MARCH - 2018 (TS)

MATHEMATICS, Paper - I(A)

Time : 3 Hrs Max. Marks : 75

SECTION - A

$10 \times 2 = 20$

I. Very Short Answer Type Questions :

(i) Answer ALL questions (ii) Each question carries Two marks.

1. If $a, b \in R$, $f : R \to R$ defined by $f(x) = ax + b$ $(a \neq 0)$ find f^{-1}. **Ch.No. 1, Ex: 1(b), Q.No.8**
2. Find the domain of real valued function : $f(x) = \sqrt{4x - x^2}$ **Ch.No. 1, Ex: 1(c), Q.No.I (V)**
3. If $A = \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}$ and $A^2 = 0$ then find the value of 'k'. **Ch.No. 3, Ex: 3(b), Q.No.7**
4. Find the rank of the following matrix : $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ **Ch.No. 3, Ex: 3(f), Q.No.II (4)**
5. Find the vector equation of the line joining the points $2\bar{i} + \bar{j} + 3\bar{k}$ and $-4\bar{i} + 3\bar{j} - \bar{k}$. **Ch.No. 4, Ex: 4(b), Q.No.I (4)**
6. If $\bar{a} = 2\bar{i} + 5\bar{j} + \bar{k}$ and $\bar{b} = 4\bar{i} + m\bar{j} + n\bar{k}$ are collinear vectors, then find 'm' and 'n'. **Ch.No. 4, Ex: 4(a), Q.No.8**
7. If $\bar{a} = 2\bar{i} - 3\bar{j} + \bar{k}$ and $\bar{b} = \bar{i} + 4\bar{j} - 2\bar{k}$, then find $(\bar{a} + \bar{b}) \times (\bar{a} - \bar{b}.)$ **Ch.No. 5, Ex: 5(b), Q.No.I(3)**
8. Prove that : $\dfrac{\cos 9^0 + \sin 9^0}{\cos 9^0 - \sin 9^0} = \cot 36^0$. **Ch.No. 6, Solved problem no.21**
9. Prove that : $\sin 50^0 - \sin 70^0 + \sin 10^0 = 0$ **Ch.No. 6, Ex:6 (e) VSAQ-1**
10. Show that : $(\cosh x + \sinh x)^n = \cosh(nx) + \sinh(nx)$, for any $n \in R$. **Ch.No. 9, Ex:9(a), Q.No.4 (II)**

SECTION - B

$5 \times 4 = 20$

II. Short Answer Type Questions :

(i) Answer ANY FIVE questions. (ii) Each question carries Four marks.

11. If $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is a non- singular matrix, then prove that $A^{-1} = \dfrac{Adj\ A}{\det A}$.

Ch.No. 3, Theorems No-1

12. If $\bar{a}, \bar{b}, \bar{c}$ are non- coplanar vectors, then prove that :

$-\bar{a} + 4\bar{b} - 3\bar{c},\ 3\bar{a} + 2\bar{b} - 5\bar{c},\ -3\bar{a} + 8\bar{b} - 5\bar{c},\ -3\bar{a} + 2\bar{b} + \bar{c}$ are coplanar. **Ch.No. 4, SAQ.No-2**

13. For any two vectors $\bar{a}$ and $\bar{b}$, show that : $(1+|\bar{a}|)^2(1+|\bar{b}|)^2 = |1-\bar{a}.\bar{b}|^2 + |\bar{a}+\bar{b}+\bar{a}\times\bar{b}|^2$.

14. Prove that : $\frac{1}{\sin 10^0} - \frac{\sqrt{3}}{\cos 10^0} = 4$. Ch.No. 6, SAQ.No-5

15. Solve the equation : $\sin x + \sqrt{3}\cos x = \sqrt{2}$ Ch.No. 7, Ex:7(a),SAQ-2(iii)

16. Prove that : $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$. Ch.No. 8, SAQ-4(iii)

17. Prove that : $\cot A + \cot B + \cot C = \frac{a^2+b^2+c^2}{4\Delta}$. Ch.No. 10, Solved problem no-21

SECTION - C

III. Long Answer Type Questions : $5 \times 7 = 35$

(i) Answer **Any Five** questions (ii) Each question carries **SEVEN** marks.

18. Let $f: A \to B$, I_A and I_B be identify functions on A and B respectively, then prove that $f \circ I_A = f = I_B \circ f$. Ch.No. 1, Imp.theorems no-3

19. Show that $49^n + 16n - 1$ is divisible by 64 for all positive integers 'n'. Ch.No. 2, Solved problem no-9

20. Show that :

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = \left(a^3+b^3+c^3-3abc\right)^2.$$

Ch.No. 3, LAQ-2

21. Solve the following system of equations by Cramer's rule :

$2x - y + 3z = 8 \qquad -x + 2y + z = 4 \qquad 3x + y - 4z = 0$. Ch.No. 3, Ex-3(b) Q.No-7

22. If $\bar{a} = \bar{i} - 2\bar{j} + 3\bar{k}$, $\bar{b} = 2\bar{i} + \bar{j} + \bar{k}$, $\bar{c} = \bar{i} + \bar{j} + 2\bar{k}$, then find $|(\bar{a}\times\bar{b})\times\bar{c}|$ and $|\bar{a}\times(\bar{b}\times\bar{c})|$.

Ref. Ch.No. 5, LAQ.No-8

23. If $A+B+C = 2s$, then prove that :

$\cos(s-A) + \cos(s-B) + \cos(s-C) + \cos s = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$. Ch.No. 6, Ex-6(f) Q.No-10(i)

24. If P_1, P_2, P_3 are altitudes drawn from vertices A, B, C to the opposite sides of a triangle respectively, then show that : (i) $\frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3} = \frac{1}{r}$ (ii) $P_1P_2P_3 = \frac{(abc)^2}{8R^3} = \frac{8\Delta^3}{abc}$.