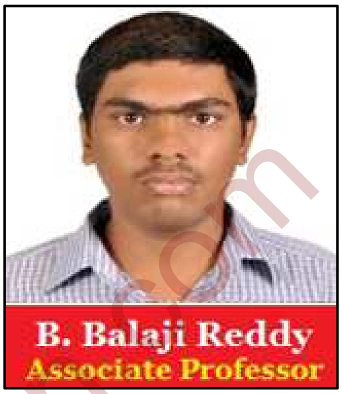
Network Topology-2 & Dual and Duality



Choice of independent branch currents and voltages:

The solution of a network involves solving of all branch currents and voltages. We know that the branch current and voltage of every branch is related to each other by its V- I relationship. Hence for branch network if we know b variables (either branch currents or branch voltages), then the other b variables can be uniquely determined. The network equations can be formulated using Kirchhoff's law using branch currents or branch voltages as variables. It can be shown that all the b variables are not independent. We can determine the number of independent branch currents and branch voltages using the concept developed by graph theory.

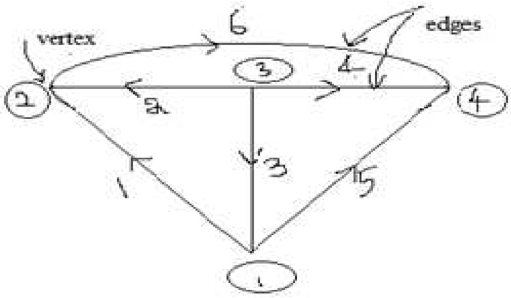
Choice of independent branch currents:

To determine the number of independent branch currents, consider the tree of a given graph which is connected sub graph with no closed path. The addition of each link branch to the tree gives rise to different closed path. Hence the opening or removal of the links destroys all closed paths, which results in forcing all branch currents to zero. Thus, if we set all link branch currents to zero, the currents in all branches of the network automatically to zero. We can conclude that tree branch currents are dependent on link branch currents and can be expressed uniquely in terms of link branch currents. This shows for a given network with b branches and n nodes, the number of independent branch currents equal to number of links is (b- (n-1)) = (b-n+1).

The dependent branch currents or tree branch currents can be expressed in a unique way in terms of link branch currents using the row of the tie-matrix [C].

Hence to solve a given network, we have to determine the independent branch currents, in terms of which other variables can be determined. To determine these independent branch currents we formulate equations by equations by applying KVL to each of the loops, and these equations are called as loop equations and the variables in which equations are formulated are called the loop currents. This method of analysis is called as loop method of analysis and is based on KVL.

To analyze this see the fallowing example, if the branch currents are $i_{1,}i_{2,\_\_\_\_\_\_,}i_8$ then they can be expressed as link branch currents$i_{1,}i_{2,}i_{3,}i_4$ as



$$[i] = [C]\,[i_1] \text{ ------------------------ } (1)$$

Where [i] = column of branch currents

$[i_1]$ = column of link branch currents

[C] = basic tie-set matrix

The tree branch currents for above example are can express as below,

	1	2	3	4
i1	+1			
i2		+1		
i3			+1	
i4				+1
i5	+1			-1
i6	-1	+1		
i7		-1	+1	
i8			-1	+1

[i1 i2 i3 i4 | i5 i6 i7 i8] = [C] [i1 i2 i3 i4]

$i_5 = i_1 - i_4$

$i_6 = -i_1 + i_2$

$i_7 = -i_2 + i_3$

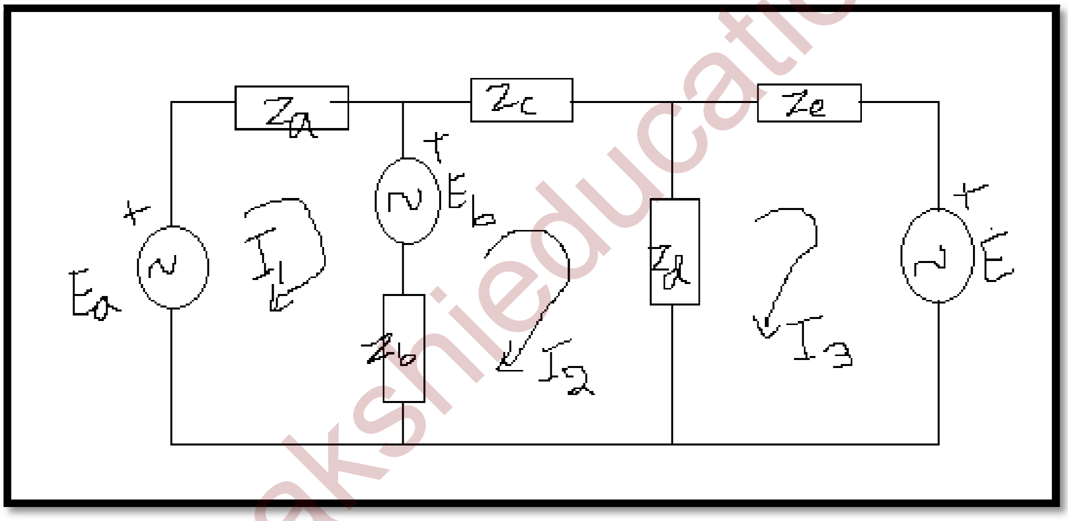
= + ----------------- (3)

The loop equations are formulated using equations (1).

