Pair of Linear Equations in Two Variables

Key Concepts

1. An equation of the form $ax+by+c=0$ where a, b, c are real numbers and where at least one of a **or** b is not zero is called a linear equation in two variables x and y.

2. A linear equation in two variables has many solutions.

3. The graph of a linear equation in two variables is a straight line.

4. Two linear equations in the same two variables are called a pair of linear equations in two variables.

   $a_1x+b_1y+c_1=0\ (a_1^{\ 2}+b_1^{\ 2}\neq 0)$
   $a_2x+b_2y+c_2=0\ (a_2^{\ 2}+b_2^{\ 2}\neq 0)$

   Where a_1,a_2,b_1b_2,c_1,c_2 are real numbers.

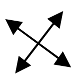
5. a_1,b_1,c_1 *and* a_2,b_2,c_2 denote the coefficients of a given pair of linear equations in two variables there exist a relation between the coefficients and nature of system of equations.

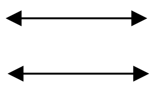
   i) If $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$, the pair of linear equations is consistent.

   ii) If $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$ the pair of linear equations is inconsistent.

   iii) If $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ the pair of linear equations is dependent and consistent.

6. **Two lines are drawn in the same plane, only one of the following three situations is possible:**

   i) The lines may intersect at one point. 

   ii) The two lines may not intersect 

   (i.e.) they are parallel.

   iii) The lines may be coincide. 

7. The graph of a pair of linear equations in two variables is represented by two lines.

   i) If the lines intersect at a point gives the unique solution of the equations. In this case, the pair of equations is consistent.

   ii) If the lines coincide, then there are infinitely many solutions – each point on the line being a solution. In this case, the pair of equations is dependent.

   iii) If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is inconsistent.

8. Methods for finding the solutions of a pair of linear equations are

   i) Model Method.

   ii) Graphical Method

   iii) Algebraic Methods – Substitution Method and Elimination method.

I. Graphical method of finding solution of a pair of linear equations

1. Check vet her following equations are consistent or inconsistent. Solve them graphically?

(a) $3x+2y=8$ (b) $2x-3y=8$

$2x-3y=1$ $4x-6y=9$

(a) $3x+2y=8 \Rightarrow 3x+2y-8=0$

$2x-3y=1 \Rightarrow 2x-3y-1=0$

$\frac{a_1}{a_2}=\frac{3}{2}; \frac{b_1}{b_2}=\frac{2}{-3}=\frac{-2}{3}$

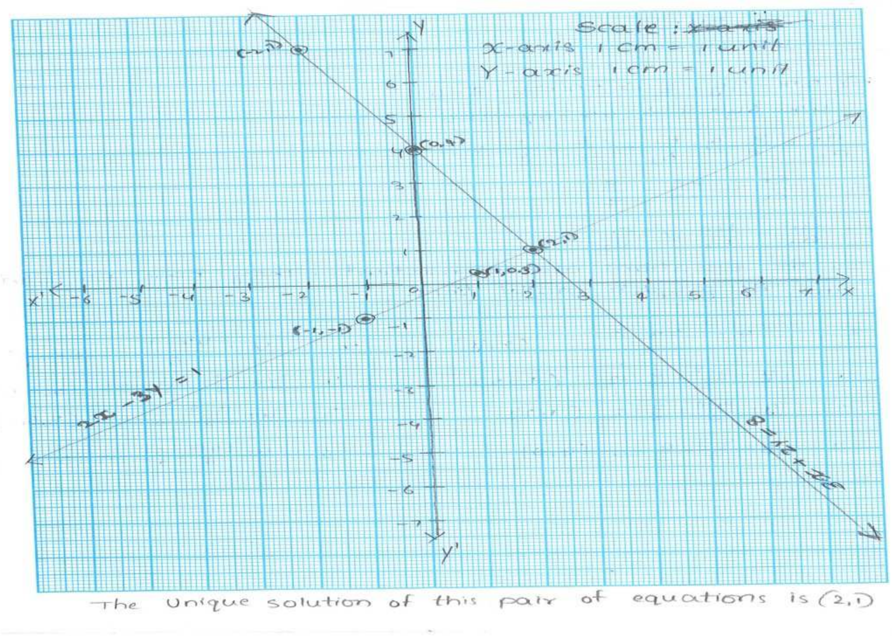
$\frac{c_1}{c_2}=\frac{-8}{-1}=\frac{8}{1}$

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\therefore$ Given lines are intersecting lines and constant pair of linear equations.

