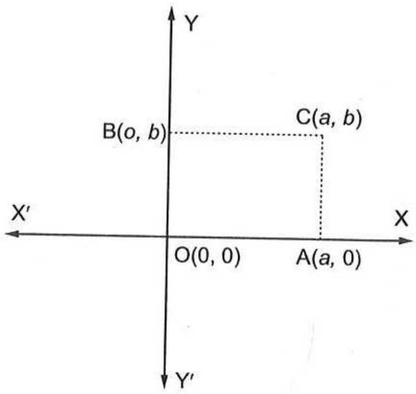
$$\left(\frac{-1+0-5}{3},\frac{3+4+2}{3}\right) i.e. (-2, 3)$$

Since G lies on the median $x-2y+k=0$. So, coordinates of G satisfy its equation.

$\therefore\ -2-6+k=0 \Rightarrow k=8.$

23 Prove that the diagonals of a rectangle bisect each other and are equal.

Solution: Let OACB be a rectangle such that OA is a along $x-$axis and Ob is along $y-$ axis. Let OA = a and OB = b.



Then, the coordinates of A and B are (a, 0) and (0, b) respectively.

Since, OACB is a rectangle. Therefore,

AC = Ob $\Rightarrow$ AC = b

Thus, we have

OA = a and AC = b

So, the coordinates of C are (a, b)

The coordinates of the mid – point of OC are $\left(\frac{a+0}{2},\frac{b+0}{2}\right)=\left(\frac{a}{2},\frac{b}{2}\right)$

Also, the coordinates of the mid – points of AB are $\left(\frac{a+0}{2},\frac{0+b}{2}\right)=\left(\frac{a}{2},\frac{b}{2}\right)$

Clearly, coordinates of the mid – point of OC and AB are same.

Hence, OC and AB bisect each other.

Also, OC = $\sqrt{a^2+b^2}$ *and* $AB=\sqrt{(a-0)^2+(0-b)^2}=\sqrt{a^2+b^2}$

$\therefore \quad OC = AB$

24 The area of a triangle, the coordinates of whose vertices are $(x_1,y_1),(x_2,y_2)$ **and** (x_3,y_3) **is**

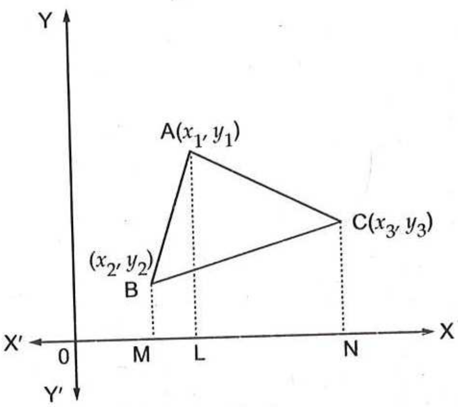
$$\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

Proof Let ABC be a triangle whose vertices are A $(x_1,y_1), B(x_2,y_2)$ *and* $C(x_3,y_3)$. Draw AL, BM and CN perpendiculars from A, B, C on the $x-$ axis.

Clearly, ABML, ALNC and BMNC are all trapeziums.

We know that

Area of trapezium = $\frac{1}{2}$ (sum of parallel sides) (Distance between them)



We have,

Area of $\Delta ABC =$ Area of trapezium ABML + Area of trapezium ALNC

-Area of trapezium BMNC

Let Δ denote the area of ΔABC. Then,

$$\Delta = \frac{1}{2}(BM + AL)\,(ML) + \frac{1}{2}(AL + CN)(LN) - \frac{1}{2}(BM + CN)(MN)$$

$$\Rightarrow \ \Delta = \left|\frac{1}{2}(y_2 - y_1)(x_1 - x_2) + \frac{1}{2}(y_1 + y_3)(x_3 - x_1) - \frac{1}{2}(y_2 + y_3)(x_3 - x_2)\right|$$

$$\Rightarrow \Delta = \left|\frac{1}{2}\{x_1(y_2 + y_1 - y_1 - y_3) + x_2(-y_2 - y_1 + y_2 + y_3) + x_3(y_1 + y_3 - y_2 - y_3)\right|$$