Another way to find loop currents:

Another way commonly employed in formulating the loop equations using loop currents as the independent variables is illustrated below. We can use the window method to arrive at the number of independent loop currents if it is planar network. This method is illustrated and loop equations are formulated using KVL in terms of loop current variables.

Consider the network shown in fig. the choice of loop currents in loop equations formulated below. Let , and are the loop currents in the loops 1, 2 and 3 flowing in the elements forming that loop. The loop equations are obtained by applying KVL for each of the loop in the network.



Loop1: - + + + (-) = 0 → (+) - = (-) – (1)

Loop2: (-) - + + (-) = → - + (+ +) - = -- (2)

Loop3: (-) + + E = 0 → - + +) = E ---- (3)

The equations 1, 2, 3 are the loop equations which are to be solved for the loop currents.

Choice of independent voltages:

In analyzing the network on voltage basis, we have to determine the number of independent branch voltages. We can use graph theory concepts to determine independent voltages.

Consider the tree of a network graph which is connected sub graph with no closed paths. Since it is connected sub graph, there exists a unique path between every pair of nodes only, through tree branches removal of tree branch voltages results in the equal potential of all nodes. Hence of all the tree branch voltages are set equal to zero, where in all the branch voltages of the network will automatically become zero. All the link branch voltages can be expressed in a unique way in terms of tree branch voltages.

Hence for any given network within n nodes and b branches, there will be (n-1) tree branches and hence there will be (n-1) independent branch voltages. The remaining branches voltages can be expressed in terms of these independent branch voltages. To determine these independent branch voltages, we formulate the (n-1) equations by applying KCL to each of the nodes and these equations are nodal equations. This method of analysis is called nodal method of analysis and is based on KCL.

The dependent branch voltages can be expressed in unique way in terms of tree branch voltages using the rows of basic cut-set matrix [B].

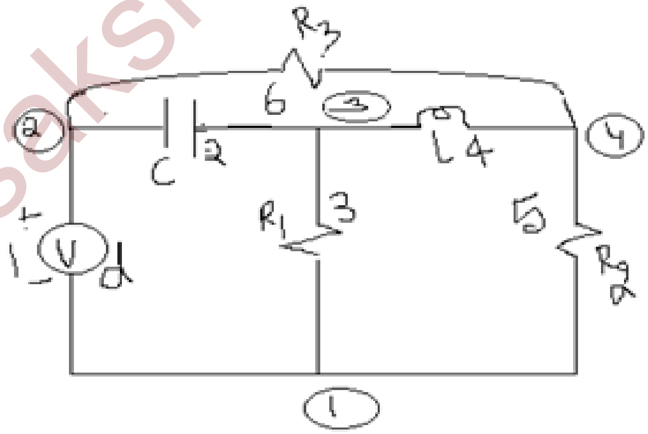
To illustrate, we consider the below example. If the branch voltages are $v_{1,}v_{2,\_\_\_\_\_\_,}v_8$ then they can be expressed in terms of tree branch voltages [$v_{5,}$ $v_{6,}$ $v_{7,}$ v_8] as

[v] = [B] [v_b] ----------- (1)

Where [v] = column branch voltages

[v_b] = column of tree branch voltages

[B] = basic cut set matrix



From that graph we can get,

$$\begin{bmatrix} v1 \\ v2 \\ v3 \\ v4 \\ \hdashline v5 \\ v6 \\ v7 \\ v8 \end{bmatrix} = \begin{array}{|c|c|c|c|} \hline \textcircled{5} & \textcircled{6} & \textcircled{7} & \textcircled{8} \\ \hline +1 & +1 & & \\ \hline & -1 & +1 & \\ \hline & & -1 & +1 \\ \hline -1 & & & -1 \\ \hline +1 & & & \\ \hline & +1 & & \\ \hline & & +1 & \\ \hline & & & +1 \\ \hline \end{array} \begin{bmatrix} v5 \\ v6 \\ v7 \\ v8 \end{bmatrix}$$

The link branch voltages are expressed in a unique way as

$$v_1 = v_5 + v_6$$

$$v_2 = -v_6 + v_7$$

$$v_3 = -v_7 + v_8$$

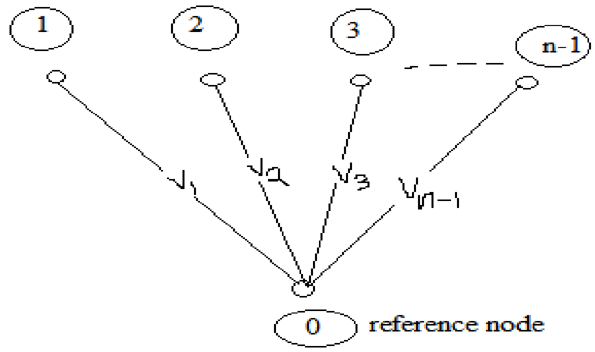
$$v_4 = -v_5 + v_8 \quad \text{---------------} \ (3)$$

The nodal equations are formulated using columns of equations (1).

