QUADRATIC EQUATIONS

1. Standard form of a quadratic equation in variable x is $ax^2 + bx + c = 0$ where a, b, c are real numbers and $a \neq 0$.

2. In general, a real number α is called a root of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ if $a\alpha^2 + b\alpha + c = 0$. The zeroes of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are the same.

3. If we can factories $ax^2 + bx + c, a \neq 0$ into a product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.

4. A quadratic equation can also be solved by the method of "Completing the square".

Quadratic Formula:

5. The roots of quadratic equation $ax^2 + bx + c = 0$ are given by $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, provided $b^2 - 4ac \geq 0$. $b^2 - 4ac$ is called the 'discriminate' of this quadratic equation.

Nature of the roots:

6. A quadratic equation $ax^2 + bx + c = 0$ has

i) Two distinct real roots, if $b^2 - 4ac > 0$

ii) Two equal roots (i.e. coincident roots), if $b^2 - 4ac = 0$ and

iii) No real roots, if $b^2 - 4ac < 0$

1. Check whether the following equations are quadratic or not?

(i) $x^2 - 6x - 4 = 0$

(ii) $x^3 - 6x^2 + 2x - 1 = 0$

(iii) $7x = 2x^2$

(iv) $x^2 + \frac{1}{x^2} = 2$

Sol: (i) $x^2 - 6x - 4 = 0$

It is in the form of $ax^2 + bx + c = 0$ Therefore the given equation is a quadratic equation.

(ii) $x^3 - 6x^2 + 2x - 1 = 0$

It is not in the form of $ax^2 + bx + c = 0$

$\therefore$ The given equation is not a quadratic equation.

(iii) $7x = 2x^2$

$\Rightarrow$ $2x^2 - 7x + 0 = 0$

It is in the form of $ax^2 + bx + c = 0$

$\therefore$ The given equation is a quadratic equation.

(iv) $x^2 + \dfrac{1}{x^2} = 2$

We have, $x^2 + \dfrac{1}{x^2} = 2$

Can be written as $x^2 + \dfrac{1}{x^2} - 2 = 0$

It is not in the form of $ax^2 + bx + c = 0$

Therefore the given equation is not a quadratic equation.

2. Check whether the following are quadratic equations.

(i) $(x-2)^2 + 1 = 2x - 3$

(ii) $x(2x+3) = x^2 + 1$

Sol: i) LHS = $(x-2)^2 + 1 = x^2 - 4x + 4 + 1 + x^2 - 4x + 5$ Therefore, $(x-2)^2 + 1 = 2x - 3$ can be written as $x^2 - 4x + 5 = 2x - 3$

i.e., $x^2 - 6x + 8 = 0$

It is in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is a quadratic equation.

(ii) Here, LHS = $x(2x+3) = 2x^2 + 3x$

So, $x(2x+3) = x^2 + 1$ can be rewritten as

$2x^2 + 3x = x^2 + 1$

Therefore, we get $x^2 + 3x - 1 = 0$

It is in the form of $ax^2 + bx + c = 0$.

So, the given equation is a quadratic equation.

3. The product of two consecutive positive integers is 306. We need to find the integers.

Sol: Let the smaller integers be x

Consecutive integer = $x + 1$

Product of the integers

$= (x(x+1) = x^2 + x$

Given product is 306

Therefore, $x^2 + x = 306$

$x^2 + x - 306 = 0$

The required quadratic equation is $x^2 + x - 306 = 0$ ($x =$ smaller integers)

4. Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360 years. We need to find Rohan's present age.

Sol : Let Rohan's age be x yrs.

Age of her mother $= (x + 26)$ yrs.

After three years Rohan's age = $(x + 3)$ yrs.

Her mother age = $x + 26 + 3 = (x + 29)$ yrs.

Product of their ages = $(x + 3)\,(x + 29)$

= $(x^2 + 29x + 3x + 87)$ yrs.

= $(x^2 + 32x + 87)$ yrs.

Given product of their ages = 360

Therefore $x^2 + 32x + 87 = 360$

$x^2 + 32x + 87 - 360 = 0$

$x^2 + 32x - 273 = 0$

$\therefore$ The required quadratic equation is $x^2 + 32x - 273 = 0$ ($x =$ Rohan's age)

5. A train travels a distance of 480 km at a uniform speed. If the speed had been 8km/ h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Sol : Let the speed of the train = x km/ h.