$$\Rightarrow \ \Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

25 Find the area of the triangle formed by the points A (5, 2), B (4, 7) and C (7, -4).

Solution: Here, $x_1 = 5, y_1 = 2, x_2 = 4, y_2 = 7, x_3 = 7$ *and* $y_3 = -4$

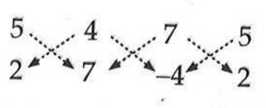
$$\therefore \ \text{Area of } \Delta ABC = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2}\left|5(7+4) + 4(-4-2) + 7(2-7)\right|$$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2}\left|(5\times 11 + 4\times -6 + 7\times -5)\right|$$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2}\left|(55 - 24 - 35)\right| = \frac{1}{2}\left|-4\right| = 2 \ \text{sq. units}$$

ALITER We have,



$$\therefore \ \textit{Area of } \Delta \ ABC = \frac{1}{2}\left|(5\times 7 + 4\times -4 + 7\times 2) - (4\times 2 + 7\times 7 + 5\times -4)\right|$$

$$\Rightarrow \ \textit{Area of } \Delta \ ABC = \frac{1}{2}\left|(35 - 16 + 14) - (8 + 49 - 20)\right|$$

$\Rightarrow Area\ of\ \Delta ABC = \frac{1}{2}|33-(37)| = \frac{1}{2}|-4| = 2$ sq. units.

26 Prove that the area of triangle whose vertices are $(t, t-2), (t+2, t+2)$ and $(t+3, t)$ is independent of t.

Solution: Let A = $(x_1, y_1) = (t, t-2), = (x_2, y_2) = (t+2, t+2)$ and C = $(x_3, y_3) = (t+3, t)$ be the vertices of the given triangle. Then,

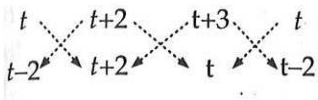
$$\therefore\ Area\ of\ \Delta ABC = \frac{1}{2}|x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)|$$

$$\Rightarrow\ Area\ of\ \Delta ABC = \frac{1}{2}|\{t(t+2-t) + (t+2)(t-t+2) + (t+3)(t-2-t-2)\}|$$

$$\Rightarrow\ Area\ of\ \Delta ABC = \frac{1}{2}|\{(2t+2t+4-4t-12)\}| = |-4| = 4\ sq.\ units$$

Clearly, area of ΔABC is independent of t.

<u>ALITER</u> We have



$$\therefore\ Area\ of\ \Delta ABC = \frac{1}{2}|\{t(t+2) + (t+2)t + (t+3)(t-2)\} - \{t+2)(t-2)|$$

$$\Rightarrow\ Area\ of\ \Delta\ ABC = \frac{1}{2}|(t^2+2t+t^2+2t+t^2+t-6) - (t^2-4+t^2+5t+6+t^2)|$$

$$\Rightarrow\ Area\ of\ \Delta ABC = \frac{1}{2}|(3t^2+5t-6) - (3t^2+5t+2)|$$

$$\Rightarrow Area\ of\ \Delta\ ABc = \frac{1}{2}|-6-2|$$

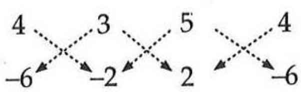
$$\Rightarrow\ Area\ of\ \Delta\ ABC = 4\ sq.\ units$$

Hence, Area of ΔABC is independent of t.

27 If A (4,-6), B (3,-2) and C (5, 2) are the vertices of ΔABC, then verify the fact that a median of a triangle ABC divides it into two triangles of equal areas.

Solution: Let D be the mid – point of BC. Then, the coordinates of D are (4, 0).

