**B**

Total No. of Questions - 24

Total No. of Printed Pages - 4

Regd. No.

## Part - III

## MATHEMATICS, Paper - II (A)

### (Algebra and Probability)

### (English Version)

*Time : 3 Hours*        *Max. Marks : 75*

**Note :** This question paper consists of **three** Sections A, B and C.

### SECTION A      $10 \times 2 = 20$

I. **Very Short Answer Type Questions.**

   i)    Answer **all** questions.

   ii)   Each question carries **two** marks.

1.    If $z = 2 - i$, then show that, $z^2 - 4z + 13 = 0$.

2.    If $z_1 = -1$ and $z_2 = i$, then find $Arg\left(\dfrac{z_1}{z_2}\right)$.

3.    If $x = Cis\,\theta$, then find the value of $\left(x^6 + \dfrac{1}{x^6}\right)$.

4.    Form a quadratic equation whose roots are $7 \pm 2\sqrt{5}$.

5.    If $-1, 2$ and $\alpha$ are the roots of $2x^3 + x^2 - 7x - 6 = 0$, then find $\alpha$.

XT-39 (DAY-6)           1           Turn Over

6. Find the number of ways of arranging the letters of the word MATHEMATICS.

7. If $^{n}C_{5} = {}^{n}C_{6}$, then find $^{13}C_{n}$.

8. Prove that $C_{0} + 2 \cdot C_{1} + 4 \cdot C_{2} + 8 \cdot C_{3} + \ldots + 2^{n} \cdot C_{n} = 3^{n}$.

9. Find the mean deviation about the median for the following data :
   4, 6, 9, 3, 10, 13, 2.

10. A Poisson variable satisfies $P(X=1) = P(X=2)$. Find $P(X=5)$.


## SECTION B

$5 \times 4 = 20$

II. **Short Answer Type Questions.**

   i)   Attempt **any five** questions.

   ii)  Each question carries **four** marks.

11. Show that the points in the Argand diagram represented by the complex numbers $2 + 2i, -2 - 2i, -2\sqrt{3} + 2\sqrt{3}\,i$ are the vertices of an equilateral triangle.

12. Prove that $\dfrac{1}{3x+1} + \dfrac{1}{x+1} - \dfrac{1}{(3x+1)(x+1)}$ does not lie between 1 and 4, if $x$ is real.

13. Find the sum of all 4 digit numbers that can be formed using the digits 1, 3, 5, 7, 9.

14. Find the number of ways of selecting a cricket team of 11 players from 7 batsmen and 6 bowlers, such that there will be atleast 5 bowlers in the team.

15. Resolve the fraction $\dfrac{2x^2 + 3x + 4}{(x-1)(x^2+2)}$ into partial fraction.

16. Suppose $A$ and $B$ are independent events with $P(A) = 0.6$, $P(B) = 0.7$. Then compute :

    i)   $P(A \cap B)$          ii)   $P(A \cup B)$

    iii) $P(B/A)$              iv)   $P(A^c \cap B^c)$

17. $A$, $B$, $C$ are three horses in a race. The probability of $A$ to win the race is twice that of $B$ and probability of $B$ is twice that of $C$. What are the probabilities of $A$, $B$ and $C$ to win the race?

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**SECTION C**                    $5 \times 7 = 35$

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III. **Long Answer Type Questions.**

   i)   Attempt **any five** questions.

   ii)  Each question carries **seven marks.**

18. If $Cos\,\alpha + Cos\,\beta + Cos\,\gamma = 0 = Sin\,\alpha + Sin\,\beta + Sin\,\gamma$, then prove that $Cos^2\,\alpha + Cos^2\,\beta + Cos^2\,\gamma = \dfrac{3}{2} = Sin^2\,\alpha + Sin^2\,\beta + Sin^2\,\gamma$.

19. Solve the equation $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.

**20.** If the coefficients of $r^{\text{th}}$, $(r+1)^{\text{th}}$ and $(r+2)^{\text{nd}}$ terms in the expansion of $(1+x)^n$ are in A.P. Then show that

$$n^2 - (4r+1)n + 4r^2 - 2 = 0$$

**21.** If $\quad x = \dfrac{1.3}{3.6} + \dfrac{1.3.5}{3.6.9} + \dfrac{1.3.5.7}{3.6.9.12} + \ldots\ldots,\quad$ then prove that $9x^2 + 24x = 11$.

**22.** Find the mean deviation about the mean for the following data :

| Marks obtained | 0–10 | 10–20 | 20–30 | 30–40 | 40–50 |
|---|---|---|---|---|---|
| No. of students | | 8 | 15 | 16 | 6 |

**23.** State and prove the addition theorem on probability.

**24.** A random variable $X$ has the following probability distribution.

| $X = x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $P(X=x)$ | 0 | $k$ | $2k$ | $2k$ | $3k$ | $k^2$ | $2k^2$ | $7k^2+k$ |

Find :

i) $k$

ii) Mean

iii) $P(0 < X < 5)$