x	$y=\frac{8-3x}{2}$	(x,y)
0	$y=\frac{8-0}{2}=4$	(0,4)
2	$y=\frac{8-6}{2}=1$	(2,1)
-2	$y=\frac{8+6}{2}=7$	(-2,7)

x	$y=\frac{2x-1}{3}$	(x,y)
-1	$y=\frac{-3}{3}=-1$	(-1, -1)
2	$y=\frac{3}{3}=1$	(2,1)
1	$y=\frac{1}{3}=0.3$	(1,0.3)



b) $2x-3y=8$

$4x-6y=9$

b) $2x-3y=8 \Rightarrow 2x-3y-8=0$

$4x-6y=9 \Rightarrow 4x-6y-9=0$

$\frac{a_1}{a_2}=\frac{2}{4}=\frac{1}{2};\ \frac{b_1}{b_2}=\frac{-3}{-6}=\frac{1}{2};$

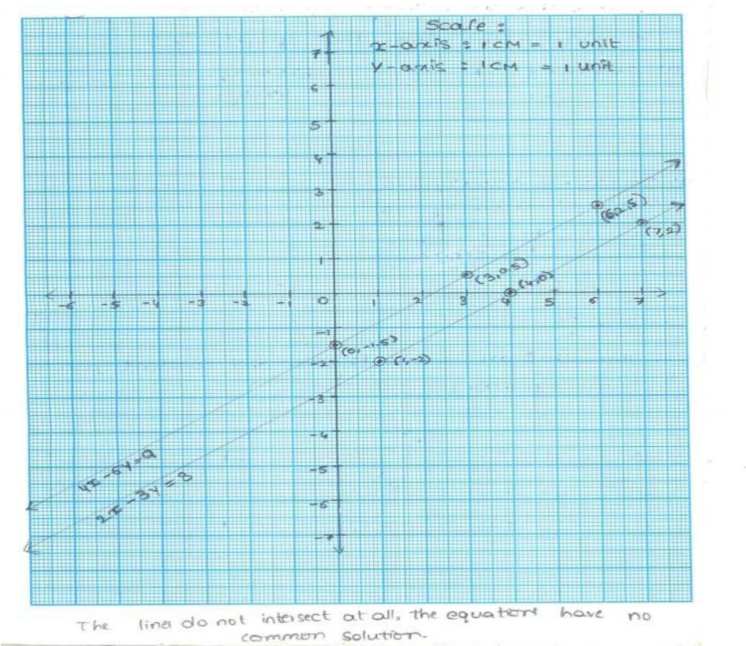
$\frac{c_1}{c_2}=\frac{-8}{-9}=\frac{8}{9}$

$\therefore \frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

$\therefore$ So the equations are inconsistent. They have no solution and its graph is parallel lines.

x	$y=\frac{2x-8}{3}$	(x,y)
1	$y=\frac{2-8}{3}=\frac{-6}{3}=-2$	(1,-2)
7	$y=\frac{14-8}{3}=\frac{6}{3}=2$	(7,2)
4	$y=\frac{8-8}{3}=\frac{0}{3}=0$	(4,0)

x	$y=\frac{4x-9}{6}$	(x,y)
0	$y=\frac{-9}{6}=\frac{-3}{2}=-1.5$	(0, -1.5)
3	$y=\frac{12-9}{6}=\frac{3}{6}=0.5$	(3,0.5)
6	$y=\frac{24-9}{6}=\frac{15}{6}=2.5$	(6,2.5)



3) Half the perimeter of a rectangular garden, whose length is 4 'm' more than its width, is 36m find the dimension of the garden?

Let the length of the garden be x m

Berth be y m

Length is 4m more than its width

x= 4+4

x-y=4 .... (1)