Distance = 480 km.

Time = $\frac{distance}{speed} = \frac{480}{x}\,hours$

If the speed is 8km/ h less

i.e., $(x-8)$ km/h

Distance = 480 km.

Time = $\frac{distance}{speed} = \frac{480}{x-8}\,hours$

If the speed had been 8 km/ h less then it would have taken 3 hours more to cover the same distance.

$$\frac{480}{x} = \frac{480}{x-8} + 3$$

$$\frac{480}{x} = \frac{480 + 3(x-8)}{x-8}$$

$$\frac{480}{x} = \frac{480 + 3x - 24}{x-8}$$

$$480(x-8) = x(480 + 3x - 24)$$

$$480x - 3840 = 480x + 3x^2 - 24x$$

$$3x^2 - 24x + 3840 = 0$$

$$x^2 - 8x + 1280 = 0$$

$\therefore$ The required quadratic equation is (x =speed of the train)

6. Verify that 1 and $\frac{3}{2}$ are the roots of the equation $2x^2 - 5x + 3 = 0$

Sol : 1 and $\frac{3}{2}$ satisfy $2x^2 - 5x + 3 = 0$

Substitute, $x = 1$ in $2x^2 - 5x + 3 = 0$

$$2(1)^2 - 5(1) + 3 = 0$$
$$2 - 5 + 3 = 0$$
$$5 - 5 = 0$$
$$0 = 0(True)$$

Substitute $x = \frac{3}{2}$ in $2x^2 - 5x + 3 = 0$

$$2\left(\frac{3}{2}\right)^2 - 5\left(\frac{3}{2}\right) + 3 = 0$$

$$2\left(\frac{9}{4}\right) - \frac{15}{2} + 3 = 0$$

$$\frac{9}{2} - \frac{15}{2} + 3 = 0$$

$$\frac{9 - 15 + 6}{2} = 0$$

15 – 15=0

0 =0 (True)

$\therefore$ 1 and $\frac{3}{2}$ satisfies the given quadratic equation $2x^2 - 5x + 3 = 0$

Therefore these are the roots of the given equation.

7. Find the roots of the following quadratic equation by factorization :

(i) $x^2 - 3x - 10 = 0$

Sol : $x^2 - 3x - 10 = 0$

$x^2 - 5x + 2x - 10 = 0$

$x(x - 5) + 2(x - 5) = 0$

$(x - 5)(x + 2) = 0$

$x-5=0$ *or* $x+2=0$

$x=5$ *or* $x=-2$

Therefore, the roots of $x^2-3x-10=0$ are 5 and -2.

(ii) $2x^2+x-6=0$

Sol : $2x^2+x-6=0$

$2x^2+4x-3x-6=0$

$2x(x+2)-3(x+2)=0$

$(x+2)\ (2x-3)=0$

$x+2=0$ *or* $2x-3=0$

$x=-2\ (or)\ 2x=3$

$$x=\frac{3}{2}$$

Therefore the roots of $2x^2+x-6=0$ are -2 and $\frac{3}{2}$

(iii) $\sqrt{2}x^2+7x+5\sqrt{2}=0$

Sol : $\sqrt{2}x^2+7x+5\sqrt{2}=0$

$\sqrt{2}x^2+2x+5x+5\sqrt{2}=0$

$\sqrt{2}x\ (x+\sqrt{2})+5(x+\sqrt{2})=0$

$(x+\sqrt{2})\ (\sqrt{2}x+5)=0$

$x+\sqrt{2}=0$ *or* $\sqrt{2}x+5=0,$

$x=-\sqrt{2}$ *or* $x=\frac{-5}{\sqrt{2}}$

Therefore the roots of $\sqrt{2}x^2+7x+5\sqrt{2}=0$ are

$-\sqrt{2}$ *and* $\frac{-5}{\sqrt{2}}$

(iv) $2x^2 - x + \frac{1}{8} = 0$

$16x^2 - 8x + 1 = 0$

$16x^2 - 4x - 4x + 1 = 0$

$4x(4x-1) - 1(4x-1) = 0$

$(4x-1)(4x-1) = 0$

$4x - 1 = 0, 4x - 1 = 0$

$x = \frac{1}{4}, x = \frac{1}{4}$

Therefore the roots of $2x^2 - x + \frac{1}{8} = 0$ are $\frac{1}{4}$ *and* $\frac{1}{4}$

