1. REAL NUMBERS
KEY CONCEPTS

1. Natural numbers (N) = {1, 2, 3, 4.......}
2. Whole numbers (W) = {0, 1, 2, 3, ...}
3. Integers (I / Z) = { .... -3, -2, -1, 0, 1, 2, 3, ...}
4. Rational Numbers (Q) = {x/x= $\frac{p}{q}$ q $\neq$ 0, p and q are Integers}
5. Euclid's Division Lemma: Given positive integers a and b, there exist unique pair of integers q and r satisfying a = bq + r, $0 \leq r < b$.
6. Euclid's division algrithm is not only useful for calculating the HCF of very large numbers, but also because it is one of the earliest examples of an algorithm that a computer had been programmed to carry out.
7. Fundamental Theorem of Arithmetic: Every composite number can be expressed (factorised) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.
8. Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form of , where p and q are coprime, and the prime factorization of q is the form 2^n5^m, where n, m are non-negative integers.
9. Let $x = \frac{p}{q}$ be a rational number, such $\frac{p}{q}$ that the prime factorization of q is of the form 2^n5^m, where n and m are non-negative integers. Then x has a decimal expansion which terminates.
10. Let $x = \frac{p}{q}$ be a rational numbers, such that the prime factorization of q is not of the form 2n5m, where n and m are non-negative integers. Then, x has a decimal expansion which is non-terminating repeating (recurring).
11. Let p be a prime number. If p divides a^2, (where a is a positive integer), then p divides a.
12. The sum of the two irrational numbers need not be irrational.
    For example, if a = $\sqrt{2}$ and b = $-\sqrt{2}$, then both a and b are irrational, but a + b =0 which is rational.
13. The product of two irrational numbers need not be irrational.
    For example, a = $\sqrt{2}$ and b = $\sqrt{8}$, then both a and b are irrational, but ab = $\sqrt{16}$ = 4 which is rational.

EXAMPLES

EXAMPLE 1 Use Euclid's division algorithm to find the HCF of 210 and 55.

SOLUTION Given integers are 210 and 55. Clearly, 210 > 55. Applying Euclid's division lemma to 210 and 55, we get

$$210 = 55 \times 3 + 45 \qquad \text{...(i)} \qquad \left[\begin{array}{c} \because 55\overline{)210}(3 \\ \underline{165} \\ 45 \end{array}\right]$$

Since the remainder 45 ‘“ 0. So, we apply the division lemma to the divisor 55 and remainder 45 to get

$$55 = 45 \times 1 + 10 \qquad \text{...(ii)} \qquad \left[\begin{array}{c} \because 45\overline{)55}(1 \\ \underline{45} \\ 10 \end{array}\right]$$

Now, we apply division lemma to the new divisor 45 and new remainder 10 to get

$$45 = 10 \times 4 + 5 \qquad \text{...(iii)} \qquad \left[\begin{array}{c} \because 10\overline{)45}(4 \\ \underline{40} \\ 5 \end{array}\right]$$

We now consider the new divisor 10 and the new remainder 5, and apply division lemma to get

$$10 = 5 \times 2 + 0$$

The remainder at this stage is zero. So, the divisor at this stage or the remainder at the previous stage i.e. 5 is the HCF of 210 and 55.

EXAMPLE 2 Use Euclid's division algorithm to find the HCF of 441, 567 and 693.

SOLUTION Applying Euclid's division lemma to 441 and 567, we obtain

$$567 = 441 \times 1 + 126$$

We apply Euclid's division lemma to 441 (divisor) and 126 (remainder) to get

$$441 = 126 \times 3 + 63$$

Now, we apply Euclid's division lemma to the divisor 126 and the remainder 63, to get

$$126 = 63 \times 2 + 0$$

The remainder at this stage is 0. So, the divisor at the previous stage i.e., 63 is the HCF of 441 and 567.

Now, we use Euclid's division lemma to find the HCF of 63 and 693. We observe that

$$693 = 63 \times 11 + 0$$

So, the HCF of the third number 693 and 63 (the HCF of first two numbers 441 and 567) is 63.

Hence, the HCF of 441, 567 and 693 is 63.

EXAMPLE 3 Two tankers contain 850 litres and 680 litres of petrol respectively. Find the maximum capacity of a container which can measure the petrol of either tanker in exact number of times.

SOLUTION Let us find the HCF of 850 and 680 by Euclid's algorithm.

```
680) 850 (1
     680
     ---
     170) 680 (4
          680
          ---
           5
```

Clearly, HCF of 850 and 680 is 170.

Hence, capacity of the container must be 170 litres.

EXAMPLE 4 Find the largest number which divides 245 and 1029 leaving remainder 5 in each case.

SOLUTION It is given that the required number when divides 245 and 1029, the remainder is 5 in each case. This means that 245 – 5 = 240 and 1029 – 5 = 1024

If follows from this that the required number is a common factor of 240 and 1024.

Therefore, it is the HCF of 240 and 1024.

Let us now find the HCF of 240 and 1024 by Euclid's algorithm.

```
240) 1024 (4
      960
      ---
       64) 240 (3
           192
           ---
            48) 64 (1
                48
                --
                16) 48 (3
                    48
                    --
                     0
```

Clearly, HCF of 240 and 1024 is the last divisor i.e. 16. Hence, required number = 16.

EXAMPLE 5 Find the largest number that divides 2053 and 967 and leaves a remainder of 5 and 7 respectively.

SOLUTION It is given that on dividing 2053 by the required humber, there is a remainder of 5. This means that 2053 -5 = 2048 is exactly divisible by the required number.

Similarly, 967 – 7 = 960 is also exactly divisible by the required number.

Also, the required number is the largest number satisfying the above property.

Therefore, it is the HCF of 2048 and 960.

Let us now find the HCF of 2048 and 960 by Euclid's algorithm.

$$
\begin{array}{l}
960)\,2048\,(2 \\
\quad\;\; 1920 \\
\quad\;\; \overline{128)}\,960\,(7 \\
\qquad\qquad 896 \\
\qquad\qquad \overline{64)}\,128\,(2 \\
\qquad\qquad\qquad \underline{128} \\
\qquad\qquad\qquad\;\; 0
\end{array}
$$

Clearly, HCF of 960 and 2048 is the last divisor i.e. 64. Hence, required number = 64.

EXAMPLE 6 If the HCF of 210 and 55 is expressible in the form 210 x 5 + 55y, find y.