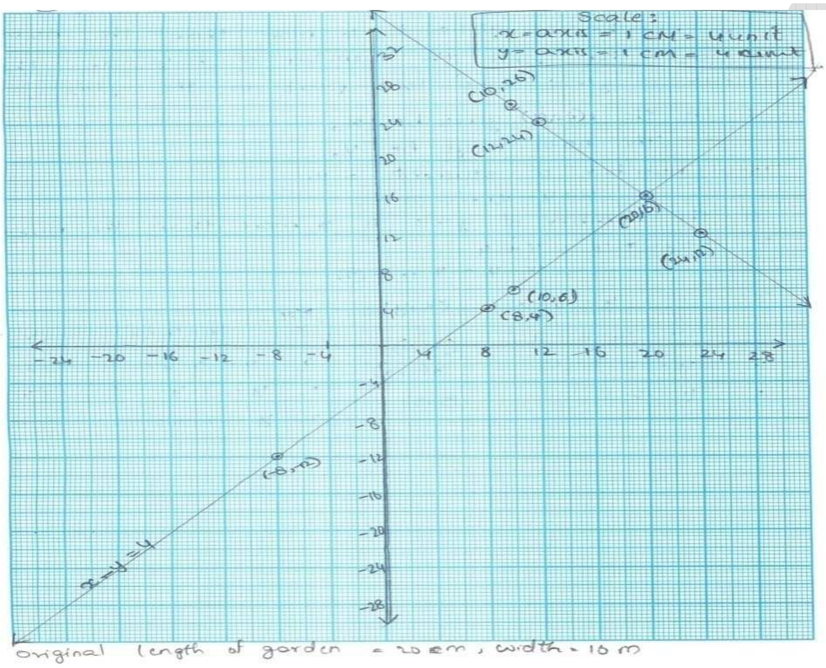
Half of the perimeter = 36 m

$$\frac{1}{2}2(x+y)=36$$

$$x+y=36 \ldots\ldots \qquad (2)$$

x	$y=x-4$	(x,y)
8	$y=8-4=4$	(8,4)
-8	$y=-8-4=-12$	(-8,-12)
10	$y=10-4=6$	(10,6)

x	$y=36-x$	(x,y)
10	$y=36-10=26$	(10,26)
12	$y=36-12=24$	(12,24)
24	$y=36-24=12$	(24,12)



4) 10 students of class – x took part in a maths quiz if the number of girls is 4 more than the number of boys then, find the number of boys and the number of girls who took part in the quiz?

Sol :- Let the no of girls be x,

The no of boys be y,

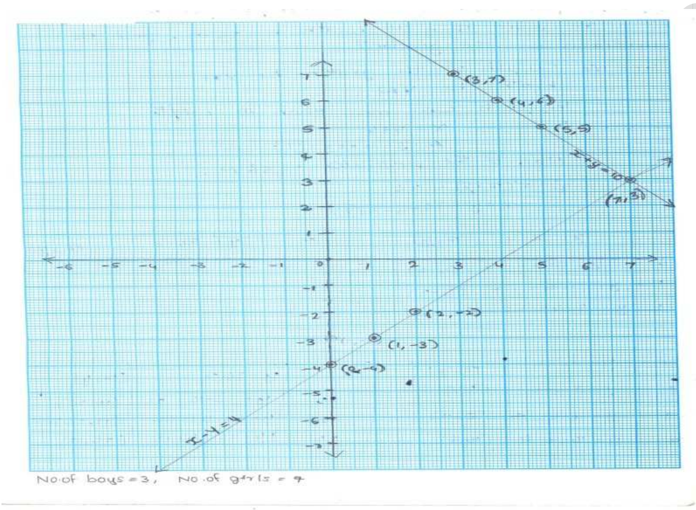
Total number of students = 10

x+y=10 .... (1)

If the number of girls is 4 more than the number of boys x =y+4 x-y=4 ..... (2)

x	$y=10-x$	(x,y)
3	$y=10-3=7$	(3,7)
4	$y=10-4=6$	(4,6)
5	$y=10-5=5$	(5,5)

x	$y=x-4$	(x,y)
0	$y=0-4=-4$	(0, -4)
1	$y=1-4=-3$	(0,-3)
2	$y=2-4=-2$	(2,-2)



II. Substitution Method

This method is useful for solving a pair of linear equations in two variables where one variable can easily be written in terms of the other variable. To understand this method, let us consider it step – wise

Step – 1: In one of the equations, express one variable in terms of the other variable. Say y in terms of x.

Step -2 : Substitute the value of y obtained in step 1 in the second equation.

Step -3 : Simplify the equation obtained in step 2 and find the value of x.

Step -4 : Substitute the value of x obtained in step 3 in either of the equations and solve it for y.

Step 5: Check the obtained solution by substituting the values of x and y in both the original equations.

Example: 1 **Solve the given pair of equations using substitution method.**

$$2x - y = 5$$
$$3x + 2y = 11$$

Solution: $2x - y = 5$ (1)

$3x + 2y = 11$ (2)

Equation (1) can be written as

y=2x-5

Substituting in equation (2) we get

$$3x + 2(2x - 5) = 11$$
$$3x + 4x - 10 = 11$$
$$7x = 11 + 10 = 21$$
$$x = 21/7 = 3.$$

Substitute x=3 in equation (1)

2(3) –y = 5

Y=6 -5 = 1

Substitute the values of x and y in equation (2), we get 3(3) + 2(1) = 9+2 =11

Both the equations are satisfied by x = 3 and y =1.

Therefore, required solution is x =3 and y = 1.

Example 2: $3x - 5y = 1$ (1)

$x - y = -1$ (2)

$$x - y = -1$$
$$x = y - 1$$

Substituting the value of x in (1)

$$3x - 5y = -1$$
$$3(y - 1) - 5y = -1$$
$$3y - 3 - 5y = -1$$
$$-2y = 2$$
$$y = -1$$

Substitute the value y = -1 in (1)

$$3x - 5y = -1$$
$$3x + 5 = -1$$
$$3x = -6$$
$$x = -2$$

Required solution x = -2 $y = -1$

III. Elimination Method

In this method, first we eliminate (remove) one of the two variables by equating its coefficients. This gives a single equation which can be solved to get the value of the other variable. To understand this method, let us consider it step wise.

