JEE (Advanced)-2016 Paper – I Mathematics Solutions

37. Let x is probability that computer turns defective.

Given that it is produced in plant T_2

Thus $x \cdot \frac{4}{5} + \frac{1}{5} \cdot 10x = \frac{7}{10} \Rightarrow x = \frac{7}{280} = \frac{1}{40}$

Required $\frac{P(A \cap B)}{P(B)} = \frac{\frac{4}{5}(1-x)}{\frac{1}{5}(1-10x) + \frac{4}{5}(1-x)} = \frac{78}{93}$ **[C]**

38. One Boy or Zero Boy

$\Rightarrow ({}^4C_1 \cdot {}^6C_3 + {}^6C_4 \cdot {}^4C_0)4 = 380$ **[A]**

39. $(i)\ x^2 - 2x\sec\theta + 1 = 0 \Rightarrow x = \frac{2\sec\theta \pm \sqrt{4\sec^2\theta - 4}}{2} = \sec\theta \pm \tan\theta$

$\alpha_1 = \sec\theta - \tan\theta, \ \ \beta_1 = \sec\theta + \tan\theta$

$(ii)\ x^2 + 2x\tan\theta - 1 = 0 \Rightarrow x = \frac{-\tan\theta \pm \sqrt{4\tan^2\theta + 4}}{2}$

$\Rightarrow x = -\tan\theta \pm \sec\theta$

$\alpha_2 = -\tan\theta + \sec\theta, \ \ \beta_2 = -\tan\theta - \sec\theta$

Thus $\alpha_1 + \beta_2 = -2\tan\theta$ **[C]**

40.

$$\frac{\sqrt{3}}{\cos x} + \frac{1}{\sin x} + 2\left(\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}\right) = 0$$

$$\Rightarrow \sqrt{3}\sin x + \cos x - 2\cos 2x = 0$$

$$\Rightarrow \sin\left(x + \frac{\pi}{6}\right) = \cos 2x$$

$$\cos\left[90 - x - \frac{\pi}{6}\right] = \cos 2x$$

$$\cos\left(\frac{\pi}{3} - x\right) = \cos 2x$$

$$\Rightarrow 2x = 2n\pi \pm \left(\frac{\pi}{3} - x\right)$$

Solutions are – 100^{o}, - 60°, 20°, 140°

Whose sum is '0' **[C]**

41. Let $y = 4\alpha x^2 + \dfrac{1}{x}$

Differentiate $\dfrac{dy}{dx} = 8\alpha x - \dfrac{1}{x^2} = 0 \Rightarrow x = \left(\dfrac{1}{8\alpha}\right)^{1/2}$

Sub in $y = 4\alpha x^2 + \dfrac{1}{x} = \dfrac{4\alpha x^3 + 1}{x} = \dfrac{3}{2} 2\alpha^{1/3} \geq 1$

$\Rightarrow \alpha \geq \dfrac{1}{27}$ **[C]**

42. $(x+2)(x+2+y)\dfrac{dy}{dx} = y^2$

Let $y = t(x+2) \Rightarrow \dfrac{dy}{dx} = (x+2)\dfrac{dt}{dx} + t.1$

Sub $t(x+2)^3 \dfrac{dy}{dx} = t(x+2)^2$

$t(x+2)^3\left[(x+2)\dfrac{dt}{dx} + t\right] = t^2(x+2)^2$

$\Rightarrow \dfrac{1+t}{t} dt = \dfrac{dx}{x+2}$

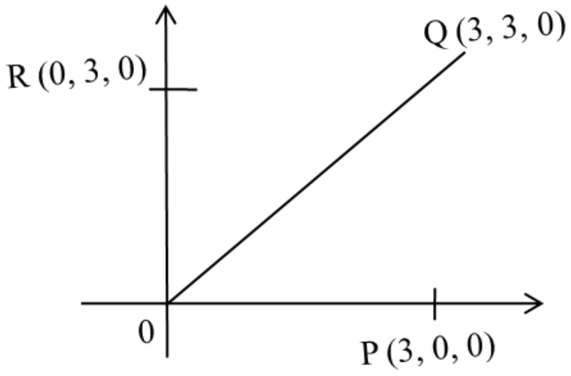
log t + t = log(x + 2) + C

(Replacing 't')

$\log\dfrac{y}{x+2} + \dfrac{y}{x+2} = \log(x+2) + C$ **[A, D]**

$\Rightarrow \log y + \dfrac{y}{x+2} = \log 3e$

43.