We have,



$$\therefore \quad Area\ of\ \Delta ABC = \frac{1}{2}\left|(4\times-2+3\times2+5\times-6)-(3\times-6+5\times-2+4\times2)\right|$$

$$\Rightarrow \quad Area\ of\ \Delta ABC = \frac{1}{2}\left|(-8+6-30)-(-18-10+8)\right|$$

$$\Rightarrow \quad Area\ of\ \Delta ABC = \frac{1}{2}\left|-32+20\right| = 6\ sq.\ units.$$

Also, we have

$$\begin{matrix} 4 & & 3 & & 4 & & 4 \\ & \times & & \times & & \times & \\ -6 & & -2 & & 0 & & -6 \end{matrix}$$

$$\therefore \quad Area\ of\ \Delta ABC = \frac{1}{2}\left|\begin{matrix}\{(4\times(-2)+3\times0+4\times(-6)\} \\ -\{3\times(-6)+4\times(-2)+4\times0\}\end{matrix}\right|$$

$$\Rightarrow \quad Area\ of\ \Delta ABC = \frac{1}{2}\left|(-8+0-24)-(-18-8+0)\right|$$

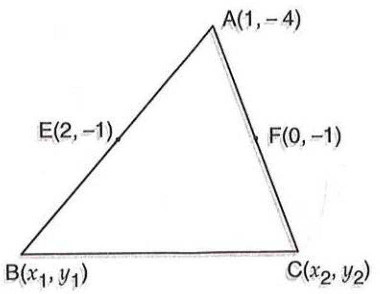
$$\Rightarrow \quad Area\ of\ \Delta ABC = \frac{1}{2}\left|(-32+26\right| = 3\ sq.\ units$$

$$\therefore \quad \frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta ABC} = \frac{6}{3} = \frac{2}{1}$$

$$\Rightarrow \quad Area\ of\ \Delta ABC = 2\ (Area\ of\ \Delta ABD)$$

28 Find the area of the triangle ABC with A (1, -4) and mid – points of sides through A being (2, -1) and (0, -1).

Solution: Let the coordinates of B and C be (x_1, y_1) *and* (x_2, y_2) respectively. It is given that the points E and F are the mid – points of AB and Ac respectively.



$$\frac{x_1+1}{2} = 2, \frac{y_1-4}{2} = -1\ and\ \frac{x_2+1}{2} = 0, \frac{y_2-4}{2} = -1$$

$$\Rightarrow \quad x_1 = 3, y_1 = 2\ and\ x_2 = -1, y_2 = 2$$

Thus, the coordinates of B and C are (3, 2) and (-1, 2) respectively.

$$\begin{matrix} 1 & & 3 & & -1 & & 1 \\ & \times & & \times & & \times & \\ -4 & & 2 & & 2 & & -4 \end{matrix}$$

$$\therefore \; Area \; of \; \Delta ABC = \frac{1}{2}\left|(2+6+4)-(-12-2+2)\right|$$

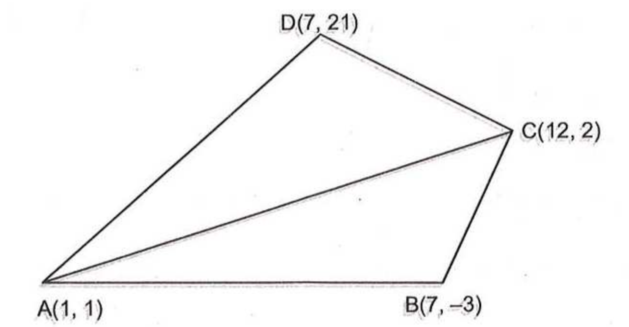
$$= \frac{1}{2}\left|12-(-12)\right| = 12 \; sq. \, units$$

Type IV. On finding the area of a quadrilateral when coordinates of its vertices are given

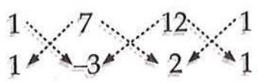
29 Find the area of the quadrilateral ABCD whose vertices are respectively A (1, 1), B (7, -3) C (12, 2) and D (7,21).

Solution: We have,

Area of quadrilateral ABCD $= \left|Area \; of \; \Delta ABC\right| + \left|Area \; of \; \Delta ACD\right|$



We have,

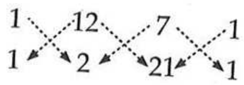


$$\therefore \; Area \; of \; \Delta ABC = \frac{1}{2}\left|(1\times -3+7\times 2+12\times 1)-(7\times 1+12\times(-3)+1\times 2)\right|$$

$$\Rightarrow \; Area \; of \; \Delta \; ABC = \frac{1}{2}\left|(-3+14+12)-(7-36+2)\right|$$

$$\Rightarrow \; Area \; of \; \Delta ABC = \frac{1}{2}\left|23+27\right| = 25 \; sq. \, units.$$

