# PROGRESSIONS

## Key Points:

1. An arithmetic progression (AP) is a list of numbers in which each term is obtained by adding a fixed number d to the preceding term. Except the first term the fixed number d is called **The Common Difference**.

   The terms of AP are $a, a+d, a+2d, a+3d, \ldots$

2. A given list of numbers $a_1, a_2, a_3, \ldots$ *is* an AP, if the differences $a_2 - a_1, a_3 - a_2, a_4 - a_3, \ldots$ give the same value, i.e., if $a_{k+1} - ak$ is the same for different values of k.

3. In an AP with first term a and common difference d, the nth term (or the general term) is given by $a_n = a + (n-1)d$.

4. The sum of the first n terms of an AP is given by:

   $$S = \frac{n}{2}[2a + (n-1)d]$$

5. If $l$ is the last term of the finite AP, say the nth term, then the sum of all terms of the AP is given by :

   $$S = \frac{n}{2}(a + l)$$

Consider the following lists of numbers:

(i) 1, 2, 3, 4,.... (ii) 100, 70, 40, 10....

(iii) -3, -2, -1, 0,..... (iv) 3, 3, 3, 3...

(v) -1.0, - 1.5, - 2.0, - 2.5...

Each of the numbers in the list is called a term.

In (i) each term is 1 more than the term preceding it.

In (ii) each term is 30 less than the term preceding it.

In (iii) each term is obtained by adding 1 to the term preceding it.

In (iv) all the terms in the list are 3, i.e., each term is obtained by adding (or subtracting) 0 to the term preceding it.

In (v), each term is obtained by adding – 0.5 to (i.e. subtracting 0.5 from) the term preceding it.

In all the lists above, we can observe that successive terms are obtained by adding or subtracting a fixed number to the preceding terms. Such list of numbers is said to form an **Arithmetic Progression (A.P)**

### What is an arithmetic progression?

We observe that an arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.

This fixed number is called the common difference of the A.P. Remember that it can be positive, negative or zero.

Let us denote the first term of an AP by $a_1$, second term by $a_2$,...., nth term by $a_n$ and the common difference by d. Then the AP becomes $a_1$, $a_2$, $a_3$, ...., $a_n$.

So, $a_2$, - $a_1 = a_3 - a_2 = \ldots = a_n - a_{n-1} = d$.

**General form of AP :** Can you see that all AP's can be written as.

A, a + d, a + 2d, a + 3d, ....

This is called general form of an A.P. where 'a' is the first term and 'd' is the common difference

For example in 1, 2, 3, 4, 5 ....

The first terms are 1 and the common difference is also 1.

**Example – 1. For the AP : $\frac{1}{4}, \frac{-1}{4}, \frac{-3}{4}, \frac{-5}{4}$...., write the first term a and the common difference d. And find the 7th term**

**Solution:** Here, $a = \frac{1}{4}; d = \frac{-1}{4} - \frac{1}{4} = \frac{-1}{2}$

Remember that we can find d using any two consecutive terms, once we know that the numbers are in AP.

The seventh term would be $\frac{-5}{4} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} = \frac{-11}{4}$

**Example -2: Which of the following forms an AP? If they form AP then write next two terms ?**

**(i) 4, 10, 16, 22, .... (ii) 1, -1, -3, -5, .... (iii) -2, 2, -2, 2, -2, .....**

**Solution:** (i) We have $a_2 - a_1 = 10 - 4 = 6$

$a_3 - a_2 = 16 - 10 = 6$

$a_4 - a_3 = 22 - 16 = 6$

i.e., $a_{k+1} - ak$ is same every time.

So, the given list of numbers forms an AP with the common difference d = 6.

The next two terms are : 22 + 6 = 28 and 28 + 6 = 34.

(ii) $a_2 - a_1 = -1 - 1 = -2$

$a_3 - a_2 = -3 - (-1) = -3 + 1 = -2$

$a_4 - a_3 = -5 - (-3) = -5 + 3 = -2$

i.e., $a_{k+1} - ak$ is same every time.

So, the given list of numbers forms an AP with the common difference d= -2.

The next two terms are:

-5+(-2)= -7 and -7 + (-2) = -9

(iii) $a_2 - a_1 = 2 - (-2) = 2 + 2 = 4$

$a_3 - a_2 = -2 - 2 = -4$

## $n^{th}$ Term of an Arithmetic Progression

So the $n^{th}$ term an of the AP with first term a and common difference d is given by $a_n = a + (n-1)d$.

$a_n$ is also called the general term of the AP.

If there are m terms in the AP, then $a_m$ represents the last term which is sometimes also denoted by $l$.

**Example: 3 Find the $10^{th}$ term of the AP : 5, 1, -3, - 7 ...**

**Solution:** Here, a = 5, d = 1 – 5 = -4 and n = 10.

We have $a_n = a + (n-1)d$

So, $a_{10} = 5 + (10-1)(-4) = 5 - 36 = -31$

Therefore, the $10^{th}$ term of the given AP is – 31.