SOLUTION Let us first find the HCF of 210 and 55.

$$
\begin{array}{l}
55)\,210\,(3 \\
\quad\;\; 165 \\
\quad\;\; \overline{45)}\,55\,(1 \\
\qquad\quad\; 45 \\
\qquad\quad\; \overline{10)}\,45\,(4 \\
\qquad\qquad\;\; 40 \\
\qquad\qquad\;\; \overline{5)}\,10\,(2 \\
\qquad\qquad\qquad \underline{10} \\
\qquad\qquad\qquad\; 0
\end{array}
$$

5 is the HCF of 210

$\therefore \quad 5 = 210 \times 5 + 55y$

$\Rightarrow \; 55y = 5 - 210 \times 5 = 5 - 1050$

$\Rightarrow \; 55y = -1045$

$$\Rightarrow y = \frac{-1045}{55} = -19$$

EXAMPLE 7 In a seminar, the number of participants in Hindi, English and Mathematics are 60, 84 and 108, respectively. Find the minimum number of rooms required if in each room the same number of participants are to be seated and all of them being in the same subject.

SOLUTION The number of participants in each room must be the HCF of 60, 84 and 108. In order to find the HCF of 60, 84 and 108, we first find the HCF of 60 and 84 by Euclid's division algorithm :

$$
\begin{array}{l}
60)\,84\,(1 \\
\quad\ 60 \\
\quad\ \overline{24})\,60\,(2 \\
\qquad\quad 48 \\
\qquad\quad \overline{12})\,24\,(2 \\
\qquad\qquad\ \ \underline{24} \\
\qquad\qquad\ \ \ 0
\end{array}
$$

Clearly, HCF of 60 and 84 is 12 $\frac{252}{12} = 21.$

Now, we find the HCF of 12 and 108

$$
\begin{array}{l}
12)\,108\,(9 \\
\quad\ \underline{108} \\
\quad\ \ \ 0
\end{array}
$$

Clearly, HCF of 12 and 108 is 12. Hence, the HCF of 60, 84 and 108 is 12.

Therefore, in each room maximum 12 participants can be seated.

We have,

Total number of participants = 60 + 84 + 108 = 252

$\therefore$ Number of rooms required =

EXAMPLE 8 Find the HCF of 81 and 237 and express it as a linear combination of 81 and 237.

SOLUTION Given integers are 81 and 237 such that 81 < 237.

Applying division lemma to 81 and 237, we get

237 = 81 x 2 + 75 ...(i) $\left[\because 81\overline{)237}(2 \\ \quad 162 \\ \quad \overline{75}\right]$

Since the remainder 75 ‘“ 0. So, consider the divisor 81 and the remainder 75 and apply division lemma to get

81 = 75 x 1 + 6 ...(ii) $\left[\because 75\overline{)81}(1 \\ \quad 75 \\ \quad \overline{6}\right]$

We consider the new divisor 75 and the new remainder 6 and apply division lemma to get

75 = 6 x 12 + 3 ...(iii) $\left[\because 6\overline{)75}(12 \\ \quad 72 \\ \quad \overline{3}\right]$

We consider the new divisor 6 and the new remainder 3 and apply division lemma to get

6 = 3 x 2 + 0 ...(iv) $\left[\because 3\overline{)6}(2 \\ \quad 6 \\ \quad \overline{0}\right]$

The remainder at this stage is zero. So, the divisor at this stage or the remainder at the earlier stage i.e. 3 is the HCF of 81 and 237.

To represent the HCF as a linear combination of the given two numbers, we start from the last but one step and successively eliminate the previous remainders as follows :

From (iii), we have

$$\begin{bmatrix} Substituting \quad 6 = 81 - 75 \times 1 \\ obtained \ \ from \ (ii) \end{bmatrix}$$

3 = 75 – 6 x 12

$$\begin{bmatrix} Substituting \quad 75 = 237 - 81 \times 2 \\ obtained \ \ from \ (i) \end{bmatrix}$$

$$\begin{bmatrix} Substituting \quad 6 = 81 - 75 \times 1 \\ obtained \ \ from \ (ii) \end{bmatrix}$$

$\Rightarrow\ 3 = 75-(81-75\times1)\times12$

$\Rightarrow\ 3 = 75-12\times81+12\times75$

$\Rightarrow\ 3 = 13\times75-12\times81$

EXAMPLE 9 Find the HCF of 65 and 117 and express it in the form 65 m + 117 n.
SOLUTION Given integers are 65 and 117 such that 117 > 65.
Applying division lemma to 65 and 117, we get

117 = 65 x 1 + 52 ...(i) $\left[\because 65\overline{)117}(1 \quad \frac{65}{52}\right]$

Since the remainder 52 ‘“ 0. So, we apply the division lemma to the divisor 65 and the remainder 52 to get

$\Rightarrow\ 3 = 13\times(237-81\times2)-12\times81$

$\Rightarrow\ 3 = 13\times237-26\times81-12\times81$

$\Rightarrow\ 3 = 13\times237-38\times81$

$\Rightarrow\ 3 = 237x+81y,\ where\ x = 13\ and\ y = -38.$

65 = 52 x 1 + 13 ...(ii) $\left[\because 52\overline{)65}(1 \quad \frac{52}{13}\right]$

We consider the new divisor 52 and the new remainder 13 and apply division lemma, to get

52 = 13 x 4 + 0

At this stage the remainder is zero. So, the last divisor or the non- zero remainder at the earlier stage i.e. 13 is the HCF of 65 and 117.

From (ii), we have

[Substituting 52 = 117 – 65 x 1 obtain from (i)]
13=65-52 x1

$\Rightarrow\ 13 = 65-(117-65\times1)$

$\Rightarrow\ 13 = 65-(117-65\times1$

$\Rightarrow\ 13 = 65\times2+117\times(-1)$

$\Rightarrow 13 = 65\,m+117\,n,\ where m = 2\ and\ n = -1.$

EXAMPLE 10 Express each of the following positive integers as the product of its prime factors :

(i) 140 (ii) 156 (iii) 234

SOLUTION (i)

7	140
2	20
2	10
	5

140 = 7 x 2 x 2 x 5

(ii)

2	156
2	78
3	39
13	13
	1

156 = 2 x 2 x 3 x13

(iii)

2	234
3	117
3	39
	13

234 = 2 x 3 x 3 x 13

EXAMPLE 11 Express each of the following positive integers as the product of its prime factors :

(i) 3825 (ii) 5005 (iii) 7429

SOLUTION

(i)

3	3825
3	1275
5	425
5	85
17	17
	1

3825 = 3 x 3 x 5 x 5 x 17

= 3^2 x 5^2 x 17

(ii)

5	5005
7	1001
11	143
	13

5005 = 5 x 7 x 11 x 13

(iii)

17	7429
19	437
23	23
	1

7429 = 17 x 19 x 23

EXAMPLE 12 Prove that there is no natural number for which 4^n ends with the digit zero.

SOLUTION We know that any positive integer ending with the digit zero is divisible by 5 and so its prime factorization must contain the prime 5.