Also, we have



$\therefore$ *Area of* $\Delta ACD = \frac{1}{2}\left|(1\times 2 + 12\times 21 + 7\times 1) - (12\times 1 + 7\times 2 + 1\times 21)\right|$

$\Rightarrow$ *Area of* $\Delta ACD = \frac{1}{2}\left|(2 + 252 + 7) - (12 + 14 + 21)\right|$

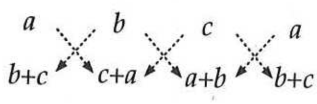
$\Rightarrow$ *Area of* $\Delta ACD = \frac{1}{2}\left|261 - 47\right| = 107$ *sq. units.*

$\therefore$ Area of quadrilateral ABCD = 25+107=132 sq. units.

30 Prove that the points $(a, b+c), (b, c, +a)$ *and* $(c, a+b)$ **are collinear.**

Solution: Let Δ be the area of the triangle formed by the points $(a, b+c), (b, c, +a)$ *and* $(c, a+b)$

We have



$\therefore \Delta = \frac{1}{2}\left|\{a(c+a) + b(a+b) + c(b+c)\} - \{b(b+c) + c(c+a) + a(a+b)\}\right|$

$\Rightarrow \Delta = \frac{1}{2}\left|(ac + a^2 + ab + b^2 + bc + c^2) - (b^2 + bc + c^2 + ca + a^2 + ab)\right|$

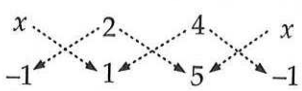
$\Rightarrow \Delta = 0$

Hence, the given points are collinear.

31 For what value of x **will the points** $(x, -1), (2, 1)$ **and (4, 5) lie on a line ?**

Solution: Given pints will be collinear if the area of the triangle formed by them is zero.

We have,



$\therefore$ *Area of the triangle* $= 0$

$\Rightarrow \left|\{x\times 1 + 2\times 5 + 4\times(-1)\} - \{(2\times -1 + 4\times 1 + x\times 5\}\right| = 0$

$\Rightarrow (x + 10 - 4) - (-2 + 4 + 5x) = 0$

$\Rightarrow (x+6) - (5x+2) = 0$

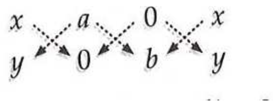
$\Rightarrow -4x + 4 = 0$

$\Rightarrow x = 1$

Hence, the given points lie on a line, if $x = 1$.

32 If $P(x,y)$ is any point on the line joining the points A $(a,0)$ and B (0, b), then show that $\frac{x}{a}+\frac{y}{b}=1.$

Solution: It is given that the point $P(x,y)$ lies on the line segment joining points A $(a,0)$ and B (0, b). Therefore, points $P(x,y)$ A $(a,0)$ and B (0, b) are collinear points.



$\therefore\ (x\times0+a\times b+0\times y)-(a\times y+0\times0+x\times b)=0$

$\Rightarrow\ ab-(ay+bx)=0$

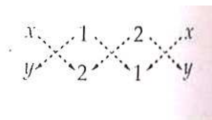
$\Rightarrow\ ab=ay+bx$ (Dividing throughout by ab)

$\Rightarrow\ \frac{ab}{ab}=\frac{ay}{ab}+\frac{bx}{ab}$

$\Rightarrow\ 1=\frac{y}{b}+\frac{x}{a}\ or\ \frac{x}{a}+\frac{y}{b}=1.$

33 If the area of ΔABC formed by A $(x,y),B(1,2)\ and\ C(2,1)$ is 6 square units, then prove that $x+y=15\ or\ ,x+y+9=0.$

Solution: We have,



Area of $\Delta ABC=6$

$\Rightarrow\ \frac{1}{2}\left|(2x+1+2y)-(x+4+y)\right|=6$

$\Rightarrow\left|x+y-3\right|=12$

$\Rightarrow x+y-3=\pm12$

$\Rightarrow x+y-15=0\ or,\ x+y+9=0$

$\Rightarrow x+y=15\ or,x+y+9=0$