Mid point of $\overline{OQ}$ = (3/2, 3/2, 0)

$$\overline{OS} = \frac{3}{2}i + \frac{3}{2}j + 3k,\ \overline{OQ} = 3i + 3j$$

$$(A)\text{.....}\ \cos 0^\circ = \frac{OQ.OS}{|OQ||OS|} = \frac{\frac{9}{2}+\frac{9}{2}+0}{\left(\sqrt{\frac{9}{4}+\frac{9}{4}+9}\right)\left(\sqrt{9+9}\right)} = \frac{1}{\sqrt{3}}$$

Unit vector $\overline{OQ} \times \overline{OS} = 2i - 2j$

(B)....... $x - y = 0 \Rightarrow x = y$

(C)..... Perpendicular (3, 0, 0) $\Rightarrow \frac{3}{\sqrt{2}}$

(D)....... RS line

$$\frac{x-0}{\frac{3}{2}} = \frac{y-3}{-\frac{3}{2}} = \frac{z}{3} = t$$

$$\Rightarrow x = \frac{3}{2}t, y = -\frac{3}{2}t + 3, z = 3t$$

$$\text{distance} = \sqrt{\frac{9t^2}{4} + \left(-\frac{3}{2}t + 3\right)^2 + 9t^2} = \left(\sqrt{\frac{15}{2}}\right)^2$$

$$\Rightarrow t = \frac{1}{3}$$

[B, C, D]

44. Solving $y^2 + 2y - 3 = 0 \Rightarrow y = 1 \Rightarrow x = \sqrt{2}$

Tangent of $(\sqrt{2}, 1)$ $\sqrt{2}x + y = 3$

Let centre of circle is [0, y]

$$\text{Distance} = \frac{\sqrt{2}.0 + y - 3}{\sqrt{\sqrt{2}+1}} = 2\sqrt{3}$$

$\Rightarrow (y - 3) = 6 \Rightarrow y = 3, -3$

(A)........ Q_2 [0, 9], Q_3 (0, -3), $Q_2 Q_3 = 12$

C_2 $x^2 + (y - 9)^2 = (2\sqrt{3})^2$

C_3 $x^2 + (y + 3)^2 = (2\sqrt{3})^2$

Foot of perpendicular from Q_2 and Q_3

R_2 $[-2\sqrt{2}, 7]$ R_3 $[2\sqrt{2}, -1]$

(B)........ $R_2 R_3 = 4\sqrt{6}$

(C)...... Area of triangle $OR_2R_3 = 6\sqrt{2}$

(D)....... $PQ_2Q_3 = 6\sqrt{2}$ **[A, B, C]**

45. $g(x) = f^{-1}(x)$

(A)........

$$f[g(x)] = r \Rightarrow f'[g(x)]g^1(x) = 1$$

$$g'(2) = \frac{1}{f'g(2)} = \frac{1}{f'(0)} = \frac{1}{3}$$

For B, D $h[g(g(x)] = x$

$\Rightarrow h\, g[g(f(x))] = f(x) \Rightarrow h[g(x)] = f(x)$

At $x = 1$, $h^1(1) = 666$ **[B, C]**

46. PQ = KI $\Rightarrow$ Q = KP^{-1}I = K$\frac{adj(p)}{|p|}$

$$Q = \frac{K}{20+12\alpha}\begin{bmatrix} 5\alpha & 3\alpha & 0 \\ 0 & 0 & -3\alpha-4 \\ - & - & - \end{bmatrix}\begin{bmatrix} 7 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

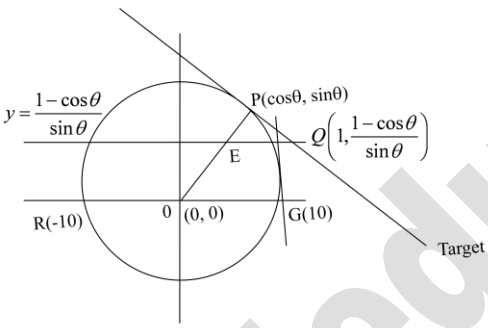
$\Rightarrow Q_{23} = -K/8 \Rightarrow \alpha = -1$ **[B, C]**

and $|Q| = \frac{K^2|I|}{|p|} \Rightarrow k = 4$

47. P (1cosθ, 1sinθ) = (cosθ , sinθ)

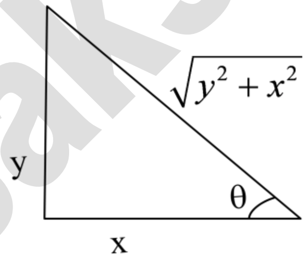
S [1, 0] $\Rightarrow$ R = [-1, 0] , Q = $\left[1, \frac{1-\cos\theta}{\sin\theta}\right]$

Parallel to RS line is y = $\frac{1-\cos\theta}{\sin\theta}$



Equation of OE, $y = \frac{1-\cos\theta}{\sin\theta}$

Normal at P, y = (tanθ)x $\Rightarrow$ tanθ = y/x



Substitute in $y = \frac{1-\cos\theta}{\sin\theta} = \frac{1-\frac{x}{\sqrt{y^2+x^2}}}{\frac{y}{\sqrt{y^2+x^2}}}$

$\Rightarrow y^2 + x = \sqrt{x^2 + y^2}$ **[A, C]**

48. $f'(x) = 2 - \dfrac{f(x)}{x} \Rightarrow \dfrac{dy}{dx} - \dfrac{y}{x} = 2$

Integration factor $e^{\int -1/x\, dx} = x$

Solution $yx = \int 2x\,dx \Rightarrow x^2 + c$

$\Rightarrow$ y = x + c/x

A) $\lim\limits_{x\to 0^+} f\left(\dfrac{1}{x}\right) = 1 - 0 = 1$ C) –c

B) $\lim\limits_{x\to 0^-} xf\left(\dfrac{1}{x}\right) = 1 + cx^2 = 1$ D) =α **[A]**