We have,

$4^n = (2^2)^n = 2^{2n}$

$\Rightarrow$ The only prime in the factorization of 4^n is 2.

$\Rightarrow$ There is no other primes in the factorization of $4^n = 2^{2n}$

(By uniqueness of the Fundamental Theorem of Arithmetic)

5 does not occur in the prime factorization of 4^n for any n.

4^n does not end with the digit zero for any natural number n.

EXAMPLE 13 Show that 12^n cannot end with digit 0 or 5 for any natural number n.

SOLUTION Expressing 12 as the product of primes, we obtain

12 = 2^2 x 3

So, only primes in the factorisation of 12^n are 2 and 3 and, not 5.

Hence, 12^n cannot end with digit 0 or 5.

1. Explain why 7 x 11 x 13 + 13 and 7 x 65 x 4 x 3 x 2 x 1 + 5 are composite numbers.

   Since 7 x 11 x 13 + 13 = (7 x 11 + 1) x 13 = 78 x 13 = 13 x 13 x 2 x 3

   and 7 x 6 x 5 x 4 x 3 x 2 x 1 +5 = (7 x 6 x 4 x 3 x 2 x 1 +1) x 5

   5 is a factor of both terms

   7 x 6 x 5 x 4 x 3 x 2 x 1 + 5 is a composite

2. Check whether 6^n can end with the digit 0 for any natural number n.

   We have, $6^n = (2 \times 3)^n = 2^n \times 3^n$. Therefore, prime factorisation of 6^n does not contain 5 as a factor. Hence, 6^n can never end with the digit 0 for any natural number.

3. Explain why 3 x 5 x 7 + 7 is a composite number.

   Since 3 x 5 x 7 +7 = (3 x 5 +1) x 7 = (15 +1) x 7 = 16 x 7. Hence, it is a composite number.

Some Important Problems

1. Use Euclid's division algorithm to find the HCF of

i) 900 and 270

Sol. When 900 is divided by 270. The remainder is 90 to get.

$900 = 270 \times 3 + 90$

Now consider the division of 270 with the remainder 90 in the above and apply the division algorithm.

$270 = 90 \times 3 + 0$

Notice that the remainder become zero and we cannot proceed any further.

The HCF of 900 and 270 is the divisor at this stage i.e., 90

ii) 196 and 38220

Sol. When 38220 is divided by 196 the remainder is 0 to get

$38220 = 196 \times 195 + 0$

Notice that the remainder become zero and we cannot proceed any further.

The HCF of 196 and 38220 is the divisor at this stage i.e., 196.

iii) 1651 and 2032

Sol. When 2032 is divided by 1651 the remainder is 381 to get

$2032 = 1651 \times 1 + 381$

Now consider the divisor of 1651 with the remainder 381 is the above and apply the division algorithm to get

$1651 = 381 \times 4 + 127$

Now consider the division of 381 with the remainder 127 in the above and apply the division algorithm to get

$381 = 127 \times 3 + 0$

Notice that the remainder become zero and we cannot proved any further.

The HCF of 1651 and 2032 is the divisor at this stage i.e., 127

2. Use Euclid division lemma to show that any positive odd integer is of the form 61 + 1 or 6q + 3 or 6q + 5, where q is some integer.

Sol. Let us start with taking 'a' where 'a' is any positive odd integer.

We apply the division lemma with a and b = 6

Since $0 \leq r < 6$, the possible remainders are 0, 1, 2, 3, 4, 5.

That is 'a' can be 6q or 6q + 1 or 6q + 2 or 6q + 3 or 6q + 4 or 6q + 5 where q is the quotient.

However, since 'a' is odd, we do not consider the cases 6q, 6q + 2 and 6q + 4 (Since allt he three are divisible by 2)

Any positive odd integer is of the form.

6q + 1 or 6q + 3 or 6q + 5

3. Use Euclid's division lemma to show that the square of any positive integer is of the form 3p, 3p + 1 or 3p + 2.

Sol. Let 'a' be any odd positive integer. We apply the division lemma with a and b = 3

Since $0 \leq r < 3$ the possible remainders are 0, 1 and 2.

That is 'a' can be 3m or 3m + 1 or 3m + 2 where m is the quotient.

Now, $(3m)^2 = 9m^2$

Which can be written in the form 3p, since 9 is divisible by 3

Again $(3m + 1)^2 = 9m^2 + 6m + 1$

$= 3 (3m^2 + 2m) 1$

Which can be written in the form 3p + 1 since $9m^2 + 6m$ i.e.,

$3 (3m^2 + 2m)$ is divisible by 3

Which can be written in the form 3p + 1 since $9m^2 + 12m + 3$ i.e., $3 (3m^2+4m+1)$ is divisible by 3

The square of any positive inteer is of the form 3p or 3p + 1 for some integer p.

4. Use Euclid's division lemma to show that the cube of any positive ineger is of the form 9m, 9m + 1 or 9m + 8.

Sol. Let 'a' be any positive integer.

We apply Euclid's division lemma with a and b = 3

Since $0 \leq r < 3$ the possible remainders are 0, 1 and 2

That is can be 3p or 3p +1 or 3p 2 where is the quotient

Now $(3p)^3 = 27p^3$ which can be written in the form 9m since 9 is divisible by 9.

Again $(3p + 1)3 = 27p^3 + 29p^2 + 9p + 1$

$= 9 (3p^3 + 3p^2 + p) + 1$

which can be written in the form 9m + 1

since $27p^3 + 27p^2 + 9p$ is $9 (3p^3 + 3p^2 + p)$ is divisible by 9.

Lastly $(3p + 2)^2 = 27p^3 + 54p^2 + 36p + 8$

$= 9 (3p^3 + 6p^2 + 4p) + 8$

which can be written in the form

9m + 8. Since $27p^3 + 54p^2 + 36p$

i.e., $9 (3p^3 + 6p^2 + 4p)$ is divisible by 9.

The cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8,.

5. Show that one and only one out of n, n + 2 or n + 4 is divisible by 3 where n is any positive integer.

Sol. We know that any positive integer is of the form 3q or 3q + 1 or 3q + 2 for some integer q and one and only one of these possibilities can occuar.

So, we have following cases:

Case (i): When n = 3q

In this case, we have

n = 3q which is divisible by 3

Now n = 3q

n + 2 = 3q + 2

n + 2 leaves remainder 2 when divided by 3.

n + 2 is not divisible by 3

Thus, n is divisible by 3 but n + 2 and n + 4 are not divisible by 3.

case (ii): When n = 3q + 1

In this case we have

n = 3q + 1

n leaves remainder 1 when divided by 3

n is not divisble by 3

Now n = 3q + 1

n + 2 = (3q + 1) + 2 = 3q + 3 = 3 (q+1)

n + 2 is divisible by 3

Again n = 3q + 1

n + 4 = 3q + 1 + 4 = 3q + 5 = 3 (q + 1) + 2

n + 4 leaves remainder 2 when divided by 3

n + 4 is not divisible by 3

Thus n + 2 is divisble by 3 but n and n + 4 are not divisible by 3.