Step -1: Write both the equations in the form of ax+by = c.

Step -2: Make the coefficients of one of the variables, say 'x', numerically equal by multiplying each equation by suitable real numbers.

Step -3: If the variable to be eliminated has the same sign in both equations, subtract the two equations to get an equation in one variable. If they have opposite signs then add.

Step -4: Solve the equation for the remaining variable.

Step -5: Substitute the value of this variable in any one of the original equations and find the value of the eliminated variable.

Example -1: Solve the following pair of linear equations using elimination method.

$$3x + 2y = 11$$
$$2x + 3y = 4$$

Solution: $3x + 2y = 11$ (1)

$2x + 3y = 4$ (2)

Let us eliminate 'y' from the given equations. The coefficients of 'y' in the given equations are 2 and 3. L.C.M. of 2 and 3 is 6. So, multiply equation (1) by 3 and equation (2) by 2.

Equation (1) x 3 9x+6y = 33

Equation (2) x 2 4x + 6y = 8

(-) (-) (-)

$$5x = 25$$

$$x = \frac{25}{5} = 5$$

Substitute x = 5, in equation (1)

$$3(5) + 2y = 11$$

$$2y = 11 - 15 = -4 \Rightarrow y = \frac{-4}{2} = -2$$

Therefore, the required solution is x=5, y = -2.

Example -2 : Rubina went to a bank to withdraw ₹2000. She asked the cashier to give the cash in ₹50 and ₹100 notes only. Rubina got 25 notes in all. Can you tell how many notes each of ₹50 and ₹100 she received?

Solution: Let the number of ₹ 50 notes be x;

Let the number of ₹ 100 notes be y;

Then, x+y = 25 (1)

And 50x + 100y = 2000 (2)

Kavitha used the substitution method.

From equation (1) x = 25 –y

Substituting in equation (2) 50(25 – y) + 100y = 2000

1250 – 50y + 100y = 2000

50y = 2000 – 1250 = 750

$$y = \frac{750}{50} = 15$$
$$x = 25 - 15 = 10$$

∴ Rubina received ten ₹ 50 notes and fifteen ₹ 100 notes.

Example: 3 **Mary told her daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be. "Find the present age of Mary and her daughter.**

Solution: Let Mary's present age be x years and her daughter's age be y years.

Then, seven years ago Mary's age was x – 7 and daughter's age was y – 7.

$$x-7=7(y-7)$$
$$x-7=7y-49 \qquad (1)$$
$$x-7y+42=0$$

Three years hence, Mary's age will be x+3 and daughter's age will be y+3.

$$x+3=3(y+3)$$
$$x+3=3y+9 \qquad (2)$$
$$x-3y-6=0$$

Elimination method

Equation 1 x- 7y = - 42

Equation 2 x – 3y = 6

(-) (+) (-) same sign for x, so subtract.

-4y = -48

$$y=\frac{-48}{-4}=12$$

Substitute the value of y in equation (2)

$$x-3\,(12)-6=0$$
$$x=36+6=42$$

Therefore, Mary's present age is 42 years and her daughter's age is 12 years.

4. The larger of two supplementary angles exceeds the smaller by 18^0. Find the angles.

Sol. Let the lager supplementary angle *be* x^o

The smaller supplementary angle be y^o

$$x=y+18 \Rightarrow x-y=18 \quad \rightarrow (1)$$

Sum of the supplementary angles

$x + y = 180^0 \rightarrow (2)$

Equation (1) x – y = 18

Equation (2) x + y = 180^0

2x = 198^0

$\therefore x = \frac{198^0}{2} = 99^0$

Substitute the value of x in equation (2)

$99^0 + y = 180^0$
$y = 180^0 - 99^0 = 81^0$

The angles are 99^0 and 81^0

5. The sum of a two digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?

Sol. Let the digit in the units place be x tens place be y

$\therefore$ The number is 10y + x

On reversing the digits 10x + y

Given the sum of a two digit number and the number obtained by reversing the digits is 66.

$\therefore (10y + x) + (10x + y) = 66$

$10y + x + 10x + y = 66$
$11x + 11y = 66 \quad x + y = 6 \quad \rightarrow (1)$

Difference of the digits = 2

$\therefore x - y = 2 \rightarrow (2)$

$x + y = 6$
$x - y = 2$

2x =8

$\therefore x = 4$

Substitute the value on x is equation (1)

$4 + y = 6$

$y = 6 - 4$

$y = 2$

$\therefore$ The number is 10 x 4 + 2 = 42

There are only two numbers to satisfy the given condition is 42 and 24.