49. $\dfrac{s-x}{4} = \dfrac{s-y}{3} = \dfrac{s-z}{2} = \dfrac{s-x+s-y+s-z}{4+3+2} = \dfrac{3}{9}$

$\Rightarrow x = \dfrac{5s}{9}\ \ y = \dfrac{2s}{3},\ z = \dfrac{7s}{9}$

$\pi r^2 = \dfrac{\Delta}{s} = \dfrac{8\pi}{3}$

$\Rightarrow \Delta^2 = \dfrac{8s^2}{3}$

$s(s-x)(s-y)(s-z) = \dfrac{8s^2}{3}$

$s\left(s - \dfrac{5s}{9}\right)\left(s - \dfrac{2s}{3}\right)\left(s - \dfrac{7s}{9}\right) = \dfrac{8s^2}{3}$

$\Rightarrow s = 9$

$\Rightarrow \Delta = 6\sqrt{6}$

$\Delta = \dfrac{abc}{4R}$

$\Rightarrow R = \dfrac{35}{4\sqrt{6}}$

$\sin\dfrac{x}{2}\sin\dfrac{y}{2}\sin\dfrac{z}{2} = \dfrac{r}{4R} = \dfrac{4}{35}$

[A, C, D]

50. Coefficient of x^2 is

$$1+{}^{3}C_2+{}^{4}C_2+{}^{5}C_2....+{}^{50}C_2m^2=(3n+1)^{51}C_3$$

$$^{3}C_3+{}^{3}C_2+{}^{4}C_2....+{}^{50}C_2m^2=(3n+1)^{51}C_3$$

$$\text{using } {}^{n}C_r+{}^{n}C_{r-1}={}^{n+1}C_r$$

This gives $^{50}C_3+{}^{50}C_2m^2=(3n+1)^{51}C_3$

Add $^{50}C_2$ $\quad {}^{50}C_2+{}^{50}C_3+{}^{50}C_2m^2=(3n+1)^{51}C_3+{}^{50}C_2$

$$^{51}C_3+{}^{50}C_2m^2-{}^{50}C_2=(3n+1)^{51}C_3$$

$$\Rightarrow m^2-1=51n$$

$$m^2=51n+1$$

This is integer for n = 5 **[5]**

51. $$\lim_{x\to 0}\frac{x^2\sin\beta x}{\alpha x-\sin x}=\frac{x^2\left[\beta x-\frac{\beta^3x^3}{\lfloor 3}+\frac{\beta^5x^5}{\lfloor 5}.....\right]}{\alpha x-\left[x-\frac{x^3}{\lfloor 3}+\frac{x^5}{\lfloor 3}.....\right]}=1$$

$$\Rightarrow\frac{x^2\left[\beta x-\frac{\beta^3x^3}{\lfloor 3}+.....\right]}{(\alpha-1)x+\frac{x^3}{\lfloor 3}-\frac{x^5}{\lfloor 3}}=1\ \Rightarrow\alpha-1=0\Rightarrow\alpha=1$$

At $\alpha = 1$ $\quad \frac{\beta}{1/\lfloor 3}=1\Rightarrow\beta=\frac{1}{6}$

52. $$P=\begin{bmatrix}(-w)^r & w^{2s}\\ w^{2s} & w^r\end{bmatrix}\Rightarrow P^2=\begin{bmatrix}w^{2r}+w^{4s} & (-1)^r w^{2s+r}+w^{r+2s}\\ (-1)^r w^{2s+r}+w^{r+2s} & w^{2r}+w^{4s}\end{bmatrix}$$

As $w^2+w=w^4+w^8=-1$ and $\left[(-1)^r+1\right]w^{2s+r}=0$

For all r, s = 1, 2, 3

At r =1, $w^{4s}=-1-w^2=w=s=1$

At r = 3, $w^{4s}=-1-1=-2$ Not possible

Number of ordered pairs (r, s) = 1 **[1]**

53. In column C_1, C_2 x, x^2 common

$$x.x^2 \begin{vmatrix} 1 & 1 & 1+x^3 \\ 0 & 2 & 6x^3-1 \\ 0 & 6 & 24x^2-2 \end{vmatrix} = 10 \Rightarrow 6x^6 + x^3 - 5 = 0$$

$$\Rightarrow \left(6x^3-5\right)\left(x^3+1\right) = 0 \Rightarrow x = -1, x = \left(5/6\right)^{1/3}$$

Number of solutions are 2 **[2]**

54. f(x) = $\int_0^x \frac{t^2}{1+t^4}$

Differentiate $f'(x) = \frac{x^2}{1+x^4} > 0 \Rightarrow$ f(x) increases

at x=0 $f(0) = 0$

$x=1$ $f(1) = \int_0^1 \frac{t^2}{1+t^4}$ *lies between 0 & 1*

$\Rightarrow$ One solution is [0 1] **[1]**