Case (iii): When n = 3q + 2

In this case we have

n = 3q + 2

n leaves remainder 2 when divided by 3

n is not divisible by 3

Now n = 3q + 2

n + 2 = 3q + 2 + 2 = 3q + 4 = 3q + 3 + 1 = 3 (q+1) + 1

n + 2 leaves remainder 1 when divided by 3

n + 2 is not divisible by 3

Against n = 3q + 2

n + 4 = 3q + 2 + 4 = 3q + 6 = 3 (q + 2)

n + 4 is divisible by 3

Thus n + 2 is divisible by 3 but n and n + 2 are not divisible by 3.

6. Find the LCM and HCF of the following integers by the prime factorization method.

i) 12, 15 and 21 ii) 17, 23 and 29 iii) 8, 9 and 25

iv) 72 and 108 v) 306 and 657

Sol. i) 12, 15 and 21 ⇒

2	12
2	6
3	3
	1

3	15
5	5
	1

We have

12 $= 2 \times 2 \times 3 = 2^2 \times 3^1$

15 $= 3^1 \times 5^1$

21 $= 3^1 \times 7^1$

HCF 12, 15 and 21 = 3

= Product of the smallest power of each common prime factors in the numbers.

LCM 12, 15 and 21 $= 2^2 \times 3^1 \times 5^1 \times 7^1$

$= 4 \times 3 \times 5 \times 7$

$= 420.$

$2^2 \times 3^1 \times 5^1 \times 7^1$ = Product of the greatest power of each prime factors in the numbers.

ii) 17, 23 and 29

We know 17, 23 and 29 are prime numbers.

HCF 17, 23 and 29 = 1

LCM (17, 23 and 29)

$= 17 \times 23 \times 29 = 391 \times 29 = 11339$

iii) 8, 9 and 25

We have $8 = 2 \times 2 \times 2 = 2^3$

$9 = 3 \times 3 = 3^2$

$25 = 5 \times 5 = 5_2$

HCF 8, 9 and 25 = 1

LCM 8, 9 and 25 $= 2^3 \times 3^2 \times 5^2$

$= 8 \times 9 \times 25$

$= 72 \times 25 = 1800$

iv) 72 and 108

We have $72 = 2 \times 36$

$= 2 \times 2 \times 18$

$= 2 \times 2 \times 2 \times 9$

$= 2 \times 2 \times 2 \times 3 \times 3$

$= 2^3 \times 3^2$

$108 = 2 \times 54$

$= 2 \times 2 \times 27$

$= 2 \times 2 \times 3 \times 9$

$= 2 \times 2 \times 3 \times 3 \times 3$

$= 2^2 \times 3^3$

2	72
2	36
2	18
3	9
	3

2	108
2	54
3	27
3	9
	3

HCL 72 and $108 = 2^2 \times 3^2 = 4 \times 9 = 36$

LCM 72 and $108 = 2^3 \times 3^3 = 8 \times 27 = 216$

v) 306 and 657

We have $306 = 2 \times 153$

$= 2 \times 3 \times 51$

$= 2 \times 3 \times 3 \times 17$

$= 2^1 \times 3^2 \times 17^1$

$657 = 3 \times 219$

$= 3 \times 3 \times 73 = 3^2 \times 73^1$

HCF (306 and 657) $= 3^2 = 9$

LCM (306 and 657) $= 2^1 \times 3^2 \times 17^1 \times 73^1$

$= 2 \times 9 \times 17 \times 73$

$= 18 \times 17 \times 73$

$= 306 \times 73 = 22338$

7. Check whether 6^n can end with the digit 0 for any natural number n.

Sol. For the number 6^n to end with digit ‘0’ for any natural number n, it should be divisible by 5.

This means that the prime factorization of 6^n should contain the prime number 5.

But it is not possible because $6^n = (3 \times 2)^n$

So 2 and 3 are the only primes in the factorization of 6^n. Since 5 is not present in the prime factorization, so there is no natural number n for which 6n ends with the digit zero.

8. How will you show that $(17 \times 11 \times 2) + (17 \times 11 \times 5)$ is a composite number? Explain.

Sol. $(17 \times 11 \times 2) + (17 \times 11 \times 5)$

$= (17 \times 11) \times [2 + 5]$

$= 17 \times 11 \times 7$

$= 17 \times 77 = 1309$

17 and 11 are factors of the first tem is $17 \times 11 \times 2$

17 and 11 are factors of the second term is $17 \times 11 \times 5$

17 and 11 are factors of both terms.

(17 x 11 x 2) + (17 x 11 x 5) is a composite number.

9. What is the last digit of 6^{100}.

Sol. $6^1 = 6$

$6^2 = 6 \times 6 = 36$

$6^3 = 6 \times 6 \times 6 = 216$

$6^4 = 6 \times 6 \times 6 \times 6 = 1296$

....

....

We see that 6n for any positive integer n ends with the digit 6.

i.e., unit digit is always 6

Unit digit of 6^{100} is 6.

10. Write the following rational numbers in their decimal form and also state which are terminating and which have non-terminating repeating decimal.

i) ii) iii) iv) v)

Sol. i)

```
8)  3 0    (0.375
    2 4
  --------
      6 0
      5 6
    --------
        4 0
        4 0
      --------
          0
      -------- (R)
```

[illegible]

= 0.375

Since the remainder is zero.

It is a Terminating decimal.

ii)

```
400)  229.0  (0.5725
       2000
    ----------
        2900
        2800
      ----------
          1000
           800
        ----------
           2000
```

```
      2000
    ----------
       0
    ---------- (R)
```

= 0.5725

Since the remainder is zero.

It is a <u>Terminating decimal.</u>

iii)

```
5)  2 1    (4.2
    2 0
  --------
      1 0
      1 0
    --------
        0

    -------- (R)
```

$\therefore 4\frac{1}{5} = 4.2$

Since the remainder is zero.