v) $x(x+4) = 12$

Sol : $x(x+4) = 12 \Rightarrow x^2 + 4x = 12$

$x^2 + 4x - 12 = 0$

$x^2 + 6x - 2x - 12 = 0$

$x(x+6) - 2(x+6) = 0$

$(x+6)\,(x-2) = 0$

$x + 6 = 0$ *or* $x - 2 = 0$

$x = -6$ *or* $x = 2$

Therefore the roots of $x(x+4) = 12$ are -6 and 2.

vi) $x - \frac{3}{x} = 2$

Sol: $x - \frac{3}{x} = 2$

$\frac{x^2 - 3}{x} = 2 \Rightarrow x^2 - 3 = 2x$

$\Rightarrow x^2 - 2x - 3 = 0$

$x^2 - 3x + x - 3 = 0$

$x(x-3)+1(x-3)=0$

$(x-3)(x+1)=0$

$x-3=0 \; or \; x+1=0$

$x=3 \; or \; x=-1$

Therefore the roots of $x-\frac{3}{x}=2$ are 3 and -1.

vii) $3(x-4)^2-5(x-4)=12$

Sol: Let $x-4=K$

$3K^2-5K=12$

$\Rightarrow 3K^2-5K-12=0$

$3K^2-9K+4K-12=0$

$3K(K-3)+4(K-3)=0$

$(K-3)(3K+4)=0$

$K-3=0 \; or \; 3K+4=0$

$K=3 \; or \; 3K=-4$

$K=\frac{-4}{3}$

Where K = 3

$x-4=3$

$x=3+4$

$x=7$

Where $K=\frac{-4}{3}$

$x-4=\frac{-4}{3}$

$x=\frac{-4}{3}+4=\frac{-4+12}{3}=\frac{8}{3}$

Therefore the roots of $3(x-4)^2-5(x-4)=12 \; are \; 7 \; and \; \frac{8}{3}$.

8.Find two numbers whose sum is 27 and product is 182.

Sol : Sum of two numbers =27

Let one number be x

Second number $= 27 - x$.....(1)

Product = 182 … (2)

$$x(27-x)=182$$
$$27x-x^2=182$$
$$27x-x^2-182=0$$
$$x^2-27x+182=0$$
$$x^2-13x-14x+182=0$$
$$x(x-13)-14(x-13)=0$$
$$(x-13)(x-14)=0$$
$$x-13=0 \text{ or } x-14=0$$
$$x-13=0 \text{ gives } x=13$$

If first number is 13 second number = 27 – 13 = 14

$x-14=0$ *gives* $x=14$

If first number is 14, second number = 27 – 14 =13

Therefore two numbers are 13 and 14.

9. Find two consecutive positive integers, sum of whose squares is 613.

Sol: Let the first positive integer be x

Then the consecutive positive integer is $x+1$

Sum of the squares = $x^2+(x+1)^2$

$$= x^2+x^2+2x+1$$

$$= 2x^2+2x+1$$

Given sum of the squares = 613

$\therefore 2x^2 + 2x + 1 = 613$

$2x^2 + 2x + 1 - 613 = 0$

$2x^2 + 2x - 612 = 0$

$x^2 + x - 306 = 0$

$x^2 + 18x - 17x - 306 = 0$

$x(x+18) - 17(x+18) = 0$

$= (x+18)(x-17)$

$x + 18 = 0 \; or \; x - 17 = 0$

$x + 18 = 0 gives \; x = -18$

$x - 17 = 0 \; gives \; x = 17$

For x = 17 its consecutive positive integer is x+1=17+1=18.

