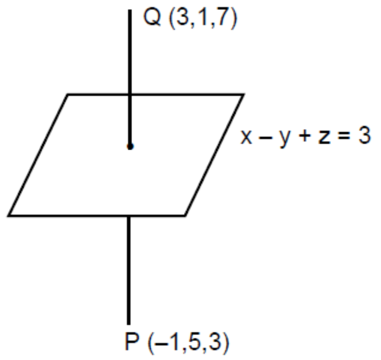
JEE(Advanced)-2016 Paper – II Mathematics Solutions

37.



$$\frac{x-3}{1}=\frac{y-1}{-1}=\frac{z-7}{1}=\frac{-2(6)}{3}=-4$$

$x=-1, y=5, z=3 \qquad P(-1,5,3)$

$a(x+1)+b(y-5)+c(z-3)=0$

$a+2b+c=0$ ....(i)

$a-5b-3c=0$ **[c]**

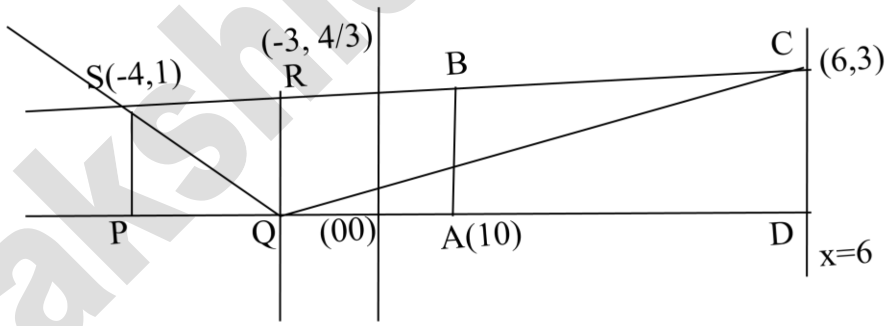
$$\frac{a}{-1}=\frac{b}{4}=\frac{c}{-7}$$

$-(x+1)+4(y-5)-7(z-3)=0$

$-x+4y-7z=0$

$x-4y+7z=0$

38. Graph given



$$\text{Area a} = \left\{\text{Area of } PQRS - \int_{-4}^{-3}\sqrt{-x-3}\right\} + \left\{\text{Trapezium } QABK - \int_{-3}^{1}\sqrt{x+3}\right\}$$

$$=\frac{3}{2}$$ **[C]**

39. Common difference d = 2 = $\log b_2 - \log b_1 = \log\frac{b_2}{b_1}$

$\Rightarrow b_2 = 2b_1$

$\Rightarrow b_1\ b_2\ b_3$ …. GP

$T = b_1 + 2b_1 + 4b_1 \ldots + 2^{50}b_1 = b_1\,(2^{51} - 1)$

$$S = \frac{51}{2}\left[a_1 + a_{51}\right] = \frac{51}{2}\left[b_1 + b_{51}\right] = \frac{51}{2}b_1\left(1 + 2^{50}\right)$$

$\Rightarrow S - T > 0$

$b_{101} = 2^{100}b_1$

$a_{101} = a_1 + 100d = 2a_1 - a_1 + 2(50d) = 2a_{51} - a_1$

$= 2b_{51} - b_1 \Rightarrow b_{101} > a_{101} \Rightarrow$ **[B]**

40.
$$2\sum_{k=1}^{13}\frac{\sin 30^\circ}{\sin\left[45^\circ + (k-1)30^\circ\right]\sin\left[45 + 30^\circ k\right]}$$

$$2\sum_{k=1}^{13}\frac{\sin\left\{\left[45^\circ + 30^\circ k\right] - \left[45^\circ + (k-1)30^\circ\right]\right\}}{\sin\left[45^\circ + (k-1)30^\circ\right]\cdot\sin\left[45 + 30^\circ k\right]}$$

Use sin (A – B) = sinA CosB – CosA sinB

$$\Rightarrow 2\sum_{k=1}^{13}\left(\cot(45 + (k-1)30^\circ - \cot(45^\circ + 30^\circ k)\right)$$

[C]

$$\Rightarrow 2\left[\cot 45^\circ - \cot(45^\circ + 30^\circ)\right] = 2\left(\sqrt{3} - 1\right)$$

41. $$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 8 & 1 & 0 \\ 48 & 8 & 1 \end{bmatrix} \text{ and } P^3 \ is \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 96 & 12 & 1 \end{bmatrix} \Rightarrow P^n = \begin{bmatrix} 1 & 0 & 0 \\ 4n & 1 & 0 \\ 8(n^2 + n) & 4n & 1 \end{bmatrix}$$

$$n = 50 \Rightarrow P^{50} = \begin{bmatrix} 1 & 0 & 0 \\ 200 & 1 & 0 \\ 20400 & 200 & 1 \end{bmatrix}$$

$P^{50} - Q = I \Rightarrow 200 - q_{21} = 0 \Rightarrow q_{21} = 200$

$q_{32} = 200,\ q_{31} = 400 \times 51 \Rightarrow \frac{q_{31} + q_{32}}{q_{21}} = 103 \quad \Rightarrow$ **[B]**

42. Replace x with –x and I + I

$$\Rightarrow 2I = \int_{-\pi/2}^{\pi/2} x^2 \cos x\, dx = 2\int_0^{\pi/2} x^2 \cos x = \frac{\pi^2}{4} - 2$$