6. Two angles are complementary. The larger angle is 3^0 less than twice the measure of the smaller angle. Find the measure of each angle.

Sol. Let the larger angle be x and the smaller angle be y

x, y are complementary angles

$\therefore x + y = 90^0 \quad \rightarrow (1)$

The large angle is 3^0 less than twice the measure of the smaller angle

Larger angle is 3 less than twice the measure of the smaller angle.

$\therefore x = 2y - 3 \Rightarrow x - 2y = -3 \rightarrow (2)$

(1) – (2)

$x+y = 90^0$

$x-2y = -3$

(-) (+) (+)

$3y=93$

$y = \frac{93}{3} = 31^0 \Rightarrow y = 31^0$

Substitute y = 31 in equation (1)

$x+31 = 90$

$x=90 - 31 = 59^0$

$\therefore$ The larger angle (x) = 59^0

The smaller angle (y) = 31^0

7. An algebra textbook has a total of 1382 pages. It is broken up into two parts. The second part of the book has 64 pages more than the first part. How many pages are in each part of the book?

Sol. Let the first part be x pages

The second art be y pages

Total number of pages = 1382

$\therefore x + y = 1382 \quad \to (1)$

Second part of the book has 64 pages more than the first part

$\therefore y = x + 64 \Rightarrow y - x = 64$

$-x + y = 64 \to (2)$

Equation (1) x+y = 1382

Equation (2) x+y = 64

2y = 1446

$\therefore \ y = \frac{1446}{2} = 723$

Substitute the value of y in (1)

x + 723 = 1382

$\therefore x = 1382 - 723 = 659$

$\therefore$ Number of pages in each part 659 and 723.

8. Solve each of the following pair of equations by reducing them to a pair of linear equations.

i) $\frac{5}{x-1} + \frac{1}{y-2} = 2; \frac{6}{x-1} - \frac{3}{y-2} = 1$

Sol. $5\left(\frac{1}{x-1}\right) + \left(\frac{1}{y-2}\right) = 2$.................(1)

$6\left(\frac{1}{x-1}\right) - 3\left(\frac{1}{y-2}\right) = 1$.................(2)

If we substitute $\frac{1}{x-1} = p$ and $\frac{1}{y-2} = q$

We get a pair of linear equations.

5p + q = 2 .... (3)

6p – 3q =1 ....(4)

Elimination Method :

Equation (3) x 3 15p+3q = 6

6p - 3q = 1

Add 21p = 7

$$P = \frac{7}{21} = \frac{1}{3}$$

Substitute the value of p in equation (3)

$$5\left(\frac{1}{3}\right) + q = 2 \Rightarrow q = 2 - \frac{5}{3} = \frac{6-5}{3} = \frac{1}{3}$$

But $\frac{1}{x-1} = p \quad \Rightarrow \frac{1}{x-1} = \frac{1}{3}$

$$x - 1 = 3$$

$$\therefore x = 3 + 1 = 4$$

$$\frac{1}{y-2} = q \Rightarrow \frac{1}{y-2} = \frac{1}{3}$$

$$y - 2 = 3; y = 3 + 2 = 5$$

$$\therefore x = 4 \text{ and } y = 5$$

IV. Equations reducible to a pair of linear equations in two variables

Now we shall discuss the solution of pairs of equations which are not linear but can be reduced to linear form by making suitable substitutions. Let us see an example

Example :1 **Solve the following pair of equations** $\frac{2}{x} + \frac{3}{y} = 13$

$$\frac{5}{x} - \frac{4}{y} = -2$$

Solution: We have $2\left(\frac{1}{2}\right) + 3\left(\frac{1}{y}\right) = 13$ (1)

$$5\left(\frac{1}{x}\right)-4\left(\frac{1}{y}\right)=-2 \qquad (2)$$

If we substitute $\frac{1}{x}=p$ and $\frac{1}{y}=q$, we get the following pair of linear equations:

$$2p+3q=13 \qquad (3)$$

$$5p-4q=-2 \qquad (4)$$

Coefficients of q are 3 and 4 and their L.C.M is 12. Using the elimination method:

Equation (3) x 4 8p+12q=52

Equation (4) x 3 15p-12q=-6 'q' terms have opposite sign, so we add the two equations.

23p=46

$$p=\frac{46}{23}=2$$

Substitute the value of p in equation (3)

$$2(2)+3q=13$$
$$3q=13-4=9$$
$$q=\frac{9}{3}=3$$

But, $\frac{1}{x}=p=2 \quad \Rightarrow x=\frac{1}{2}$

$\frac{1}{y}=q=3 \Rightarrow y=\frac{1}{3}$

Example -2. A man travels 370 km partly by train and partly by car. If he covers 250 km by train and the rest by car, it takes him 4 hours but if he travels 130 km by train and the rest by car, it takes 18 minutes more. Find the speed of the train and that of the car.