$\therefore$ Two consecutive positive integers are 17, 18

10. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Sol: Let the base of the right triangle is x cm

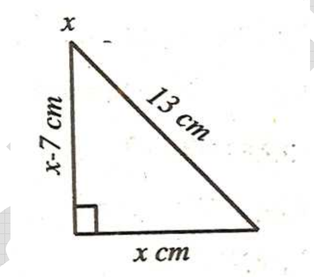
Its altitude is 7 cm less than its base

$\therefore$ Altitude is (x -7) cm

Hypotenuse = 13 cm

In a right angle triangle

(Hypotenuse)^2 = (side)^2 + (side)^2



$(13)^2 = x^2 + (x-7)^2$

$x^2 + x^2 - 14x + 49 = 169$

$2x^2 - 14x + 49 - 169 = 0$

$2x^2 - 14x - 120 = 0$

$x^7 - 7x - 60 = 0$

$x^2 - 12x + 5x - 60 = 0$

$x(x-12)+5(x-12)=0$

$(x-12)(x+5)=0$

$x-12=0 \text{ or } x+5=0$

$x=12 \text{ (or) } x=-5$

Base of triangle can't be negative

∴ Base of the triangle = 12 cm

Altitude of the triangle = 12 – 7 = 5 cm

∴ Other two sides are 5 cm and 12 cm.

11. Find the dimensions of a rectangle whose perimeter is 28 meters and whose area is 40 square meters.

Sol: Area of rectangle = 40 sq. m

Let the length of rectangle be = x m.

$x \times$ breadth = 40 sq. m

Breadth $=\dfrac{40}{x}m$

Perimeter of rectangle = 28 m.

$2(l+b)=28$

$2\left(x+\dfrac{40}{x}\right)=28$

$\dfrac{x^2+40}{x}=\dfrac{28}{2}=14$

$x^2+40=14x$

$x^2-14x+40=0$

$(x-10)(x-4)=0$

$x-10 \quad (or) \quad x-4=0$

$x=10 \quad (or) \quad x=4$

If length = 10 m then breadth

$=\dfrac{40}{10}=4m$

If length = 4 m then breadth

$=\dfrac{40}{4}=10m$

$\therefore$ Dimensions of rectangle = 10 m, 4 m

12. The base of a triangle is 4 cm longer than its altitude. If the area of the triangle is 48 sq. cm then find its base and altitude.

Sol : Let the altitude of a triangle be x cm

Its base = (x +4) cm

Area of the triangle = $\frac{1}{2}\times$ base $\times$ altitude

$=\frac{1}{2}\times x\times(x+4)$

$=\frac{1}{2}(x^2+4x)sq.cm$

Given the area of the triangle = 48 sq.com

$\frac{1}{2}(x^2+4x)=48$

$x^2+4x=48\times 2$

$x^2+4x=96 \Rightarrow x^2+4x-96=0$

$x^2+12x-8x-96$

$(x+12)(x-8)=0$

Either x + 12 =0 or x -8 = 0

x + 12 = 0 gives x = -12

x -8=0 gives x = 8

x = - 12 is not possible

$\therefore$ Altitude = x = 8 cm

$\therefore$ Base = x +4 = 8+ 4 = 12 cm

Base = 12 cm : Altitude = 8 cm

13. In a class of 60 students, each boy contributed rupees equal to the number of girls and each girl contributed rupees equal to the number of boys. If the total money then collected was ₹1600. How many boys are there in the class?

Sol : Number of students = 60

Let the number of boys be x

$\therefore$ Number of girls = $60 - x$

Each boy contributed is rupees equal to the number of girls.

$\therefore$ Contribution of boys = $x(60 - x)$ ₹ = $(60x - x^2)$ ₹

Each girl contributed rupees equal to the number of boys.

$\therefore$ Contribution of girls = $(60 - x)x = (60x - x^2)$ ₹

Total money collected = 1600 ₹

$\therefore 60x - x^2 + 60x - x^2 = 1600$

$$120x - 2x^2 = 1600$$
$$2x^2 - 120x + 1600 = 0$$
$$x^2 - 60x + 800 = 0$$
$$x^2 - 40x - 20x + 800 = 0$$
$$x(x-40) - 20(x-40) = 0$$
$$(x-40)(x-20) = 0$$

$$x - 40 = 0 \text{ or } x - 20 = 0$$
$$x = 40 \ (or)\ x = 20$$

Number of boys in the class are 20 (or) 40.

14. A motor boat heads upstream a distance of 24 km and a river whose current is running at 3 km per hour. The trip up and back takes 6 hours. Assuming that the motor boat maintained a constant speed, what was its speed ?

Sol : Let the speed of the motor boat be x km/ hour.

Distance travelled = 24 km.

Speed of the running water = 3 km/ hour.

