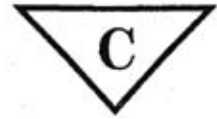


166
(TS)



Total No. of Questions - 24

Total No. of Printed Pages - 4

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Part - III
MATHEMATICS, Paper - I (A)
(Algebra, Vector Algebra and Trigonometry)
(English Version)

Time : 3 Hours

Max. Marks : 75

Note : This question paper consists of three sections A, B and C.

SECTION A

10 × 2 = 20

I. Very short answer type questions.

- i) Answer all questions.
- ii) Each question carries two marks.

1. If $A = \{-2, -1, 0, 1, 2\}$ and $f: A \rightarrow B$ is a surjection defined by $f(x) = x^2 + x + 1$, then find B .
2. Find the domain of the real valued function $f(x) = \text{Log}_e(x^2 - 4x + 3)$.
3. Solve the following system of homogeneous equations $x - y + z = 0$, $x + 2y - z = 0$, $2x + y + 3z = 0$.
4. Define Triangular Matrix.
5. Let $\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{b} = 3\vec{i} + \vec{j}$. Find the unit vector in the direction of $\vec{a} + \vec{b}$.

6. Find the vector equation of the line joining the points $2\vec{i} + \vec{j} + 3\vec{k}$ and $-4\vec{i} + 3\vec{j} - \vec{k}$.
7. If the vectors $\lambda\vec{i} - 3\vec{j} + 5\vec{k}$ and $2\lambda\vec{i} - \lambda\vec{j} - \vec{k}$ are perpendicular to each other, find λ .
8. If $A + B = \frac{\pi}{4}$, then prove that $(1 + \tan A)(1 + \tan B) = 2$.
9. Eliminate ' θ ' from $x = a \cos^3 \theta$, $y = b \sin^3 \theta$.
10. If $\sinh x = 3$, then show that $x = \log_e (3 + \sqrt{10})$.

SECTION B

5 × 4 = 20

II. Short answer type questions.

- i) Attempt **any five** questions.
- ii) Each question carries **four** marks.
11. If $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $E = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ then show that $(aI + bE)^3 = a^3I + 3a^2bE$ where I is identity matrix of order 2.
12. Show that the line joining the pair of points $6\vec{a} - 4\vec{b} + 4\vec{c}$, $-4\vec{c}$ and the line joining the pair of points $-\vec{a} - 2\vec{b} - 3\vec{c}$, $\vec{a} + 2\vec{b} - 5\vec{c}$ intersect at the point $-4\vec{c}$ when \vec{a} , \vec{b} , \vec{c} are non-coplanar vectors.

13. Find λ in order that the four points
 $A(3, 2, 1)$, $B(4, \lambda, 5)$, $C(4, 2, -2)$ and $D(6, 5, -1)$ be coplanar.

14. If none of the denominators is zero, prove that

$$\left(\frac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\frac{\sin A + \sin B}{\cos A - \cos B}\right)^n = \begin{cases} 2 \cot^n\left(\frac{A-B}{2}\right), & \text{if } n \text{ is even} \\ 0, & \text{if } n \text{ is odd} \end{cases}$$

15. If θ_1, θ_2 are solutions of the equation $a \cos 2\theta + b \sin 2\theta = c$,
 $\tan \theta_1 \neq \tan \theta_2$ and $a + c \neq 0$, then find the values of

i) $\tan \theta_1 + \tan \theta_2$ ii) $\tan \theta_1 \cdot \tan \theta_2$

16. Prove that $\sin^{-1} \frac{4}{5} + \sin^{-1} \frac{7}{25} = \sin^{-1} \frac{117}{125}$.

17. If $a = (b-c) \sec \theta$, then prove that $\tan \theta = \frac{2\sqrt{bc}}{b-c} \sin \frac{A}{2}$.

SECTION C

5 × 7 = 35

III. Long answer type questions.

- i) Attempt any five questions.
 ii) Each question carries seven marks.

18. If $f: A \rightarrow B$, $g: B \rightarrow C$ are bijections, then prove that

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$$

19. Using Mathematical induction, for all $n \in \mathbb{N}$, show that

$$a + (a+d) + (a+2d) + \dots \dots \dots n \text{ upto } n \text{ terms} = \frac{n}{2} [2a + (n-1)d]$$

20. Show that
$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

21. Solve the following system of equations by using Matrix inversion method. $2x - y + 3z = 9$, $x + y + z = 6$, $x - y + z = 2$

22. If $\vec{a} = \vec{i} - 2\vec{j} + 3\vec{k}$, $\vec{b} = 2\vec{i} + \vec{j} + \vec{k}$, $\vec{c} = \vec{i} + \vec{j} + 2\vec{k}$ then find $|(\vec{a} \times \vec{b}) \times \vec{c}|$ and $|\vec{a} \times (\vec{b} \times \vec{c})|$.

23. If $A + B + C = 2S$, then prove that

$$\sin(S - A) + \sin(S - B) + \sin C = 4 \cos\left(\frac{S - A}{2}\right) \cos\left(\frac{S - B}{2}\right) \sin\frac{C}{2}$$

24. If $r_1 = 2$, $r_2 = 3$, $r_3 = 6$ and $r = 1$ then prove that $a = 3$, $b = 4$ and $c = 5$.
