Example – 1 **Write the first five terms of the sequence defined by** $a_n = (-1)^{n-1}.2^n$

Sol: We have, $a_n = (-1)^{n-1}.2^n$

Putting n =1, 2, 3, 4, and 5, we get

$$a_1 = (-1)^{1-1} \times 2^1 = (-1)^0 \times 2 = 2$$
$$a_2 = (-1)^{2-1} \times 2^2 = (-1)^1 \times 4 = -4$$
$$a_3 = (-1)^{3-1} \times 2^3 = (-1)^2 \times 8 = 8$$
$$a_4 = (-1)^{4-1} \times 2^4 = (-1)^3 \times 16 = -16.$$

and $a_5 = (-1)^{5-1} \times 2^5 = (-1)^4 \times 32 = 32$

Example – 2 **What is 18th term of the sequence defined by** $a_n = \dfrac{n(n-3)}{n+4}$

Sol : We have, $a_n = \dfrac{n(n-3)}{n+4}$

Putting n = 18, we get

$$a_{18} = \frac{18 \times (18-3)}{18+4} = \frac{18 \times 15}{22} = \frac{135}{11}$$

Example -3 **Show that the sequence defined by** $a_n = 4n+5$ **is an A.P. Also, find its common difference.**

Sol : We have, $a_n = 4n+5$

Replacing n *by* $(n+1)$, we get

$$a_{n+1} = 4(n+1)+5 = 4n+9$$

Now , $a_{n+1} - a_n = (4n+9)-(4n+5) = 4$

Clearly, $a_{n+1} - a_n$ is independent of n and is equal to 4.

So, the given sequence is an A.P. with common difference 4.

Example – 4 **Write an A.P. whose first term is 10 and common difference is 3.**

Sol : We know that if a is the first term and d is the common difference, then the arithmetic progression is

a, a + d, a + 2d, a + 3d, ...

Here, a =10 and d= 3.

So, the arithmetic progression is 10, 13, 16, 19, 22, ...

Example – 5 Write an A.P. having 4 as the first term and -3 as the common difference.

Sol : The arithmetic progression with first term a and common difference d is given by

a, a +d, a + 2d, a + 3d, ...

i.e., each term is obtained by adding 'd' to the preceding term.

Here, a =4 and d = -3.

So, the arithmetic progression is

4, 4 + (-3), 4+2 x (-3), 4 +3 (-3), 4 + 4 (-3) , ...

or, 4, 1, -2, -5, -8, ...

Example – 6 Find the 12th, 24th and nth term of the A.P. given by 9, 13, 17, 21, 25, ...

Sol : We have, a = First term = 9

and, d = Common difference = 4 $[\because 13-9=4, 17-13=4, 21-17=4\ etc.]$ We know that the nth term of an A.P. with first term a and common difference d is given by

$$a_n = a+(n-1)d$$

$$\therefore \quad a_{12} = a+(12-1)d \ = a+11d = 9+11\times 4 = 53$$

$$a_{24} = a+(24-1)d \ = a+23d = 9+23\times 4 = 101$$

and, $a_n = a+(n-1)d = 9+(n-1)\times 4 = 4n+5$

Thus, we have

$$a_{12} = 53, a_{24} = 101\ and\ a_n = 4n+5$$

Example – 7 Show that the sequence 9, 12, 15, 18, ... is an A.P. Find its 16th term and the general term.

Sol: We have,

(12 – 9) = (15 – 12) = (18 – 15) = 3

Therefore, the given sequence is an A.P. with common difference 3.

a = First term = 9

$\therefore$ $16th\ term = a_{16} = a + (16-1)d = a + 15d$ $[\because a_n = a + (n-1)d]$

$\Rightarrow$ $a_{16} = 9 + 15 \times 3 = 54$

$\because$ $General\ term = nth\ term = a + (n-1)d$

$\therefore$ $a_n = 9 + (n-1) \times 3 = 3n + 6$

Example – 8 Which term of the sequence 4, 9, 14, 19, ... is 124 ?

Sol : Clearly, the given sequence is an A.P. with first term a (=4) and common difference d (=5)

Let 124 be the nth term of the given sequence. Then,

$a_n = 124 \Rightarrow a + (n-1)d = 124 \Rightarrow 4 + (n-1) \times 5 = 124 \Rightarrow 5n - 1 = 124 \Rightarrow 5n = 125 \Rightarrow n = 25$

Hence, 25th term of the given sequence is 124.