Time taken for 24 km down stream

$= \frac{24}{x+3} hours$

Time takes for 24 km upstream

$\frac{24}{x-3} hours$

The total time is given to be 6 hours.

$\therefore \frac{24}{x+3} + \frac{24}{x-3} = 6$

We get

$$\frac{24(x-3)+24(x+3)}{(x+3)(x-3)}$$

$$24(x-3)+24(x+3) = 6(x+3)(x-3)$$

Dividing by 6

$4(x-3)+4(x+3) = (x+3)(x-3)$

$4x-12+4x+12 = x^2-9$

$8x = x^2-9$

$x^2-8x-9=0$

$x^2-9x+x-9=0$

$x(x-9)+1(x-9)=0$

$(x-9)(x+1)=0$

Either $x-9=0$ *or* $x+1=0$

x =9 or x = -1

$\therefore$ The speed can't be negative, the speed of the motor boat = 9 km/ hour.

15. Find the roots of $4x^2+3x+5=0$ by the method of completing the square.

Sol : Given : $4x^2+3x+5=0$

$x^2+\frac{3}{4}x+\frac{5}{4}=0$

$x^2+\frac{3}{4}x=\frac{-5}{4}$

$x^2+\frac{3}{4}x+\left(\frac{3}{8}\right)^2=\frac{-5}{4}+\left(\frac{3}{8}\right)^2$

$\left(x+\frac{3}{8}\right)^2=\frac{-5}{4}+\frac{9}{64}$

$$\left(x+\frac{3}{8}\right)^2 = \frac{-71}{64} < 0$$

But $\left(x+\frac{3}{8}\right)^2$ cannot be negative for any real value of x. So, there is no real value of x satisfying the given equation. Therefore, the given equation has no real roots.

16. Find two consecutive odd positive integers, sum of whose squares is 290.

Sol : Let first odd positive integers be x. Then, the second integer will be x +2. According to the question,

$$x^2 + (x+2)^2 = 290$$
$$i.e., x^2 + x^2 + 4x + 4 = 290$$
$$i.e., 2x^2 + 4x - 286 = 0$$
$$i.e., x^2 + 2x - 143 = 0$$

Which is a quadratic equation in x. Using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2}$$

We get, $x = \dfrac{-2 \pm \sqrt{4+572}}{2}$

$$= \frac{-2 \pm \sqrt{576}}{2} = \frac{-2 \pm 24}{2}$$

i..e, x = 11 (or) x = - 13

but x is given to be an odd positive integer.

Therefore, $x \neq -13, x = 11.$

Thus, the two consecutive odd integers are 11 and $(x+2)$ =11+2=13.

17. Find the roots of the following quadratic equations, if they exist, using the quadratic formula :

i) $x^2 + 4x + 5 = 0$ $\quad$ $ii)\ 2x^2 - 2\sqrt{2}x + 1 = 0$ h

Sol : i) $x^2 + 4x + 5 = 0$ Here, $a = 1, b = 4, c = 5.$

So, $b^2 - 4ac = 16 - 20 = -4 < 0.$

Since the square of a real number cannot be negative, therefore $\sqrt{b^2-4ac}$ will not have any real value.

So, there are no real roots for the given equation.

ii) $2x^2-2\sqrt{2}x+1=0$

Here $a=2, b=-2\sqrt{2}, c=1.$

So, $b^2-4ac=8-8=0$

Therefore, $x=\dfrac{2\sqrt{2}+\sqrt{0}}{4}=\dfrac{\sqrt{2}}{2}\pm 0$

i.e., $x=\dfrac{1}{\sqrt{2}}.$

so, the roots are $\dfrac{1}{\sqrt{2}},\dfrac{1}{\sqrt{2}}.$

18. A motor boat whose speed is 18 km/ h in still water. It takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Sol : Let the speed of the stream be x km/ h

Therefore, the speed of the boat upstream

$=(18-x)$ km/ h and the speed of the boat

downstream $=(18+x)$ km/ h

the time taken to go upstream

$\dfrac{distance}{speed}=\dfrac{24}{18-x}\,hours.$

Similarly, the time taken to go down

Stream = $\dfrac{24}{18+x}\,hours.$

According to the question,

$=\dfrac{24}{18-x}-\dfrac{24}{18+x}=1$

$i.e., 24(18+x)-24(18-x)$
$=(18-x)(18+x)$
$i.e., x^2+48x-324=0$

Using the quadratic formula, we get

$$x=\frac{-48\pm\sqrt{48^2+1296}}{2}=\frac{-48\pm\sqrt{3600}}{2}$$
$$=\frac{-48\pm 46}{2}=6\,(or)-54$$

Since x is the speed of the stream, it cannot be negative. So, we ignore the root x = - 54. Therefore, x =6 gives the speed of the stream as 6 km/ h.

