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Total No. of Questions—**24**

Total No. of Printed Pages—**4** **Regd. No.** | | | | | | | | | | |

Part III

MATHEMATICS

Paper I(A)

(English Version)

Time : 3 Hours ***Max. Marks : 75***

Note :—This question paper consists of THREE sections A, B and C.

SECTION A 10×2=20

I. Very Short Answer Type questions.

(*i*) Answer ALL questions.

(*ii*) Each question carries TWO marks.

1. If $A = \left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right\}$ and $f : A \to B$ is a surjection defined by $f(x) = \cos x$, then find B.

2. If $f : Q \to Q$ is defined by $f(x) = 5x + 4$ for all $x \in Q$, then find f^{-1}.

3. If $A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$, then show that $A^2 = -I$, $(i^2 = -1)$.

4. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & x & 7 \end{bmatrix}$ is a symmetric matrix, then find x.

5. If $\vec{a} = 2\vec{i} + 5\vec{j} + \vec{k}$ and $\vec{b} = 4\vec{i} + m\vec{j} + n\vec{k}$ are collinear vectors, then find m and n.

6. Find the vector equation of the line joining the points $2\vec{i} + \vec{j} + 3\vec{k}$ and $-4\vec{i} + 3\vec{j} - \vec{k}$.

7. Find the area of the parallelogram having $\vec{a} = 2\vec{j} - \vec{k}$ and $\vec{b} = -\vec{i} + \vec{k}$ as adjacent sides.

8. If

$$\tan 20° = \lambda,$$

then show that :

$$\frac{\tan 160° - \tan 110°}{1 + \tan 160° . \tan 110°} = \frac{1 - \lambda^2}{2\lambda}.$$

9. Find a sine function whose period is $\frac{2}{3}$.

10. If $\cosh x = \frac{5}{2}$, then find the values of :

(*a*) cosh (2*x*)

(*b*) sinh (2*x*).

SECTION B 5×4=20

II. Short Answer Type Questions.

(*i*) Answer ANY FIVE questions.

(*ii*) Each question carries FOUR marks.

11. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then show that :

$$A^2 - 4A - 5I = 0.$$

12. If ABCDEF is a regular hexagon with centre 'O', then show that :

$$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = 3\overrightarrow{AD} = 6\overrightarrow{AO}$$

13. Find the volume of the tetrahedron having the edges :

$$\vec{i} + \vec{j} + \vec{k},\ \vec{i} - \vec{j} \text{ and } \vec{i} + 2\vec{j} + \vec{k}.$$

14. Prove that :

$$\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4.$$

15. Solve :

$$\sqrt{2}(\sin x + \cos x) = \sqrt{3}.$$

16. Prove that :

$$\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}.$$

17. In Δ ABC, if $\sin\theta = \frac{a}{b+c}$, then show that :

$$\cos\theta = \frac{2\sqrt{bc}}{b+c}\cos\frac{A}{2}.$$

SECTION C 5×7=35

III. Long Answer Type Questions.

(*i*) Answer ANY FIVE questions.

(*ii*) Each question carries SEVEN marks.

18. If $f : A \to B$ and $g : B \to C$ are bijections, then prove that : $g \circ f : A \to C$ is bijection.

19. Using mathematical induction, prove that :

$$1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots\ldots\ldots \text{ upto } n \text{ terms}$$

$$= \frac{n(n+1)^2 (n+2)}{12} \text{ for all } n \in \mathbf{N}.$$

20. Show that :

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3.$$

21. Solve the following system of equations by using Cramer's rule

$$2x - y + 3z = 9, x + y + z = 6, x - y + z = 2.$$

22. If

$$\vec{a} = \vec{i} - 2\vec{j} + \vec{k},\ \vec{b} = 2\vec{i} + \vec{j} + \vec{k},\ \vec{c} = \vec{i} + 2\vec{j} - \vec{k}$$, then find $\vec{a} \times (\vec{b} \times \vec{c})$

and $|(\vec{a} \times \vec{b}) \times \vec{c}|$.

23. If A, B, C are angles of a triangle, then prove that :

$$\sin^2\frac{A}{2} + \sin^2\frac{B}{2} - \sin^2\frac{C}{2} = 1 - 2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}.$$

24. In Δ ABC, if $a = 13$, $b = 14$, $c = 15$, then show that :

$$R = \frac{65}{8},\ r = 4,\ r_1 = \frac{21}{2},\ r_2 = 12 \text{ and } r_3 = 14.$$