Example – 9 How many terms are there in the sequence 3, 6, 9, 12, ..., 111 ?

Sol : Clearly, the given sequence is an A.P. with first term a =3 and common difference d = 3. Let there be n terms in the given sequence. Then,

nth term = 111

$\Rightarrow$ $a + (n-1)d = 111$

$\Rightarrow$ $3 + (n-1) \times 3 = 111 \Rightarrow n = 37$

Thus, the given sequence contains 37 terms.

Example – 10 Find the middle term (s) of the A.P. 7, 13, 19, ..., 241.

Sol : Clearly, 7, 13, 19, ..., 241 is an A.P. with first term a = 7 and common differenced d = 6. Let there be n terms in the A.P. Then,

$a_n = 241$

$\Rightarrow$ $a + (n-1)d = 241$

$\Rightarrow 7 + 6(n-1) = 241$

$\Rightarrow 6n = 240 \Rightarrow n = 40$

Clearly, n is even. So, $\left(\frac{n}{2}\right)^{th} = 20^{th}\ and\ \left(\frac{n}{2}+1\right)^{th} = 21^{th}$ are middle terms and are given by

$a_{20} = a + (20-1)d = a + 19d = 7 + 19 \times 6 = 121$

and $a_{21} = a + (21-1)d = a + 20d = 7 + 20 \times 6 = 127$

Example – 11 If the 8th term of an A.P. is 31 and the 15th term is 16 more than the 11th term, find the A.P.

Sol : Let a be the first term and d be the common difference of the A.P.

We have,

$a_8 = 31 \; and \; a_{15} = 16 + a_{11}$

$\Rightarrow \; a + 7d = 31 \; and \; a + 14d = 16 + a + 10d$

$\Rightarrow a + 7d = 31 \; and \; 4d = 16$

$\Rightarrow \; a + 7d = 31 \; and \; d = 4$

$\Rightarrow \; a + 7 \times 4 = 31 \Rightarrow a + 28 = 31 \Rightarrow a = 3$

Hence, the A.P. is $a, a + d, a + 2d, a + 3d$, ... i.e., 3, 7, 11, 15, 19, ...

Example – 12 Which term of the arithmetic progression 5, 15, 25, ... will be 130 more than its 31st term?

Sol : We have, a = 5 and d = 10

$\therefore \; a_{31} = a + 30d = 5 + 30 \times 10 = 305$

Let nth term of the given A.P. be 130 more than its 31st term. Then,

$a_n = 130 + a_{31}$

$\therefore \; a + (n-1)d = 130 + 305$

$\Rightarrow 5 + 10(n-1) = 435$

$\Rightarrow 10(n-1) = 430$

$\Rightarrow n - 1 = 43$

$\Rightarrow n = 44$

Hence, 44th term of the given A.P. is 130 more than its 31st term.

Example – 13 Is 184 a term of the sequence 3, 7, 11, ...?

Sol : Clearly, the given sequence is an A.P. with first term a (=3) and common difference d (=4).

Let the nth term of the given sequence be 184. Then,

$$a_n = 184$$

$$\Rightarrow \; a + (n-1)d = 184$$

$$\Rightarrow \; 3 + (n-1)\times 4 = 184 \Rightarrow 4n = 185 \Rightarrow n = 46\frac{1}{4}$$

Since n is not a natural number. So, 184 is not a term of the given sequence.

Example – 14 If five times the fifth term of an A.P. is equal to 8 times its eighth term, show that its 13^{th} term is zero.

Sol : Let $a_1, a_2, a_3, ...a_n, ...$ be the A.P. with its first term a and common difference d. It is given that

$$5a_5 = 8a_8$$

$$\Rightarrow \; 5(a+4d) = 8(a+7d)$$

$$\Rightarrow \; 5a + 20d = 8a + 56d$$

$$\Rightarrow \; 3a + 36d = 0$$

$$\Rightarrow \; 3(a+12d) = 0$$

$$\Rightarrow \; a + 12d = 0 \Rightarrow a + (13-1)d = 0 \Rightarrow a_{13} = 0$$

Hence, 13^{th} term is zero.