It is a <u>Terminating decimal.</u>

iv)

$\therefore 4\frac{1}{5} = \frac{21}{5}$

```
11)  2 0    (0.181818
     1 1
   --------
       9 0
       8 8
     --------
         2 0
         1 1
       --------
           9 0
           8 8
         --------
             2 1
             1 1
           --------
               9 0
               8 8
              ------
                 2
              ------
```

= 0.181818........... =

This division does not end.
Hence it is <u>non terminating, repeating decimal.</u>

v)

```
125 ) 8 0 0 ( 0.064
      7 5 0
     ---------
        5 0 0
        5 0 0
       ----------
          0
       ----------
```

Since the remainder is zero.
It is a <u>Terminating decimal.</u>

11. Without performing division, state whether the following rational numbers will have a terminating decimal form or a non-terminating, repeating decimal form.

i) ii) iii) iv) v)

vi) vii) $\frac{129}{2^2\,5^7\,7^5}$ viii) $\frac{9}{15}$ ix) x)

[illegible]

Sol. i)

$= \frac{13}{5^5} \rightarrow$ Terminating decimal

ii) Non terminating, repeating decimal.

iii) $\frac{64}{455} = \frac{64}{5\times7\times13} \rightarrow$ Non-terminating, repeating decimal.

iv) $\frac{15}{1600} = \frac{15}{2^6\times5^2} \rightarrow$ Terminating , decimal

v) $\frac{29}{343} = \frac{29}{7\times7\times7} = \frac{29}{7^3} \rightarrow$ Non-terminating repeating decimal.

vi) $\frac{23}{2^3\,5^2} \rightarrow$ Terminating decimal

vii) $\frac{129}{2^5\,5^7\,7^5} \rightarrow$ Non-terminating, repeating decimal.

viii) $\frac{9}{15}=\frac{9}{3\times5}=\frac{3}{5}$ Terminating decimal

ix) $\frac{36}{100}=\frac{36}{2^2\times5^2}\rightarrow$ Terminating decimal

x) $\frac{77}{210}=\frac{77}{2\times3\times5\times7}\rightarrow$ Non-terminating, repeating decimal.

12. Write the following rationals in decimal form

i) $\frac{13}{25}$ ii) iii)

iv) $\frac{7218}{3^2.5^2}$ v) $\frac{143}{110}$

Sol. i)

ii) $\frac{15}{16}=\frac{15}{2\times2\times2\times2}=\frac{15}{2^4}=\frac{15\times5^4}{2^4\times5^4}=\frac{15\times625}{(10)^4}=\frac{9375}{(10)^4}=0.9375$

iii) $\frac{23}{2^3.5^2}=\frac{23\times5}{2^3.5^2\times5}=\frac{115}{2^3.5^3}=\frac{23\times5}{2^3.5^2\times5}=\frac{115}{2^3.5^3}=\frac{115}{(10)^3}=0.115$

iv) $\frac{7218}{3^2.5^2}=\frac{7218}{9\times25}=\frac{802}{5^2}=\frac{802\times2^2}{5^2\times2^2}=$ [illegible] $= 32.08$ $\frac{13\times2^2}{5\times5\times2^2}=\frac{13\times4}{5^2\times2^2}=\frac{52}{10^2}=0.52$

v) $\frac{143}{110}=\frac{143}{11\times10}=\frac{13}{10}=1.3$

13. The decimal form of some real numbers are given below. In each case, decide whether the numer is reational or not. If it is rational, and expressed in the form $\frac{p}{q}$, what can you say about the prime factors of q?

i) 43.123456789
ii) 0.120120012000120000....
iii) $43.\overline{123456789}$

Sol. i) 43. 123456789 Rational
ii) 0.120120012000120000.........
Not a Rational
iii) 43. Rational

14. Prove that the following are irrational.

i) ii) $\sqrt{3}+\sqrt{5}$ iii) $6+\sqrt{2}$ iv) $\sqrt{5}$ v) $3+2\sqrt{5}$

Sol. i) $\frac{1}{\sqrt{2}}$

Let us assume, the contrary that $\frac{1}{\sqrt{2}}$ is rational.

i.e., we can find co-primes a and b ($b \neq 0$)

Such that,

Rearranging, we get $\sqrt{2}$ a = b

$$\sqrt{2} = \frac{b}{a}$$

a and b are integers, is rational, and so is irrational.

But this contradicts the fact that is irrational.

So, we conclude that $\frac{1}{\sqrt{2}}$ is irrational.

ii) $\sqrt{3}+\sqrt{5}$

Let us suppose that $\sqrt{3}+\sqrt{5}$ irrational

Let $\sqrt{3}+\sqrt{5} = \frac{a}{b}$, where is rational, b 0

$\sqrt{3} = \frac{a}{b} - \sqrt{5}$; $3 = \frac{a^2}{b^2} + 5 - 2\frac{a}{b}\sqrt{5}$

Squaring on both sides, we get

Rearranging,

$$\frac{2a}{b}\sqrt{5} = \frac{a^2}{b^2} + 5 - 3 \; ; \; \frac{2a}{b}\sqrt{5} = \frac{a^2}{b^2} + 2$$

$$\frac{2a\sqrt{5}}{b} = \frac{a^2+2b^2}{b^2} ; \sqrt{5} = \frac{a^2+2b^2}{2ab}$$

Since a,b are integers $\frac{a^2+2b^2}{2ab}$ is rational and so, $\sqrt{5}$ is rational

This contradicts the fact that $\sqrt{5}$ is irrational.

Hence $\sqrt{3}+\sqrt{5}$ is irrational.

iii) $6+\sqrt{2}$

Let us assume to the contrary, that $6+\sqrt{2}$ is rational.

That is we can find coprimes a and b ($b \neq 0$)

Such that

Rearranging this equation.

we get

Since a and b are integers,

we get is rational

So is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $6+\sqrt{2}$ is rational.

So, we conclude that $6+\sqrt{2}$ is irrational.

iv) $\sqrt{5}$

Proof: Since we are using proof by contradiction, let us assume the contrary, i.e., $\sqrt{5}$ is rational.

If it is rational, then there must exist two integers r and s (s $\neq$ 0) such that

Suppose r and s have a common factor other than 1. Then, we divide by the common factor to get where a and b are co-prime.

So, b

Squaring on both sides and rearranging, we get $5b^2 = a^2$. Therefore, 5 divides a^2.
Now, by statement 1, it follows that if 5 divides a^2 it also divides a.
So, we can write a = 5c for some integer c.
Substituting for a, we get $5b^2 = 25c^2$, that is, $b^2 = 5c^2$.
This means that 5 divides b^2, and no 5 divides b
Therefore, both a and b have 5 as a common factor.
But this contradicts the fact that a and b are co-prime and have no common factors other than 1.
This contradiction has arisen because of our assumption that $\sqrt{5}$ is rational.

So, we conclude that $\sqrt{5}$ is irrational.

v) $3+2\sqrt{5}$

Let us suppose that $3+2\sqrt{5}$ is irritional.

Let $3+2\sqrt{5}=\frac{a}{b}$ where is rational, (b 0)

$\therefore \sqrt{5} = \frac{a-3b}{2b}$

Since a, b are integers is rational and so is rational.

This contradicts the fact that $\sqrt{5}$ is irrational.

Hence $3+2\sqrt{5}$ is irrational.

