166
(TS)

C

Total No. of Questions - **24**
Total No. of Printed Pages - **4**

Regd. No. [ ]

Part - III
MATHEMATICS, Paper - I (A)
(Algebra, Vector Algebra and Trigonometry)
(English Version)

Time : 3 Hours ***Max. Marks : 75***

Note : This question paper consists of **three** sections *A*, *B* and *C*.

SECTION A $10 \times 2 = 20$

I. **Very short answer** type questions.

i) Answer **all** questions.

ii) Each question carries **two** marks.

1. If $A = \{-2, -1, 0, 1, 2\}$ and $f : A \to B$ is a surjection defined by $f(x) = x^2 + x + 1$, then find B.

2. Find the domain of the real valued function
$f(x) = Log_e\left(x^2 - 4x + 3\right)$.

3. Solve the following system of homogeneous equations
$x - y + z = 0,\ x + 2y - z = 0,\ 2x + y + 3z = 0$.

4. Define Triangular Matrix.

5. Let $\bar{a} = \bar{i} + 2\bar{j} + 3\bar{k}$ and $\bar{b} = 3\bar{i} + \bar{j}$. Find the unit vector in the direction of $\bar{a} + \bar{b}$.

6. Find the vector equation of the line joining the points $2\bar{i} + \bar{j} + 3\bar{k}$ and $-4\bar{i} + 3\bar{j} - \bar{k}$.

7. If the vectors $\lambda\bar{i} - 3\bar{j} + 5\bar{k}$ and $2\lambda\bar{i} - \lambda\bar{j} - \bar{k}$ are perpendicular to each other, find λ.

8. If $A + B = \frac{\pi}{4}$, then prove that $(1 + TanA)(1 + TanB) = 2$.

9. Eliminate 'θ' from $x = a\,Cos^3\theta$, $y = b\,Sin^3\theta$.

10. If $Sinh\,x = 3$, then show that $x = Log_e\left(3 + \sqrt{10}\right)$.

SECTION B 5 × 4 = 20

II. **Short answer** type questions.

i) Attempt **any five** questions.

ii) Each question carries **four** marks.

11. If $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $E = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ then show that $(aI + bE)^3 = a^3 I + 3a^2 bE$ where I is identity matrix of order 2.

12. Show that the line joining the pair of points $6\bar{a} - 4\bar{b} + 4\bar{c}$, $-4\bar{c}$ and the line joining the pair of points $-\bar{a} - 2\bar{b} - 3\bar{c}$, $\bar{a} + 2\bar{b} - 5\bar{c}$ intersect at the point $-4\bar{c}$ when $\bar{a}$, $\bar{b}$, $\bar{c}$ are non-coplanar vectors.

13. Find λ in order that the four points $A(3,2,1)$, $B(4,\lambda,5)$, $C(4,2,-2)$ and $D(6,\ 5,-1)$ be coplanar.

14. If none of the denominators is zero, prove that

$$\left(\frac{Cos\,A+Cos\,B}{Sin\,A-Sin\,B}\right)^n+\left(\frac{Sin\,A+Sin\,B}{Cos\,A-Cos\,B}\right)^n=\begin{cases}2\ \ Cot^n\left(\frac{A-B}{2}\right), & \text{if } n \text{ is even}\\ 0, & \text{if } n \text{ is odd}\end{cases}$$

15. If θ_1, θ_2 are solutions of the equation $a\,Cos2\theta+bSin2\theta=c$, $Tan\theta_1 \neq Tan\theta_2$ and $a+c\neq 0$, then find the values of

i) $Tan\theta_1+Tan\theta_2$ ii) $Tan\theta_1 \cdot Tan\theta_2$

16. Prove that $Sin^{-1}\frac{4}{5}+Sin^{-1}\frac{7}{25}=Sin^{-1}\frac{117}{125}$

17. If $a=(b-c)\,Sec\theta$, then prove that $Tan\theta=\frac{2\sqrt{bc}}{b-c}\,Sin\frac{A}{2}$.

SECTION C

5 × 7 = 35

III. **Long answer** type questions.

i) Attempt **any five** questions.

ii) Each question carries **seven** marks.

18. If $f:A\to B,\ g:B\to C$ are bijections, then prove that $(gof)^{-1}=f^{-1}og^{-1}$

19. Using Mathematical induction, for all $n\in N$, show that

$$a+(a+d)+(a+2d)+\ldots\ldots\ldots\ n \text{ upto } n \text{ terms } =\frac{n}{2}\left[2a+(n-1)d\right]$$

20. Show that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

21. Solve the following system of equations by using Matrix inversion method. $2x-y+3z=9,\ x+y+z=6,\ x-y+z=2$

22. If $\bar{a}=\bar{i}-2\bar{j}+3\bar{k},\ \bar{b}=2\bar{i}+\bar{j}+\bar{k},\ \bar{c}=\bar{i}+\bar{j}+2\bar{k}$ then find $|(a\times b)\times c|$ and $|a\times(b\times c)|$.

23. If $A+B+C=2S$, then prove that

$$Sin(S-A)+Sin(S-B)+SinC=4\,Cos\left(\frac{S-A}{2}\right)Cos\left(\frac{S-B}{2}\right)Sin\frac{C}{2}$$

24. If $r_1=2;\ r_2=3,\ r_3=6$ and $r=1$ then prove that $a=3,\ b=4$ and $c=5$.