43. $\underset{x\to 2}{\text{Lt}} \dfrac{f(x)g(x)}{f'(x)g'(x)} = 1$ Apply L' Hospital Rule

$\Rightarrow$ f″ (2) = f(2) $\Rightarrow$ **[D]**

Range of f(x) $\in$ (0, α)

f ′ (x) = f ′ (2) > 0

Gives minimum at x = 2 $\Rightarrow$ **[A]**

44. C [2, 8], r = 2,

Normal equation y = mx – 2m – m^2 passes through center [2, 8]

$\Rightarrow$ 8 = 2m -2m – m^2 $\Rightarrow$ m = -2

Normal at P [am^2, - 2am] = [4, 4]

Gives y = -2x + 12

Slope of Tangent = $\frac{1}{2}$ **[A, C, D]**

45. if a = 0, b = 1 f(x) = x sin(x^3 + x) is differentiable $\Rightarrow$ **A**

if a = 0, b = 1 f(x) = cos (x^3 - x), differentiable at 1, 0 $\Rightarrow$ **B**

$\Rightarrow$ [C, D] wrong

46. f(x) = [x^2] – 3, it is discontinues at x = 1, √2, √3, 2

g(x)	= 15x – 21	x < 0
	= 9x – 21,	0 ≤ x ≤ 1
	= 6x – 14	1 ≤ x < √2
	= 3x – 7	√2 ≤ x < √3
	= 0	√3 ≤ x < 2
	= 3	x = 2

Thus at x = 0, 1, √2, √3

g(x) is not differentiable

$\Rightarrow$ **[B C]**

47. f (x)

$$= \left[\frac{n^n \cdot n^n (1 + x/n)\left(x/n + \frac{1}{2}\right)\left(x/n + \frac{1}{3}\right)....\left(x + \frac{1}{n}\right)}{\left| n\left(n^2\right)^{2n}\left[1 + \frac{x^2}{n^2}\right]\left[\frac{1}{4} + \frac{x^2}{n^2}\right]....\left[1 + \frac{1}{n^2}\right] \right.} \right]^{x/n}$$

$$\Rightarrow f\left(\frac{1}{3}\right) = f\left(\frac{2}{3}\right) \quad and \quad f'(2) \le 0$$

$\Rightarrow$ **[B, C]**

48. ax + 2y = λ

3x – 2y = μ $\quad \Rightarrow \Delta = \begin{vmatrix} a & 2 \\ 3 & -2 \end{vmatrix} = -2a - 6 = 0$

$$\Rightarrow a = -3$$

$$\Delta_1 = \begin{vmatrix} \lambda & 2 \\ \mu & -2 \end{vmatrix} = -2(\lambda + \mu)$$

$$\Delta_2 = \begin{vmatrix} -3 & \lambda \\ 3 & \mu \end{vmatrix} = -3(\lambda + \mu)$$

$\Rightarrow$ **[B, C, D]**

49. Perpendicular of u, v is $\bar{u} \times \bar{v}$

$\bar{w}$ is perpendicular to $\bar{u} \times \bar{v}$

w. (u ×v) = 1 $\Rightarrow$ infinitely many solutions. **[B, C]**

50.

$$z = \frac{1}{a + ibt} \times \frac{a - ibt}{a - ibt}$$

$$x + iy = \frac{a - ibt}{a^2 + b^2t^2}$$

[A, C, D]

$$\Rightarrow x = \frac{a}{a^2 + b^2t^2} \qquad y = \frac{-bt}{a^2 + b^2t^2}$$

$$\Rightarrow x^2 + y^2 = \frac{x}{a} \Rightarrow \left(x - \frac{1}{2a}\right)^2 + y^2 = \left(\frac{1}{2a}\right)^2$$

51. P(x > y) implies = $\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{2}=\frac{5}{12}$ **[B]**

52. P(x = y) = $\left(\frac{1}{2}\times\frac{1}{3}\right)2+\frac{1}{6}\times\frac{1}{6}=\frac{13}{36}$ **[C]**

53. a = 3, $e=\frac{1}{3}$ F_1 = (-1, 0) F_2 (1, 0) ⇒ parabola $y^2 = 4x$

Point of intersection $\left(\frac{3}{2},\pm\sqrt{6}\right)$

⇒ Or the center $\left(\frac{-9}{10},0\right)$ ⇒ **[A]**

54. Equation tangent $\frac{3x}{18}+\frac{y\sqrt{6}}{8}=1$

For x – axis, y = 0

⇒ R [6, 0]

Normal at M $\sqrt{\frac{3}{2}}x+y=2\sqrt{\frac{3}{2}}+\left(\sqrt{\frac{3}{2}}\right)^2$

At y = 0 $Q\left(\frac{7}{2},0\right)$

Area of Δ^{le} $\frac{1}{2}\left(6-\frac{7}{2}\right)\times\sqrt{6}=\frac{5\sqrt{6}}{4}$

Quadrilateral area = 2Δ F_1 F_2 M

$=2\left(\frac{1}{2}\right)(2)\left(\sqrt{6}\right)=2\sqrt{6}$

Ratio = $\frac{\frac{5}{4}\sqrt{6}}{2\sqrt{6}}=\frac{5}{8}\Rightarrow 5:8$ **[c]**