15. Prove that $\sqrt{p} + \sqrt{q}$ is irrational, where p, q are premises.

Sol. Let us suppose that $\sqrt{p} + \sqrt{q}$ is irrational.

Let $\sqrt{p}+\sqrt{q}=\frac{a}{b}$, where $\frac{a}{b}$ is a rational, b 0

Squaring on both sides, we get

$(\sqrt{p})^2 = \left(\frac{a}{b} - \sqrt{q}\right)^2$, $p = \frac{a^2}{b^2} + q^2 - \frac{2a}{b}\sqrt{q}$ $\sqrt{5} = \frac{a-3b}{b}$

Rearranging

$$\frac{2a}{b}\sqrt{q} = \frac{a^2}{b^2} + q - p\ ;$$

$$\frac{2a}{b}\sqrt{q} = \frac{a^2 + b^2\ q - pb^2}{b^2}$$

$$\sqrt{q} = \frac{a^2 + b^2\ q - pb^2}{2ab}$$

Since a, b are integers, is rational and so, $\sqrt{q}$ is rational.

This contradicts the fact that $\sqrt{q}$ is irrational.

Hence $\sqrt{p} + \sqrt{q}$ is irrational.

16. From the above questions in do this, what is the nature of q and r?

Sol. q and r are pairs of positive integers.

17. Can you find the HCF of 1.2 and 0.12? Justify your answer?

Sol. No. We cannot find the HCF of 1.2 and 0.12. HCF can be find only for positive integers.

18. If r=0, then what is the relationship between a, b and q in a = bq + r of Euclid divison lemma?

Sol. Given : r=0 in $a = bq + 0$

If q = 0 then $a = bq$

i.e., b divides a completely

i.e., b is factor of a.

19. Find q and r for the following pairs of positive integers a and b, satisfying a=bq+r.

i) $a = 13$; $b = 3$

Sol. $a = bq + r$

$13 = 3q + r$

When

$q = 4,\ 13 = 3(4) + r \Rightarrow r = 13 - 12 = 1$

$q = 3,\ 13 = 3(3) + r \quad r = 13 - 9 = 4$

$q = 2,\ 13 = 3(2) + r \quad r = 13 - 6 = 7 \Rightarrow$

$q = 1,\ 13 = 3(1) + r \quad r = 13 - 3 = 10$

ii) $a = 8$; $b = 80$

Sol. $a = bq + r$

$8 = 80q + r$

No positive integers of a and r which satisfies $a = bq + r$ when $a = 8$ and $b = 80$.

iii) $a = 125$; $b = 5$

Sol. $a = bq + r$

$125 = 5q + r$

When

$q = 5,\ 125 = 5(5) + r \quad r = 125 - 25 = 100$

$q = 10,\ 125 = 5(10) + r \quad r = 125 - 50 = 75$

$q = 15,\ 125 = 5(15) + r \quad r = 125 - 75 = 50$

$q = 20,\ 125 = 5(20) + r \quad r = 125 - 100 = 25$

iv. $a = 132$; $b = 11$

Sol. $a = bq + r$

$132 = 11q + r$

When

$q = 4$, $132 = 11(4) + r$ $r = 132 - 44 = 88$

$q = 6$, $132 = 11(6) + r$ $r = 132 - 66 = 66$

$q = 8$, $132 = 11(8) + r$ $r = 132 - 88 = 4$

$q = 10$, $132 = 11(10) + r$ $r = 132 - 110 = 22$

20. Find the HCF of the following by using Euclid division lemma.

i) 50 and 70

Sol. When 70 is divided by 50, the remainder is 20 to get

$70 = 50 \times 1 + 20$

Now consider the division of 50 with the remainder 20 in the above and apply the division lemma to get

$50 = 20 \times 2 + 10$

Now consider the division of 20 with the remainder 10 in the above and apply the division lemma to get

$20 - 10 \times 2 + 0$ $\Rightarrow$

Notice that the remainder become zero and we cannot proceed any further.

The HCF of 50 and 70 is the divisor at this state i.e., 10.

ii) 96 and 72.

Sol. When 96 is divided by 72 the remainder is 24 to get

$96 = 72 \times 1 + 24$

Now consider the division of 72 with the remainder 24 in the above and apply the division lemma to get

$72 = 24 \times 3 + 0$

Notice that the remainder become zero and we cannot proceed any further.

The HCF of 96 and 72 is the divisor at this stage is 24.

iii) 300 and 550

Sol. When 550 is divided by 300, the remainder i 250 to get

$550 = 300 \times 1 + 250$

Now consider the division of 300 with the remainder 250 in the above and apply the division lemma to get

$300 = 250 \times 1 + 50$

Now consider the division of 250 with the remainder 50 in the above and apply the division lemma to get

$250 = 50 \times 5 + 0$

Notice that the remainder become zero and we cannot proceed any further.

The HCF of 300 and 550 is the divisor at this stage is 50.

iv) 1860 and 2015

Sol. When 2015 is divided by 1860, the remainder is 155 to get

$2015 = 1860 \times 1 + 155$

Now consider the division of 1860 with remainder 155 in the above and apply the division lemma to get

$1860 = 155 \times 12 + 0$

Notice that the remainder become zero and we cannot proceed any further.

The HCF of 1860 and 2015 is the divisor at this stage is 155.

21. Find HCF and LCM of the following given pairs of numbers by prime factorisation.

(i) 120 , 90

Sol. We have
$$\begin{aligned} 120 &= 2 \times 60 \\ &= 2 \times 2 \times 30 \\ &= 2 \times 2 \times 3 \times 10 \\ &= 2 \times 2 \times 3 \times 2 \times 5 \\ &= 2^3 \times 3^1 \times 5^1 \\ 90 &= 2 \times 45 \\ &= 2 \times 3 \times 15 \\ &= 2 \times 3 \times 3 \times 5 \\ &= 2^1 \times 3^2 \times 5^1 \end{aligned}$$

HCF (120, 90) $= 2^1 \times 3^2 \times 5^1 = 6 \times 5 = 30$

LCM (120, 90) $= 2^3 \times 3^2 \times 5^1 = 8 \times 9 = 5$

$= 360$

(ii) 50, 60

Sol. We have
$$\begin{aligned} 50 &= 2 \times 25 \\ &= 2 \times 5 \times 5 = 2^1 \times 5^2 \end{aligned}$$

$$
\begin{aligned}
60 &= 2 \times 30 \\
&= 2 \times 2 \times 15 \\
&= 2 \times 2 \times 3 \times 5 \\
&= 2^2 \times 3^1 \times 5^1
\end{aligned}
$$

HCF (50, 60) = $2^1 \times 5^1 = 10$

LCM (50, 60) = $2^2 \times 5^2 = 3^1$

= $4 \times 25 \times 3 = 300$

(iii) 37 , 49

Sol. We have 37 = $37^1 \times 1^1$

49 = $1 \times 7 \times 7 = 1 \times 7^2$

HCF (37, 49) = 1

LCM (37, 49) = $37^1 \times 7^2$

= $37 \times 49 = 1813$

22. Write the following terminating decimals in the form of $\frac{p}{q}, q \neq 0$ and p, q are co-primes.

i) 15.265 ii) 0.1255 iii) 0.4 iv) 23.34 v) 1215.8

What can you conclude about the denominators through this process?