**Example: 4 Which term of the AP : 21, 18, 15, .... is – 81 ?**

Is there any term 0 ? Give reason for your answer.

**Solution:** Here, $a = 21, d = 18 - 21 = -3$ *and if* $a_n = -81,$ we have to find n.

As $a_n = a + (n-1)d.$

We have $-81 = 21 + (n-1)(-3)$

$-81 = 24 - 3n$

$-105 = -3n$

So, $n = 35$

Therefore, the $35^{th}$ term of the given AP is -81.

Next, we want to know if there is any n for which $a_n = 0.$ If such an n is there, then

$21 + (n-1)(-3) = 0.$

i.e., $3(n-1) = 21$

i.e., $n = 8$

So, the eighth term is 0.

**Example – 5 : Determine the AP whose 3rd term is 5 and the 7th term is 9.**

**Solution:** We have

$$a_3 = a + (3-1)d = a + 2d = 5$$

and $$a_7 = a + (7-1)d = a + 6d = 9$$

Solving the pair of linear equations (1) and (2), we get

$a = 3, d = 1$

Hence, the required AP is 3, 4, 5, 6, 7, .....

**Example: 6 How many two – digit numbers are divisible by 3 ?**

**Solution:** The list of two – digit numbers divisible by 3 is:

12, 15, 18 ,..... 99

Is this an AP ? Yes it is. Here, $a = 12, d = 3, a_n = 99.$

As $a_n = a + (n-1)d,$

We have $99 = 12 + (n-1) \times 3$

i.e., $87 = (n-1) \times 3$

i.e., $n - 1 = \frac{87}{3} = 29$

i.e., $n = 29 + 1 = 30$

So, there are 30 two – digit numbers divisible by 3.

**Example: 7 Find the 11th term from the last of the AP series given below :**

**AP : 10, 7, 4, .... – 62.**

**Solution:** Here, $a = 10, d = 7 - 10 = -3, l = -62.$

Where $l = a + (n-1)d$

To find the 11th term from the last term. We will find the total number of terms in the AP.

So, $-62 = 10 + (n-1)(-3)$

i.e., $-72 = (n-1)(-3)$

i.e., $n - 1 = 24.$

or $n = 25$

So, there are 25 terms in the given AP.

The 11th term from the last will be the 15th term of the series. (Note that it will not be the 14th term. Why ? )

$an = a + (n-1)d$

So, $a_{15} = 10 + (15-1)(-3) = 10 - 42 = -32$

i.e. the 11$^{th}$ term from the end is – 32.

**Note:** The 11$^{th}$ term from the last is also equal to 11$^{th}$ term of the AP with first term – 62 and the common difference 3.

**Example – 8 : In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed ?**

**Solution:** The number of rose plants in the 1$^{st}$, 2$^{nd}$, 3$^{rd}$ .... rows are :

23, 21, 19, …., 5

Let the number of rows in the flower bed be n.

Then a= 23, d = 21 – 23 = - 2, $a_n$ = 5

As, $a_n$ + a+ (n-1)d

We have, 5 = 23 + (n-1) (-2)

i.e., - 18 = (n-1) (-2)

i.e., n= 10

so, there are 10 rows in the flower bed.

### Sum of n Terms of an AP

We will now use the same technique that was used by transs to find the sum of the first n terms of an AP :

a, a+d, a + 2d, ….

The nth term of this AP is a + (n-1) d.

Let $S_n$ denote the sum of the first n terms of the A.P. Whose n$^{th}$ term is

$a_n$ = a + (n-1) d

$\therefore S_n = a + (a+d) + (a+2d) + .... + (a+(n-1)d)$

$S_n = (a+(n-1)d) + (a+(n-2)d + ... + a$

$Adding\ \ 2S_n = (2a+(n-1)d) + (2a+(n-1)d) + .... + (2a+(n-1)d)(n\ times)$

$= n(2a+(n-1)d)$

$\therefore S_n = \frac{n}{2}[2a+(n-1)d] = \frac{n}{2}[a+a+(n-1)d] = \frac{n}{2}[\,first\ term + nth\ term] = \frac{n}{2}(a+a_n)$

$S_n = \frac{n}{2}(a+a_n)$ is

If the first and last term of an A.P. are given and the common difference is not given then very useful to find $S_n$.

**Example – 9: If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20th term.**

**Solution:** Here, $S_n = 1050; n = 14, a = 10$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$1050 = \frac{14}{2}[2a+13d] = 140 + 91d$$

$$910 = 91d$$

$$\therefore d = 10$$

$$\therefore a_{20} = 10 + (20-1)10 = 200$$

**Example – 10 : How many terms of the AP : 24, 21, 18,… must be taken so that their sum is 78?**

**Solution:** Here, a = 24, d = 21 – 24 = -3, Sn = 78. We need to find n.

We know that $S_n = \frac{n}{2}[2a+(n-1)d]$

So, $78 = \frac{n}{2}[48+(n-1(-3)] = \frac{n}{2}[51-3n]$

or $3n^2 - 51n + 156 = 0$

or $n^2 - 17n + 52 = 0$

or $(n-4)(n-13) = 0$

or n = 4 or 13

**Example – 11: Find the sum of :**

**(i) The first 1000 positive integers** **(ii) The first n positive integers**

**Solution:**

(i) Let S = 1 + 2 + 3 + …. + 1000

Using the formula $S_n = \frac{n}{2}(a+l)$ for the sum of the first n terms of an AP, w have

$$S_{1000} = \frac{1000}{2}(1+1000) = 500 \times 1001 = 500500$$

So, the sum of the first 1000 positive integers is 500500.

