0166/TS

C

Total No. of Questions – 24

Total No. of Printed Pages - 4

Regd. No.

Part – III
MATHEMATICS, Paper – I(A)
(English Version)

Time : 3 Hours] **[Max. Marks : 75**

Note : This question paper consists of **three** Sections – **A**, **B** and **C**.

SECTION – A

10 × 2 = 20

I. Very Short Answer Type questions.

(i) Answer **all** questions.

(ii) Each question carries **two** marks.

1. If $A = \left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right\}$ and $f : A \to B$ is a surjection defined by $f(x) = \cos x$ then find B.

2. If $f(x) = 2$, $g(x) = x^2$, $h(x) = 2x$ for all $x \in R$, then find $(fo(goh))(x)$.

3. If $A = \begin{bmatrix} 3 & 2 & -1 \\ 2 & -2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & -1 & 0 \\ 2 & 1 & 3 \\ 4 & -1 & 2 \end{bmatrix}$ and $X = A + B$ then find X.

4. If $A = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$ then find AA'.

5. $a = 2i + 5j + k$ and $b = 4i + mj + nk$ are collinear vectors then find m and n.

6. Find the vector equation of the line passing through the point $2i + 3j + k$ and parallel to the vector $4i - 2j + 3k$.

7. Find the angle between the vectors $i + 2j + 3k$ and $3i - j + 2k$.

8. If $\sin\theta = \frac{4}{5}$ and θ is not in the first quadrant, find the value of $\cos\theta$.

9. Prove that $\cos 48° \cdot \cos 12° = \frac{3+\sqrt{5}}{8}$.

10. If $\cosh x = \frac{5}{2}$, find the values of (i) $\cosh(2x)$ and (ii) $\sinh(2x)$.

SECTION – B

$5 \times 4 = 20$

II. Short Answer Type questions.

(i) Attempt any **five** questions.

(ii) Each question carries **four** marks.

11. Show that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$.

12. If a, b, c are non-coplanar find the point of intersection of the line passing through the points $2a + 3b - c$, $3a + 4b - 2c$ with the line joining the points $a - 2b + 3c$, $a - 6b + 6c$

13. If $a = 2i + j - k$, $b = -i + 2j - 4k$ and $c = i + j + k$ then find $(a \times b) \cdot (b \times c)$.

14. (i) Find the range of $13\cos x + 3\sqrt{3}\sin x - 4$.

(ii) Evaluate $\sin^2 82\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ$

15. Solve $1 + \sin^2\theta = 3 \sin\theta \cdot \cos\theta$.

16. Show that $\cot\left(\sin^{-1}\sqrt{\frac{13}{17}}\right) = \sin\left(\tan^{-1}\frac{2}{3}\right)$.

17. In ΔABC, if

$\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$, show that $C = 60°$.

SECTION – C $5 \times 7 = 35$

III. Long Answer Type questions :

(i) Attempt any **five** questions.

(ii) Each question carries **seven** marks.

18. (i) If $f : Q \to Q$ is defined by $f(x) = 5x + 4$, $\forall\, x \in Q$, show that f is a bijection and find f^{-1}.

(ii) If $f = \{(4, 5), (5, 6), (6, -4)\}$ and $g = \{(4, -4), (6, 5), (8, 5)\}$ then find $f + g$ and fg.

19. Using mathematical induction, prove $1\cdot2\cdot3 + 2\cdot3\cdot4 + 3\cdot4\cdot5 +$ ..... upto n terms $= \frac{n(n+1)(n+2)(n+3)}{4}$, $\forall\, n \in N$.

20. Solve the following system of equations by using Cramer's rule :

$2x - y + 3z = 9$

$x + y + z = 6$

$x - y + z = 2$

21. (i) Show that $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ is non-singular and find A^{-1}.

(ii) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ then show that $A^2 - 4A - 5I = 0$

22. Find the shortest distance between the skew lines :

$\vec{r} = (6\hat{i} + 2\hat{j} + 2\hat{k}) + t(\hat{i} - 2\hat{j} + 2\hat{k})$

and $\vec{r} = (-4\hat{i} - \hat{k}) + s(3\hat{i} - 2\hat{j} - 2\hat{k})$.

23. If $A + B + C = 2S$, then prove that

$$\cos(S - A) + \cos(S - B) + \cos C = -1 + 4\cos\frac{S-A}{2}\cos\frac{S-B}{2}\cos\frac{C}{2}$$

24. Show that in a ΔABC, $\frac{1}{r^2} + \frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} = \frac{a^2 + b^2 + c^2}{\Delta^2}$.