19. Find the roots of the quadratic equations by applying the quadratic formula.

(i) $2x^2+x-4=0$

(ii) $4x^2+4\sqrt{3}x+3=0$

(iii) $5x^2-7x-6=0$

(iv) $x^2+5=-6x$

Sol : $2x^2+x-4=0$

Here $a=2; b=1; c=-4$

So, $b^2-4ac=(1)^2-4(2)(-4)$

$=1+32=33>0$

Therefore $x=\frac{-1\pm\sqrt{33}}{2\times 2}=\frac{-1\pm\sqrt{33}}{4}$

$x=\frac{-1+\sqrt{33}}{4}; x=\frac{-1-\sqrt{33}}{4}$

$x=\frac{-1+\sqrt{33}}{4}; x=\frac{-1-\sqrt{33}}{4}$

The roots are $=\frac{-1+\sqrt{33}}{4},\frac{-1-\sqrt{33}}{4}$

(ii) $4x^2+4\sqrt{3}x+3=0$

Here $a=4; b=4\sqrt{3}; c=3$

$b^2 - 4ac = (4\sqrt{3})^2 - 4(4)(3) = 48 - 48 = 0$

Therefore $x = \dfrac{-4\sqrt{3} \pm \sqrt{0}}{2 \times 4} = \dfrac{-4\sqrt{3}}{8} \pm 0$

$= \dfrac{-\sqrt{3}}{2}$

So the roots are $\dfrac{-\sqrt{3}}{2}, \dfrac{-\sqrt{3}}{2}$

(iii) $5x^2 - 7x - 6 = 0$

Here $a = 5; b = -7; c = -6$

$b^2 - 4ac = (-7)^2 - 4(5)(-6)$
$= 49 + 120 = 169 > 0$

Therefore $x = \dfrac{-(-7) \pm \sqrt{169}}{2 \times 5}$

$= \dfrac{7 \pm 13}{10}$

$x = \dfrac{7 + 13}{10}$

$x = \dfrac{20}{10} = 2$

$x = \dfrac{7 - 13}{10} = \dfrac{-6}{10} = \dfrac{-3}{5}$

∴ The roots are $2, \dfrac{-3}{5}$

(iv) $x^2 + 5 = -6x$

$x^2 + 6x + 5 = 0$

Here $a = 1; b = 6; c = 5$

$b^2 - 4ac = (6)^2 - 4(1)(5)$

$= 36 - 20 = 16 > 0$

Therefore $x = \dfrac{-6 \pm \sqrt{16}}{2 \times 1} = \dfrac{-6 \pm 4}{2}$

$x = \dfrac{-6 + 4}{2} = \dfrac{-2}{2} = -1$

$x = \dfrac{-6 + 4}{2} = \dfrac{-10}{2} = -5$

∴ The roots are -1, -5

20. The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Sol : Let the present age of Rehman's is x yrs. 3 years ago Rehman's age is $(x-3)$ yrs. 5 years from now Rehman's age is $(x+5)$ yrs.

Sum of the Reciprocals of Rehman's ages 3 years ago and 5 years from now is $\frac{1}{3}$.

$$\frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$$

$$\frac{x+5+x-3}{(x-3)(x+5)}=\frac{1}{3}$$

$$\frac{2x+2}{x^2+5x-3x-15}=\frac{1}{3}$$

$$\frac{2x+2}{x^2+2x-15}=\frac{1}{3}$$

$$3(2x+2)=1(x^2+2x-15)$$

$$6x+6=x^2+2x-15$$

$$x^2+2x-15-6x-6=0$$

$x^2-4x-21=0$ Which is a quadratic equation.