Sol. i) 15.265

ii) 0.1255

$$= \frac{251}{2\times2\times2\times2\times5\times5\times5} = \frac{251}{2^4\times5^3}$$

iii) 0.4

iv) 23.34 $= \frac{2334}{100} = \frac{1167\times2}{50\times2} = \frac{1167}{50}$

$$= \frac{1167}{2\times5\times5} = \frac{1167}{2^1\times5^2}$$

v) 1215.8

Conclusion :

Decimal expression is expressed in its simplest rational form, p and q are co primes ; $q \neq 0$.

The denominator (i.e., q) has only powers of 2 or powers of 5 or both. This is because the powers of 10 can only have powers of 2 and 5 as factors.

23. Write the following rational numbes in the form of where q is of the form 2^n5^m where n, m are non-negative integers and then write the numbers in their decimal form.

i) ii) iii) iv) v)

Sol. i)

ii) $\frac{7}{5} = \frac{7}{5\times5} = \frac{7\times2^3}{5^2\times2^2} = \frac{7\times4}{10^2} = \frac{28}{10^2} = 0.28$

$\frac{6079\times2}{5\times2} = \frac{3\times5^2}{2^2\times5^2} = \frac{75}{(2\times5)^2} = \frac{75}{10^2} = 0.75$

iii) $\frac{51}{64} = \frac{51}{2\times2\times2\times2\times2\times2} = \frac{51\times5^6}{2^6\times5^6} = \frac{796875}{10^6} = 0.796875$

iv) $\frac{14}{25} = \frac{14}{5^2} = \frac{14\times2^2}{5^2\times2^2} = \frac{56}{100} = 0.56$

v) $\frac{80}{100} = \frac{80}{5^2\times2^2} = \frac{80}{10^2} = \frac{80}{100} = 0.8$

24. Write the following rational numbers as decimals and find out the block of digits, repeating in the quotient.

i) $\frac{1}{3}$ ii) iii) iv)

Sol. i)

```
3) 10000000 (0.3333333
     9
     10
       9
       10
```

9
10
9
10
9
10
9
10
9
1

Which is non - terminating and recurring decimal.

The block of digits <u>‘3’</u> is repeating in the Quotient.

Sol. ii)

7) 20000000 (0.28517428
14
60
56
40
35
50
49
10
7
30
28
20
14
60
56
4

$\frac{1}{3} = 0.333... = 0.\bar{3}$

$\frac{2}{7} = 0.28517428... = 0.\overline{285171428}$

Which is non - terminating and recurring decimal.
The block of digits <u>‘285714’</u> is repeating in the Quotient.

Sol. iii) $\frac{5}{11}$

11) 5.0000000 (0.4545
44
60

55
50
44
60
55
5

Which is non - terminating and recurring decimal.
The block of digits ‘45’ is repeating in the Quotient.

Sol. iv) $\frac{10}{3}$

13) 100000 (0.76923076
91
90
78
120
117
30
26
40
39
100
91
90
78
12

$\frac{5}{11} = 0.4545...... = 0.\overline{45}$

$\frac{10}{13} = 0.76923076...... = 0.\overline{76923076}$

Which is non - terminating and recurring decimal.
The block of digits ‘769230’ is repeating in the Quotient.

25. Verify the statment proved above (i.e., Text page no 11, 12 statement 1) for p=2 and p=5 and for a^2 = 1, 4, 9, 25, 36, 49, 64 annd 81.

Sol. P = 2 is a prime number : P divide a^2 for a^2 = 1, 9, 25, 49, 81 P = 2 is not a prime factor of

P doesnot divides a.

P = 2 is a prime factor of 4, 36, 64,

P divides a.

P = 5 is not a prime factor of 1, 4, 9, 36, 49, 64, 81

P doesnot divides a.

P = 5 is a prime factor of 25

P divides a.

26. Show that every positive even integer is of the form 2q and that every positive odd integer is of the form 2q + 1 where q is some integer.

Sol. Let 'a' be any positive integer and b=2. Then, by Ecluid's division lemma. There exists integers q and r such that

a = 2q + r where

Now (r is an integer)

a = 2q or a = 2q+1

If a = 2q and 'a' is an even integer.

We know that an integer can be either even or odd.

Therefore, any positive odd integer is of the form 2q+1.

27. Show that any positive odd integer is of the form 4q + 1 or 4q + 3 where q is some integer.

Sol. Let 'a' be any odd positive integer a and b = 4

By division lemma there exists integers q and r such that a = 4q + r where

a = 4q or a = 4q +1 or a = 4q + 4 or a = 4q +3

$\Rightarrow$ a= 4q +1 or a = 4q +3 (a is an odd integer)

Hence, any odd integer is of the form 4q+1 or 4q+3.

28. Find the LCM and HCF of 12 and 18 by the prime factorization method.

Sol. We have 12 = 2 x 2 x 3 = 2^2 x 3^1

18 = 2 x 3 x 3 = 2^1 x 3^2

Note that HCF (12, 18) = 2^1 x 3^1 = 6

= Product of the smallest power of each common prime factos of the numbers.

LCM (12, 18) = 2^2 x 3^2 = 36

= Product of the greatest power of each prime fators, of the numbers.

HCF = 6 ; LCM = 36

29. Using the above theorems, without actual division, state whether the following rational numbers are terminating or non-terminating, repeating decimals.

i) ii) iii) iv)

Sol. i) = terminating decimal

ii) $\frac{25}{32} = \frac{25}{2\times2\times2\times2\times2} = \frac{25}{2^5}$ = terminating decimal

iii) $\frac{100}{81} = \frac{100}{3\times3\times3\times3} = \frac{100}{3^4}$ = non-terminating , repeating decimal

iv) $\frac{41}{75} = \frac{41}{3\times5\times5} = \frac{41}{3\times5^2}$ = non-terminating , repeating decimal

30. Write the decimal expansion of the following rational numbers without actual division.

i) ii) iii)

Sol. i)

ii) $\frac{21}{25} = \frac{21}{5\times5} = \frac{21\times2^2}{5\times5\times2^2} = \frac{21\times4}{5^2\times2^2} = \frac{84}{10^2} = 0.84$

iii) $\frac{7}{8} = \frac{7}{2\times2\times2} = \frac{7}{2^3} = \frac{7\times5^3}{\left(2^3\times5^3\right)} = \frac{7\times125}{\left(2\times5^3\right)} = \frac{875}{\left(10^3\right)} = 0.875$

31. Show that $5-\sqrt{3}$ is irrational.

$\frac{7\times5}{2\times5\times5} = \frac{7}{2\times5} = \frac{7}{10^1} = 0.7$

Sol. Let us assume, to the contrary, that $5-\sqrt{3}$ is rational.