Solution: Let the speed of the train be x km. per hour and that of the car be y km. per hour.

Also, we know that time = $\frac{Distance}{Speed}$

In situation 1, time spent travelling by train $=\frac{250}{x}hrs.$

And time spent travelling by car $=\frac{120}{y}hrs.$

So, total time taken = time spent in train + time spent in car $= \frac{250}{x} + \frac{120}{y}$

But, total time of journey is 4 hours, so

$$\frac{250}{x} + \frac{120}{y} = 4$$

$$\frac{125}{x} + \frac{60}{y} = 2 \qquad \rightarrow (1)$$

Again, when the travels 130 km by train and the rest by car

Time taken by him to travel 130 km by train $= \frac{130}{x}$ *hrs.*

Time taken by him to travel 240 km (370-130) by car $= \frac{240}{y}$ *hrs.*

Total time taken $\frac{130}{x} + \frac{240}{y}$

3

But given, time of journey is 4 hrs 18 min i.e., $4\frac{18}{60} hrs. = 4\frac{3}{10} hrs.$

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So, $\frac{130}{x} + \frac{240}{y} = \frac{43}{10}$ (2)

Substitute $\frac{1}{x} = a$ *and* $\frac{1}{y} = b$ in equations (1) and (2)

$125a + 60b = 2$ (3)

$130a + 240b = 43/10$ (4)

For 60 and 240, L.C.M. is 240. Using the elimination method,

Equation (3) x 4 500a+240b=8

Equation (4) x 1 130a+240b=$\frac{43}{10}$ (Same sign, so subtract)

(-) (-) (-)

$$370a = 8 - \frac{43}{10} = \frac{80-43}{10} = \frac{37}{10}$$

$$a = \frac{37}{10} \times \frac{1}{\frac{370}{10}} = \frac{1}{100}$$

Substitute $a = \frac{1}{100}$ in equation (3)

$$\left[125 \times \frac{1}{100}\right] + 60b = 2$$

$$60b = 2 - \frac{5}{4} = \frac{8-5}{4} = \frac{3}{4} \Rightarrow b = \frac{3}{4} \times \frac{1}{60} = \frac{1}{80}$$

$$So\ a = \frac{1}{100}\ and\ b = \frac{1}{80}$$
$$\frac{1}{x} = \frac{1}{100}\ and\ \frac{1}{y} = \frac{1}{80}$$

x=100 km/ hr and y = 80 km/ hr.

So, speed of train was 100 km/ hr and speed of car was 80 km / hr.

On solving simultaneous linear Equations in two variables

Example: 1 Solve the following system of equations:

$$8a - 3b = 5ab$$
$$6a - 5b = 2ab$$

Solution: Clearly the given equations are not linear in the variables a and b but can be reduced into linear equations by the appropriate substitution.

If we put $a = 0$ in either of the two equations, we get $b = 0$.

Thus, $a = 0, b = 0$ form one solution of the given system of equations.

To find the other solutions, we assume that $a \neq 0, b \neq 0$.

Since $a \neq 0, b \neq 0$. therefore, $ab \neq 0$.

On dividing each of the given equations by ab, we get

$$\frac{8}{a} - \frac{3}{b} = 5$$
$$\frac{6}{a} - \frac{5}{b} = -2$$

Taking $\frac{1}{a} = x$ *and* $\frac{1}{b} = y$, the above equations become

$$8x - 3y = 5$$
$$6x - 5y = -2$$

Multiplying equation (i) by 3 and equation (ii) by 4, we get

$$24x - 9y = 15$$
$$24x - 20y = -8$$

Subtracting equation (vi) from equation (v), we get

$$11y = 23 \Rightarrow y = \frac{23}{11}$$

Putting $y = \frac{23}{11}$ in equation (iv), we get

$$6x - 5\left(\frac{23}{11}\right) = -2 \Rightarrow 6x = \frac{93}{11} x = \frac{93}{11} \times \frac{1}{6} = \frac{31}{22}$$

Now, $x = \frac{31}{22} \Rightarrow \frac{1}{a} = \frac{31}{22} \Rightarrow a = \frac{22}{31}$

And, $y = \frac{23}{11} \Rightarrow \frac{1}{b} = \frac{23}{11} \Rightarrow b \frac{11}{23}$

Hence, the given system of equations has two solutions given by

(i) a=0, b=0 (ii) $a = \frac{22}{31}, b = \frac{11}{23}$

Type IV Equations of the form $ax + by = c$ **and** $bx + ay = d$ **where** $a \neq b$

To solve the above type of equations, following algorithm may be used.

Algorithm

STEP I Obtain the two equations.