Here $a=1; b=-4, c=-21.$

$$b^2-4ac=(-4)^2-4(1)(-21)$$

$$=16+84=100>0$$

Therefore $x=\frac{-(-4)\pm\sqrt{100}}{2\times 1}=\frac{4\pm 10}{2}$

$$x=\frac{4+10}{2}=\frac{14}{2}=7$$

$$x=\frac{4-10}{2}=\frac{-6}{2}=-3$$

$\therefore$ The age cannot be negative. $\therefore x=7$

Present age of Rehman is 7 years.

21. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Sol : Let the larger number be x;

The square of the smaller number is 8 times the larger number.

Square of the smaller number = 8 x

The difference of squares of two numbers = 180

$$\therefore x^2 - 8x = 180$$
$$x^2 - 8x - 180 = 0$$

$$x^2 - 18x + 10x - 180 = 0$$
$$x(x-18) + 10(x-18) = 0$$
$$(x-18)(x+10) = 0$$
$$x - 18 = 0 \; or \; x + 10 = 0$$
$$x = 18 \; or \; x = -10$$

-10 is not real number.

Larger number = 18

Square of smaller number $= 8 \times 18 = 144$

Small number $= \sqrt{144} = \pm 12$

Required numbers = 18, 12 (or) 18; - 12

22. A train travels 360 km at a uniform speed. If the speed had been 5 km/ hr more it would have taken 1 hour less for the same journey. Find the speed of the train.

Sol : Let the speed of the train be x km/ hr.

Distance travelled by train = 360 km.

Time taken = $\frac{360}{x}$ *hours.*

If speed had been 5 km/ hr

More, time taken for journey = $\frac{360}{x+5}$

As per problem $\frac{360}{x} - \frac{360}{x+5} = 1$

$$\frac{360(x+5)-360x}{x(x+5)}=1$$

$360x+1800-360x=x(x+5)$

$1800=x^2+5x$

$x^2+5x-1800=0$

$x^2+45x-40x-1800=0$

$x(x+45)-40(x+45)=0$

$(x+45)(x-40)=0$

$x+45=0$

$x=-45$

$x-40=0$

$x=40$

Speed can't be negative

$\therefore$ Required speed = 40 km/ hr.

23. Sum of the areas of two squares is 468 m². If the difference of their perimeters is 24m find the sides of two squares.

Sol : Let the sides of two squares be xm and ym

Area of the first square is x^2 sq. m

Area of the second square is y^2 sq. m

Sum of the areas of two squares is x^2+y^2 Given sum of the areas is 468

$x^2+y^2=468$ ...(1)

Perimeter of the first square = 4 xm

Perimeter of the second square = 4 ym

Difference of their perimeters = 24

$4x-4y=24$

$x-y=6$

$x=6+y$ ...(2)

Substitute x value in (1)

$(6+y)^2 + y^2 = 468$

$36+12y+y^2+y^2 = 468$

$2y^2+12y+36-468=0$

$2y^2+12y-432=0$

$y^2+6y-216=0$ Which is a quadratic equation.

Here $a=1; b=6; c=-216.$

$b^2-4ac = (6)^2 - 4\times1(216)$

$= 36+864 = 900$

Therefore $y = \frac{-6\pm\sqrt{900}}{2\times1} = \frac{-6\pm30}{2}$

$y = \frac{-6+30}{2}, y = \frac{-6-30}{2}$

$= \frac{24}{2} = 12$

$= \frac{-36}{2} = -18$

Side of a square cannot be negative

$\therefore\ y = 12$

$x = 6+y \Rightarrow x = 6+12 = 18$

∴ Sides of the two square are 12m and 18m.

24. Find the values of k for $2x^2+kx+3$ quadratic equations, so that they have two equal roots.

Sol : $2x^2+kx+3=0$

The given equation is in the form of $ax^2+bx+c=0$

Where $a=2; b=k; c=3$

Given that the quadratic equation has two equal roots.

∴ Discriminate = 0

$b^2 = 4ac = 0$

$(K)^2 - 4 \times 2 \times 3 = 0 \quad \Rightarrow K^2 - 24 = 0$

$= k^2 = 24$

$k = \sqrt{24} = \sqrt{4 \times 6} = \pm 2\sqrt{6}$

$\therefore k = \pm 2\sqrt{6}$

25. Is it possible to design a rectangular mango grove whose length is twice its breadth and the area is 800 m^2, If so find its length and breadth.