Example – 15 If the m^{th} term of an A.P. be 1/ n and n^{th} term be 1/m, then show that its $(mn)^{th}$ term is 1.

Sol : Let a and d be the first term and common difference respectively of the given A.P. Then.

$$\frac{1}{n} = m\,th\,term \Rightarrow \frac{1}{n} = a + (m-1)d$$

$$\frac{1}{m} = n\,th\,term \Rightarrow \frac{1}{m} = a + (n-1)d$$

On subtracting equation (ii) from equation (i), we get

$$\frac{1}{n}-\frac{1}{m}=(m-n)d \Rightarrow \frac{m-n}{mn}=(m-n)d \Rightarrow d=\frac{1}{m}$$

Putting d= $\frac{1}{mn}$ in equation (i), we get

$$\frac{1}{n}=a+\frac{(m-1)}{mn} \Rightarrow \frac{1}{n}=a+\frac{1}{n}-\frac{1}{mn} \Rightarrow a=\frac{1}{mn}$$

$$\therefore \quad (mn)th\ term=a+(mn-1)d=\frac{1}{mn}+(mn-1)\frac{1}{mn}=1 \quad \left[\because a=\frac{1}{mn}=d\right]$$

Example – 16 The sum of three numbers in A.P. is -3, and their product is 8. Find the numbers.

Sol : Let the numbers be $(a-d), a, (a+d)$. It is given that the sum of the numbers is -3.

$\therefore\ (a-d)+a+(a+d)=-3 \Rightarrow 3a=-3 \Rightarrow a=-1$

It is also given that the product of the product of the numbers is 8.

$$\therefore \quad (a-d)\,(a)\,(a+d)=8$$
$$\Rightarrow a(a^2-d^2)=8$$
$$\Rightarrow (-1)\,(1-d^2)=8 \qquad [\because a=-1]$$
$$\Rightarrow d^2=9 \Rightarrow d=\pm 3$$

If d =3, the numbers are -4, -1, 2. If d = -3, the numbers are 2, -1, -4.

Thus, the numbers are -4, -1, 2, or 2, -1, -4.

Example – 17 Find four numbers in A.P. whose sum is 20 and the sum of whose squares is 120.

Sol : Let the numbers be $(a-3d),(a-d),(a+d),(a+3d).\ Then,$

Sum of numbers = 20

$\Rightarrow\ (a-3d)+(a-d)+(a+d)+(a+3d)=20 \Rightarrow 4a=20 \Rightarrow a=5$

It is given that, sum of the squares = 120

$$\Rightarrow\ (a-3d)^2+(a-d)^2+(a+d)^2+(a+3d)^2=120$$
$$\Rightarrow 4a^2+20d^2=120$$
$$\Rightarrow a^2+5d^2=30$$
$$\Rightarrow 25+5d^2=30$$
$$\Rightarrow 5d^2=5 \Rightarrow d=\pm 1$$

If d=1, then the numbers are 2, 4, 6, 8. If d = -1, then the numbers are 8, 6, 4, 2.

Thus, the numbers are 2, 4, 6, 8 or 8, 6, 4, 2.

Example – 18 Divide 32 into four parts which are in A.P. such that the product of extremes is to the product of means is 7:15.

Sol : Let the four parts be $(a-3d),(a-d),(a+d)$ *and* $(a+3d)$. Then,

Sum of the numbers = 32

$\Rightarrow (a-3d)+(a-d)+(a+d)+(a+3d)=32 \Rightarrow 4a=32 \Rightarrow a=8$

It is given that

$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)}=\frac{7}{15}$$

$$\Rightarrow \frac{a^2-9d^2}{a^2-d^2}=\frac{7}{15}$$

$$\Rightarrow \frac{64-9d^2}{64-d^2}=\frac{7}{15} \Rightarrow 128d^2=512 \Rightarrow d^2=4 \Rightarrow d=\pm 2$$

Thus, the four parts are $a-3d, a-d, a+d$ *and* $a+3d$ *i.e.,* 2,6,10,14.

Example – 19 Determine k so that $k^2+4k+8, 2k^2+3k+6, 3k^2+4k+4$ are three consecutive terms of an A.P.