Let the equation be $ax+by=c \text{ and } bx+ay=d$

STEP II Adding and subtracting the two equations, we obtain

$$(a+b)x+(a+b)y=c+d \Rightarrow x+y=\frac{c+d}{a+b}$$
$$(a-b)x-(a-b)y=c-d \Rightarrow x-y=\frac{c-d}{a-b}$$

STEP III Add and subtract equations (i) and (ii) to get the values of $x \text{ and } y$

Type IV Equations of the Form

$$a_1x+b_1y+c_1=d_1$$
$$a_2x+b_2y+c_2=d_2$$
$$a_3x+b_3y+c_3=d_3$$

To solve the above type of equations, following algorithm may be used.

Algorithm

STEP I Take any one of the three equations.

STEP II Obtain the value of one of the variable, say z from it.

STEP III Substitute the value of z obtained in Step II in the remaining two equations to obtain two linear equations in x, y

STEP IV Solve the equations in x, y obtained in Step III by elimination method.

STEP V Substitute the values of x, y obtained in Step IV and step II to get the value of z.

1. Method of cross – Multiplication

Theorem Let $a_1x+b_1y+c_1=0$

$$a_2x+b_2y+c_2=0$$

be a system of simultaneous linear equations in two variables $x \text{ and } y$ such that $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$ i.e. $a_1b_2\neq 0$. Then the system has a unique solution given by

$$x = \frac{(b_1c_2 - b_2c_1)}{(a_1b_2 - a_2b_1)} \text{ and } y = \frac{(c_1a_2 - c_2a_1)}{(a_1b_2 - a_2b_1)}$$

or

$$\frac{x}{\begin{matrix} b_1 & c_1 \\ b_2 & c_2 \end{matrix}} = \frac{-y}{\begin{matrix} a_1 & c_1 \\ a_2 & c_2 \end{matrix}} = \frac{1}{\begin{matrix} a_1 & b_1 \\ a_2 & b_2 \end{matrix}}$$

Example : 1 **Solve the each of the following systems of equations by using the method of cross- multiplication :**

(*i*) $x + y = 7$

$5x + 12y = 7$

Solution: (i) The given system of equations is

$$x + y - 7 = 0$$
$$5x + 12y - 7 = 0$$
$$a_1 = 1, b_1 = 1, c_1 = -7$$
$$a_2 = 5, b_2 = 12, c_2 = -7$$

By cross – multiplication, we get

$$\frac{x}{\begin{matrix} 1 & -7 \\ 12 & -7 \end{matrix}} = \frac{-y}{\begin{matrix} 1 & -7 \\ 5 & -7 \end{matrix}} = \frac{1}{\begin{matrix} 1 & 1 \\ 5 & 12 \end{matrix}}$$

$$\Rightarrow \frac{x}{1\times -7 - 12\times -7} = \frac{-y}{1\times -7 - 5\times -7} = \frac{1}{1\times 12 - 5\times 1}$$

$$\Rightarrow \frac{x}{-7+84} = \frac{-y}{-7+35} = \frac{1}{12-5}$$

$$\Rightarrow \frac{x}{77} = \frac{-y}{28} = \frac{1}{7}$$

$$\Rightarrow x = \frac{77}{7} \text{ and } y = -\frac{28}{7} \Rightarrow x = 11 \text{ and } y = -4$$

Hence, the solution of the given system of equations is x = 11, y = - 4.

Example: 2 **Solve** : $\frac{x}{a} + \frac{y}{b} = a + b$

$$\frac{x}{a^2} + \frac{y}{b^2} = 2$$

Solution The given system of equations may be written as

$$\frac{1}{a}.x+\frac{1}{b}.y-(a+b)=0$$
$$\frac{1}{a^2}.x+\frac{1}{b^2}.y-2=0$$

By cross – multiplication, we have

$$\frac{x}{\begin{array}{cc}1/b & -(a+b)\\ 1/b^2 & -2\end{array}}=\frac{-y}{\begin{array}{cc}1/a & -(a+b)\\ 1/a^2 & -2\end{array}}=\frac{1}{\begin{array}{cc}1/a & 1/b\\ 1/a^2 & 1/b^2\end{array}}$$

$$\Rightarrow \frac{x}{\frac{1}{b}\times(-2)-\frac{1}{b^2}\times-(a+b)}=\frac{-y}{\frac{1}{a}\times-2-\frac{1}{a^2}\times-(a+b)}=\frac{1}{\frac{1}{a}\times\frac{1}{b^2}-\frac{1}{a^2}\times\frac{1}{b}}$$

$$\Rightarrow \frac{x}{-\frac{2}{b}+\frac{a}{b^2}+\frac{1}{b}}=\frac{-y}{-\frac{2}{a}+\frac{1}{a}+\frac{b}{a^2}}=\frac{1}{\frac{1}{ab^2}-\frac{1}{a^2b}}$$

$$\Rightarrow \frac{x}{\frac{a}{b^2}-\frac{1}{b}}=\frac{-y}{-\frac{1}{a}+\frac{b}{a^2}}=\frac{1}{\frac{1}{ab^2}-\frac{1}{a^2b}}$$

$$\Rightarrow \frac{x}{\frac{a-b}{b^2}}=\frac{y}{\frac{a-b}{a^2}}=\frac{1}{\frac{a-b}{a^2b^2}}$$

$$\Rightarrow x=\frac{a-b}{b^2}\times\frac{1}{\frac{a-b}{a^2b^2}}=a^2 \text{ and } y=\frac{a-b}{a^2}\times\frac{1}{\frac{a-b}{a^2b^2}}=b^2$$