That is, we can find coprimes a and b $(b \neq 0)$ such that .

Therefore,

Rearranging this equation,

we get $\sqrt{3} = 5-\frac{a}{b} = \frac{5b-a}{b}$

Since a annd b are integers, we get $5-\frac{a}{b}$ is rational so is rational.

But this contradicts the fact that is irrational.

This contradiction has arisen because of our incorrect assumption that is rational.

So, we conclude that $5-\sqrt{3}$ is irrational.

32. Show that 3 $\sqrt{2}$ is irrational.

Sol Let us assume, the contrary, that 3 is rational.

i.e., we can find co-primes a and b such that

Rearranging, we get

Since 3, a and b are integers, $\frac{a}{3b}$ is rational, and so is irrational.

But this contradicts the fact that is irrational.

So, we conclude that 3 is irrational.

33. Prove that is irrational.

Sol. Let us suppose that $\sqrt{2} + \sqrt{3}$ is rational.

Let $\sqrt{2} + \sqrt{3} = \frac{a}{b}$, where a, b are integers and

Therefore,

Squaring on both sides, we get

$$\left(\frac{a}{b}\right)^2 + \left(\sqrt{3}\right)^2 - 2.\frac{a}{b}.\sqrt{3}$$

$$2 = \frac{a^2}{b^2} + 3 - \frac{2a}{b}\sqrt{3}$$

Rearranging

$$\frac{2a}{b}\sqrt{3} = \frac{a^2}{b^2} + 3 - 2 = \frac{a^2}{b^2} + 1 = \frac{a^2 + b^2}{b^2}$$

$$\sqrt{3} = \frac{a^2 + b^2}{b^2} \times \frac{b}{a}\ ;\ \sqrt{3} = \frac{a^2 + b^2}{2ab}$$

Since a, b are integers, $\frac{a^2 + b^2}{2ab}$ is rational, and so, is rational.

This contradicts the fact that is irrational. Hence, is irrational.

34. Show that $3^n \times 4^m$ cannot end with digits 0 5 for any natural numbers 'n' and 'm'.

Sol. For the number to end with digits odd 0 or 5 for any natural numbers n and m it should be divisible by 2 and 5. This means that the prime factorisation of should contain. The prime number 5 and 2. But it is not possible because

So 2 and 3 are the only prime is the factorisation of $3^n \times 4^m$

5 is not present in the prime factorisation, there is no natural numbers of n and m for which ends with the digit 0 or 5.

1. Real Numbers

Fill in the blanks with suitable answers

1. Express 3825 as a product of prine fadors ... (A)

   a) $5 \times 5 \times 3 \times 3 \times 17$ b) $5 \times 5 \times 3 \times 17$

   c) $5 \times 5 \times 3 \times 3 \times 3 \times 17$ d) $5 \times 5 \times 17$

2. Express 7429 as a product of prine fadors ... (A)

   a) $17 \times 19 \times 23$ b) $18 \times 19 \times 23$

   c) $23 \times 21 \times 25$ d) $21 \times 24 \times 25$

3. L.C.M. of 12 and 18 is .... (D)

   12 బి) 18 సి) 63 డి) 36

4. L.C.M. of 24 and 36 is .... (C)

   a) 24 బి) 36 సి) 72 డి) 27

5. Any two positive integers a,b HCF (a,b) x LCM (a,b) = ... (A)

   a) Product of two numbers b) Sum of two numbers

   c) Difference of two numbers d) $\frac{p}{q}$ None

6. is a .... (A)

   a) Irrational b) Rational c) Natural d) None

7. G.C.D. of 12 and 18 is .... (B)

   a) 12 బి) 6 సి) 18 డి) 36

8. G.C.D. of 120 and 90 is .... (D)

   a) 90 బి) 120 సి) 360 డి) 30

9. Numbers that are of the form whose p and q are integers $(q \neq 0)$ are called .... numbers (B)

   a) Irrational b) Rational

   c) Negative Integers d) Positive Integers

10. Numbers which cannot be expressed in the form of where p, q are integers $(q \neq 0)$ .... numbers (B)

a) Rational b) Irrational c) Natural d) Whole

11. L.C.M. of 120 and 90 is .... (C)

a) 90 b) 120 c) 360 d) 30

12. L.C.M. of 50 and 60 is .... (A)

a) 300 b) 30 c) 60 d) 10

13. .... is a neither prime nor composite number (A)

a) 1 b) 2 c) 0 d) -1

14. The difference of rational and irrational is (A)

a) Irrational b) Rational

c) Negetive Integers d) Positive Integers

15. L.C.M. of 37 and 49 is .... (A)

a) 1813 b) 1318 c) 1138 d) 1

16. G.C.D. of 50and 60 is .... (D)

a) 300 b) 30 c) 60 $\frac{11}{12}$ d) 10

17. 7x11 x 13 +13 is a .... number (C)

a)Prime b) Real c) Composite d) None

18. The last digit of 6^{100} is .... (C)

a) 0 b) 1 c) 6 d) -6

19. G.C.D. of 37 and 49 is .... (D)

a) 1813 b) 1318 c) 1138 d) 1

20. L.C.M. and G.C.D. of 12, 15 and 21 is .... (A)

a) 420, 3 b) 3, 420 c) 1, 420 d) 420, 1

21. Which of the following rational numbers will have a terminating decimal

a) b) $\frac{77}{210}$ c) $\frac{64}{455}$ d) $\frac{9}{15}$ (D)

22. $3+2\sqrt{5}$ is a .... number (B)

a) Rational b) Irrational c) Positive Integer d) None

23. L.C.M. and G.C.D. of 17, 23, and 29 is (A)

a) 11339, 1 b) 1, 11339 c) 1, 8 d) 8, 12

24. p, q are primes then $\sqrt{p}+\sqrt{q}$ is a .... (B)

a) Rational b) Irrational c) Positive Integer d) None

25. G.C.D. of 1.2 and 0.12 is .... (D)

a) 1.2 b) 0.12 c) 2.2 d)None of these

26. 0.1201 1200 1200 .... is a .... (A)

a) Irrational b) Rational c) Positive Integer d) None

27. (17x11x2) + (17x11x5) is a .... number (A)

a) Comosite b) Prime c) Real d) None