Sol : Let the breadth of the rectangle be x mts.

Length = $2x$ mts

Area of the rectangle = Length x Breadth

$= 2x \times x = 2x^2 m^2$

Given area of the rectangle = 800 m^2

$2x^2 = 800$

Yet it is possible to design a rectangular mango glove

$2x^2 = 800 \Rightarrow x^2 = \frac{800}{2} = 400$

$x = \sqrt{400} = \pm 20$

Sides of a rectangle cannot be negative.

$\therefore \ x = 20$

Breadth = 20m

Length = 2 x 20 = 40m;

26. The sum of the ages of two friends is 20 years. Four years ago the product of their ages in years was 48. Is the situation possible? If so determine their present ages.

Sol : Let the present age be x yrs.

Age of his friend is $(20 - x)$ yrs.

Four years ago his age is $(x - 4)$ yrs.

His friends age is $20 - x - 4 = (16 - x)$ yrs.

Product of their ages = 48

$(x-4)(16-x)=48$

$16x-x^2-64+4x=48$

$-x^2+20x-64-48=0$

$-x^2+20x-112=0$

$x^2-20x+112=0$ Which is a quadratic equation.

Here $a=1, b=-20, c=112$

discriminate $b^2-4ac=(-20)^2-4\times1\times112.$

$=400-448=-48<0$

So, the given equation has no real roots.

$\therefore$ The above situation is not possible.

27. Is it possible to design a rectangular park of perimeter 80m and area 400 m^2. If so, find its length and breadth.

Sol : Area of rectangular park = 400 m^2

Let the length be x m.

The breadth $=\dfrac{400}{x}m$

Perimeter = 80 m

$2\left(x+\dfrac{400}{x}\right)=80$

$\dfrac{x^2+400}{x}=\dfrac{80}{2}=40$

$x^2+400=40x$

$x^2-40x+400=0$

$(x-20)(x-20)=0$

$x=20m$

$Length=x=20m$

$breadth=\dfrac{400}{20}=20m$

28. A two digit number is such that the product of the digits is 8 when 18 is added to the number they inter change their places. Determine the number.

Sol : Let the number in the tens place be x and the number in the units place be y.

The value of the number is $10x+y$.

The Product of the digits is 8

$xy=8$ ..(1)

When 18 is added to the number, the digits get interchanged

$\therefore\ 10x+y+18=10y+x$

$10x+y+18-10y-x=0$

$9x-9y=-18 \Rightarrow x-y=-2$

$y=x+2$ ..(2)

Substitute $y=x+2$ in equation (1) we get

$x(x+2)=8$

$x^2+2x-8=0 \Rightarrow x^2+2x=8$

$x^2+2x.1+(1)^2=8+(1)^2$

$(x+1)^2=8+1 \Rightarrow (x+1)^2=9$

$x+1=\sqrt{9}=\pm 3 \Rightarrow x+1=+3\ or\ x+1=-3$

$x=3-1\ or\ x=-3-1$

$x=2\ or\ x=-4$

The digits are positive integers.

$xy=8 \Rightarrow 2y=8 \Rightarrow y=\frac{8}{2}=4$

$10x+y=(10\times 2)+4=20+4=24$

$\therefore$ The two digits number is 24

29. Find the roots of $x^2-x-12=0$ using factorization method.

Sol : $x^2-x-12=0$

$x^2-x-12=x^2-4x+3x-12$

$=x(x-4)+3(x-4)$

$=(x-4)(x+3)$

The roots of $x^2-x-12=0$ are the values of x for which $(x-4)(x+3)=0$

$x-4=0\ or\ x+3=0$

$x=4\ or\ x=-3$

Therefore the roots of $x^2 - x - 12 = 0$ are 4 and -3.

30. Find the value of k for the quadratic equation $9x^2 - kx + 4 = 0.$ so that this has two equal roots.

Sol : The equation is in the form of $ax^2 + bx + c = 0$

Where $a = 9; b = -k; c = 4$

Given that the quadratic equation has two equal roots.

$\therefore$ Discriminate = 0

$b^2 - 4ac = 0$

$(-k)^2 - 4(9)(4) = 0$

$k^2 - 144 = 0 \Rightarrow k^2 = 144$

$\Rightarrow k = \sqrt{144} = \pm 12$