Hence, $x=a^2, y=b^2$ is the solution f the given system of equations.

Example: 3 **Solve :** $\frac{x}{a}+\frac{y}{b}=2$

$$ax-by=a^2-b^2$$

Solution: The given system of equations may be written as

$$bx+ay-2ab=0$$
$$ax-by-(a^2-b^2)=0$$

By cross – multiplication, we have

$$\frac{x}{-a(a^2-b^2)-(-b)(-2ab)} = \frac{-y}{-b(a^2-b^2)-a(-2ab)} = \frac{1}{b\times -b - a\times a}$$

$$\Rightarrow \frac{x}{-a(a^2-b^2)-2ab^2} = \frac{-y}{-b(a^2-b^2)+2a^2b} = \frac{1}{-b^2-a^2}$$

$$\Rightarrow \frac{x}{-a(a^2-b^2+2b^2)} = \frac{-y}{-(a^2-b^2-2a^2)} = \frac{1}{-(a^2+b^2)}$$

$$\Rightarrow \frac{x}{-(a^2+b^2)} = \frac{-y}{-b(-a^2-b^2)} = \frac{1}{-(a^2+b^2)}$$

$$\Rightarrow x = \frac{-a(a^2+b^2)}{-(a^2+b^2)} = a \text{ and } y = \frac{-b(a^2+b^2)}{-(a^2+b^2)} = b$$

Hence, the solution of the given system of equations is $x = a, y = b$.

Example: 4 **Find the value of k for which the following system of linear equations has infinite solutions:**

$$x + (k+1)y = 5$$
$$(k+1)x + 9y = 8k - 1$$

Solution: The given system of equations will have infinite solutions, if

$$\frac{1}{k+1} = \frac{k+1}{9} = \frac{5}{8k-1}$$

$$\Rightarrow \frac{1}{k+1} = \frac{k+1}{9} \text{ and } \frac{k+1}{9} = \frac{5}{8k-1}$$

$$\Rightarrow (k+1)^2 = 9 \text{ and } (k+1)(8k-1) = 45$$

$$Now,\ (k+1)^2 = 9$$

$$\Rightarrow k+1 = \pm 3 \Rightarrow k = 2, -4$$

We observe that k = 2 satisfies the equation (k+1) (8k-1) =45 but, k=-4 does not satisfy it.

Hence, the given system of equations will have infinitely many solution, if k=2.

Applications to word problems

In this section, we shall learn about some applications of simultaneous linear equations in solving problems related to our day – to – day life. There is a wide variety of such problems which are generally called 'word problems'. In solving such problems, we may use the following algorithm.

Algorithm

Step I Read the problem carefully and identify the unknown quantities. Give these quantities a variable name like x, y, a, b, w etc.

Step II Identify the variables to be determined.

Step III Read the problem carefully and formulate the equations in terms of the variables to be determined.

Step IV Solve the equations obtained in step III using any one of the methods learnt earlier.

Applications to problems based on articles and their costs

Following examples will illustrate the applications of linear equations in solving word problems based on articles and their costs.

Example : 1 **4 Chairs and 3 tables cost ₹ 2100 and 5 chairs and 2 tables cost ₹ 1750. Find the cost of a chair and a table separately.**

Solution Let the cost of a chair be ₹ x and that of a table be ₹ y. Then.

$$4x + 3y = 2100$$

And, $$5x + 2y = 1750$$

This system of equations can be written as

$$4x + 3y - 2100 = 0$$
$$5x + 2y - 1750 = 0$$

By using cross – multiplication, we have

$$\frac{x}{3\times -1750 - 2\times -2100} = \frac{-y}{4\times -1750 - 5\times -2100} = \frac{1}{4\times 2 - 3\times 5}$$

$$\Rightarrow \frac{x}{-5250 + 4200} = \frac{-y}{-7000 + 10500} = \frac{1}{8-15}$$

$$\Rightarrow \frac{x}{-1050} = \frac{y}{-3500} = \frac{1}{-7}$$

$$\Rightarrow x = \frac{-1050}{-7} = 150 \ and \ y = \frac{-3500}{-7} = 500.$$