Sol : We now that if a, b, c are three consecutive terms of an A.P., then

$b-a=c-b$ *i.e.* $2b=a+c$

Thus, if $k^2+4k+8, 2k^2+3k+6$ *and* $3k^2+4k+4$ are three consecutive terms of an A.P. then

$$2(2k^2+3k+6)=(k^2+4k+8)+(3k^2+4k+4)$$

$\Rightarrow 4k^2+6k+12=4k^2+8k+12$

$\Rightarrow 2k=0 \Rightarrow k=0.$

Example – 20 Find the sum of 20 terms of the A.P. 1, 4, 7, 10 ...

Sol : Let a be the first term and d be the common difference of the given A.P. Then, we have

$a = 1$ and d = 3.

We have to find the sum of 20 terms of the given A.P.

Putting a =1, d = 3, n = 20 in $S_n = \frac{n}{2}\{2a + (n-1)d\}$, we get

$$S_{20} = \frac{20}{2}\{2 \times 1 + (20-1) \times 3\} = 10 \times 59 = 590$$

Example – 21 Find the sum of first 30 terms of an A.P. whose second term is 2 and seventh term is 22.

Sol : Let a be the first term and d be the common difference of the given A.P. Then,

$$a_2 = 2 \text{ and } a_7 = 22$$

$$\Rightarrow a + d = 2 \text{ and } a + 6d = 22$$

Solving these two equations, we get a =-2 and d = 4. Putting n = 30, a =-2 and d = 4 in $S_n = \frac{n}{2}\{20 + (n-1)d\}$, we obtain

$$\therefore S_{30} = \frac{30}{2}\{2 \times (-2) + (30-1) \times 4\}$$

$$\Rightarrow S_{30} = 15(-4 + 116) = 15 \times 112 = 1680$$

Hence, the sum of first 30 terms is 1680.

Example – 22 Find the sum of all natural numbers between 250 and 1000 which are exactly divisible by 3.

Sol : Clearly, the numbers between 250 and 1000 which are divisible by 3 are 252, 255, 258, ..., 999. This is an A.P. with first term a = 252, Common difference = 3 and last term = 999. Let there be n terms in this A.P. then,

$$a_n = 999$$

$\Rightarrow\ a+(n-1)d = 999$

$\Rightarrow\ 252+(n-1)\times 3 = 999$

$\Rightarrow\ n = 250$

$\therefore$ Required sum $= S_n = \frac{n}{2}(a+l) = \frac{250}{2}(252+999) = 156375$

Example – 23 If the sum of m terms of an A.P. is the same as the sum of its n terms, show that the sum of its (m=n) terms is zero.

Sol : Let a be the first term and d be the common difference of the given A.P. Then, $S_m = S_n$

$$\Rightarrow \frac{m}{n}\{2a+(m-1)d\} = \frac{n}{2}\{2a+(n-1)d\}$$

$$\Rightarrow 2a(m-n)+\{m(m-1)-n(n-1)\}d = 0$$

$$\Rightarrow 2a(m-n)+\{m^2-n^2)-(m-n)\}d = 0$$

$$\Rightarrow (m-n)\{2a+(m+n-1)d\} = 0$$

$$\Rightarrow 2a+(m+n-1)d = 0 \qquad [\because m-n \neq 0]$$

Now,

$$S_{m+n} = \frac{m+n}{2}\{2a+(m+n-1)d\} = \frac{m+n}{2}\times 0 = 0$$

Example – 24 The ratio of the sum of n terms of two A.P's is (7n+1) : (4n+27). Find the ratio of their m^th terms.

Sol : Let a_1, a_2 be the first terms and d_1, d_2 the common differences of the two given A.P.'s. Then, the sums of their n terms are given by

$$S_n = \frac{n}{2}\{2a_1+(n-1)d_1\} \text{ and , } S_n' = \frac{n}{2}\{2a_2+(n-1)d_2\}$$

$$\therefore \frac{S_n}{S_n'} = \frac{\frac{n}{2}\{2a_1+(n-1)d_1\}}{\frac{n}{2}\{2a_2+(n-1)d_2\}} = \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2}$$

It is given that

$$\frac{S_n}{S_n'} = \frac{7n+1}{4n+27}$$

$$\Rightarrow \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{7n+1}{4n+27}$$

To find the ratio of the m^{th} terms of the two given A.P.'s, we replace n by (2m-1) in equation (i).