(ii) Let $S_n = 1+2+3+....+n$

Here a = 1 and the last term $l$ *is* $n$.

Therefore, $S_n = \frac{n(1+n)}{2}$ $(or)$ $S_n = \frac{n(n+1)}{2}$

So, the sum of first n positive integers is given by

$$S_n = \frac{n(n+1)}{2}$$

**Example – 12 : Find the sum of first 24 terms of the list of numbers whose nth term is given by**

$$a_n = 3+2n$$

**Solution:** As $a_n = 3+2n$

So, $a_1 = 3+2 = 5$

$a_2 = 3+2\times 2 = 7$

$a_3 = 3+2\times 3 = 9$

List of numbers becomes 5, 7, 9, 11, ….

Here, 7-5=9-7=11-9=2 and so on.

So, it forms an AP with common difference d = 2.

To find $S_{24}, we\ have\ n = 24, a = 5, d = 2.$

Therefore $S_{24} = \frac{24}{2}[2\times5+(24-1)\times2] = 12(0+46) = 672$

So, sum of first 24 terms of the list of numbers is 672.

$\therefore A$ list of numbers $a_1, a_2, a_3, \ldots. a_n \ldots is$ called a geometric progression (GP), if each term is non zero and

$$\frac{a_n}{a_{n-1}} = r$$

Where n is a natural number and n $\geq$ 2.

**Example – 13. Write the GP. If the first term a=3, and the common ratio r= 2.**

**Solution:** Since 'a' is the first term it can easily be written

We know that in GP. Every succeeding term is obtained by multiplying the preceding term with common ratio 'r' . So to get the second term we have to multiply the first term a =3 by the common ratio r = 2.

$\therefore$ Second term = ar = 3 x 2 = 6

Similarly the third term = second term x common ratio

= 6 x 2 = 12

If we proceed in this way we get the following GP.

3, 6, 12, 24, …..

**Example – 14. Write GP. If a = 256, r = $\frac{-1}{2}$**

**Solution:** General form of GP = $a, ar, ar^2, ar^3, \ldots..$

$$= 256, 256\left(\frac{-1}{2}\right), 256\left(\frac{-1}{2}\right)^2, 256\left(\frac{-1}{2}\right)^3$$

$$= 256, -128, 64, -32\ldots..$$

**Example – 15. Find the common ratio of the GP 25, - 5, 1, $\frac{-1}{5}$.**

**Solution:** We know that if the first, second, third .. terms of a GP are $a_1, a_2, a_3...$ respectively the common ratio $r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = .....$

Here $a_1 = 25, a_2 = -5, a_3 = 1.$

So common ratio $r = \frac{-5}{25} = \frac{1}{-5} = \frac{-1}{5}$

**Example - 16 Find the 20th and nth term of the GP.**

$$\frac{5}{2}, \frac{5}{4}, \frac{5}{8}.....$$

**Solution:** Here $a = \frac{5}{2}$ $and\ r = \frac{\frac{5}{4}}{\frac{5}{2}} = \frac{1}{2}$

Ten $a_{20} = ar^{20-1} = \frac{5}{2}\left(\frac{1}{2}\right)^{19} = \frac{5}{20^{20}}$

and $a_n = ar^{n-1} = \frac{5}{2}\left(\frac{1}{2}\right)^{n-1} = \frac{5}{2^n}$

**Example – 17. Which term of the GP : 2,** $2\sqrt{2}, 4....$ **is 128?**

**Solution:** Here a = 2 $r = \frac{2\sqrt{2}}{2} = \sqrt{2}$

Let 128 be the nth term of the GP.

Then $a_n = ar^{n-1} = 128$

$$2.(\sqrt{2})^{n-1} = 128$$

$$(\sqrt{2})^{n-1} = 64$$

$$(2)^{\frac{n-1}{2}} = 2^6$$

$$\Rightarrow \frac{n-1}{2} = 6$$

$$\therefore n = 13.$$

Hence 128 is the 13th term of the GP.

**Example – 18. In a GP the 3rd term is 24 and 6th term is 192. Find the 10th term.**

**Solution:** Here $a_3 = ar^2 = 24$

$a_6 = ar^5 = 195$

Dividing (2) by (1) we get $\frac{ar^5}{ar^2} = \frac{195}{24}$

$\Rightarrow r^3 = 8 = 2^3$

$\Rightarrow r = 2$

Substituting r=2 in (1) we get a = 6.

$\therefore\ a_{10} = ar^9 = 6(2)^9 = 3072.$