Replacing n by (2m-1) in equation (i), we get

$$\therefore \quad \frac{2a_1+(2m-2)d_1}{2a_2+(2m-2)d_2}=\frac{7(2m-1)+1}{4(2m-1)+27}$$

$$\Rightarrow \frac{a_1+(m-1)d_1}{a_2+(m-1)d_2}=\frac{14m-6}{8m+23}$$

Hence, the ratio of the m^{th} terms of the two A.P.'s is (14m-6) : (8m+23).

Example – 25 The ratio of the sums of m and n terms of an A.P. is $m^2 : n^2$. Show that the ratio of the m^{th} and n^{th} terms is (2m-1) : (2n-1).

Sol : Let a be the first term and d the common difference of the given A>p. Then, the sums of m and n terms are given by

$$S_m=\frac{m}{2}\{2a+(m-1)d\}\ and,\ S_n=\frac{n}{2}\{2a+(n-1)d\} \text{ respectively.}$$

Then,

$$\frac{S_m}{S_n}=\frac{m^2}{n^2}$$

$$\Rightarrow \frac{\frac{m}{2}\{2a+(m-1)d\}}{\frac{n}{2}\{2a+(n-1)d\}}=\frac{m^2}{n^2}$$

$$\Rightarrow \frac{2a+(m-1)d}{2a+(n-1)d}=\frac{m}{n}$$

$$\Rightarrow \{2a+(m-1)d\}n=\{2a+(n-1)d\}m$$

$$\Rightarrow 2a(n-m)=d(n-m)$$

$$\Rightarrow d=2a$$

$$\therefore \frac{T_m}{T_n}=\frac{a+(m-1)d}{a+(n-1)d}=\frac{a+(m-1)2a}{a+(n-1)2a}=\frac{2m-1}{2n-1}$$

Example – 26 **Solve the equation : 1+4+7+10+.....+** $x = 287$.

Sol : Here, 1, 4, 7, 10, ... x, is an A.P. with first term a =1 and common difference d = 3. Let there the n terms in the A.P. Then, $x = n^{th}\ term \Rightarrow x = 1 + (n-1)\times 3 = 3n - 2$

Now,

$$1+4+7+10+....+x = 287$$

$$\Rightarrow \frac{n}{2}(1+x) = 287 \qquad \left[Using\ S_n = \frac{n}{2}(a+l)\right]$$

$$\Rightarrow \frac{n}{2}(1+3n-2) = 287$$

$$\Rightarrow 3n^2 - n = 574 \Rightarrow 3n^2 - n - 574 = 0 \Rightarrow 3n^2 - 42n + 41n - 574 = 0$$

$$\Rightarrow 3n(n-14) + 41(n-14) = 0$$

$$\Rightarrow (n-14)(3n+41) = 0 \Rightarrow n - 14 = 0 \qquad [\because 3n + 4 \neq 0]$$

$$\Rightarrow n = 14$$

Putting n = 14 in (i), we get $x = 3\times 14 - 2 = 40$.

Example – 27 **A man repays a long of ₹3250 by paying ₹ 20 in the first month and then increases the payment by ₹15 every month. How long will it take him to clear the loan ?**

Sol : Suppose the loan is cleared in n months. Clearly, the amounts form an A.P. with first term 20 and the common difference 15.

$\therefore$ Sum of the amounts = 3250

$$\Rightarrow \frac{n}{2}\{2\times 20 + (n-1)\times 15\} = 3250$$

$$\Rightarrow \frac{n}{2}(40 + 15n - 15) = 3250$$

$$\Rightarrow n(15n + 25) = 6500$$

$$\Rightarrow 15n^2 + 25n - 6500 = 0$$

$$\Rightarrow 3n^2 + 5n - 1300 = 0$$

$$\Rightarrow (n-20)(3n+65) = 0$$

$$\Rightarrow n = 20\ or,\ n = -\frac{65}{3} \Rightarrow n = 20 \qquad \left[\because n \neq -\frac{65}{3}\right]$$

Thus, the loan is cleared in 20 months.