Another way of choosing independent voltages:

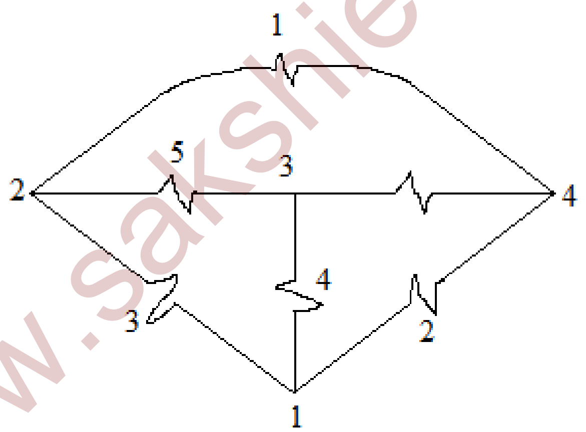
This method is commonly employed in practice instead of using graph theory. In this method, we employ node pair voltages or nodal voltages as the variables in terms of which nodal equations are obtaining KCL at each node.

Consider a network having nodes and choose one of the nodes as the reference node. Theoretically any one of the nodes can be chosen as reference node and the potentials of all other nodes are measured with respect to node. A node to which more number of elements is connected is generally taken as reference node. Consider below example



The reference node is numbered zero and assumed to be at zero potential. The other nodes are numbered as 1, 2, -----, n-1and the potentials of these nodes with reference to node (0) are v_1,v_2, -----, v_{n-1} respectively. Thus there will be (n-1) node pair voltages which are independent voltages. All other voltages can be expressed in terms of these node pair voltages. The nodal equations are formulated in terms of these variables, by applying KCL at each node but expect at reference node. This method is known as nodal method of analysis.

Ex 1: For the given network draw the graph and choose a possible tree. Construct the basic tie-set schedule. Write the equation for the branch currents and in terms of the link currents and write separately the independent equations.

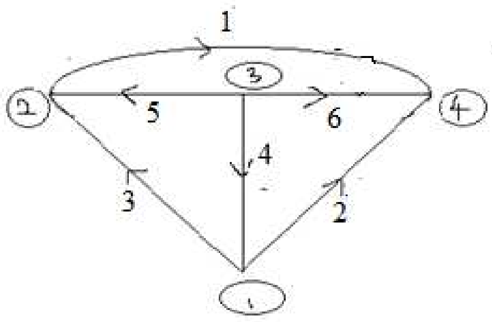


No of nodes of a tree, n_t = 4

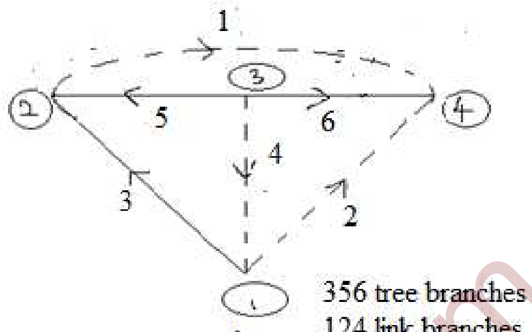
No of tree branches, n= 4-1 = 3

Total no of branches, b = 6

No of links, l = no of independent loop currents = b – n = 6 – 3 = 3

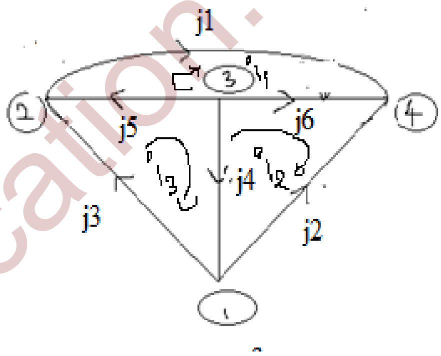


a



b

Link No (i)	Branch No (j)					
	1	2	3	4	5	6
1	+1	0	0	0	+1	-1
2	0	+1	+1	0	-1	+1
3	0	0	+1	+1	-1	0



Branch currents in terms of independent link currents from fig c

$j_1 = i_1$

$j_2 = i_2$

$j_4 = i_3$ identities

$j_3 = i_2 + i_3$

$j_3 = i_1 - (i_2 + i_3)$

$j_6 = i_2 - i_1$ independent equations

The loop equations from the rows of tie-set matrix are,

$v_1 + v_5 - v_6 = 0$

$v_2 + v_3 - v_5 + v_6 = 0$

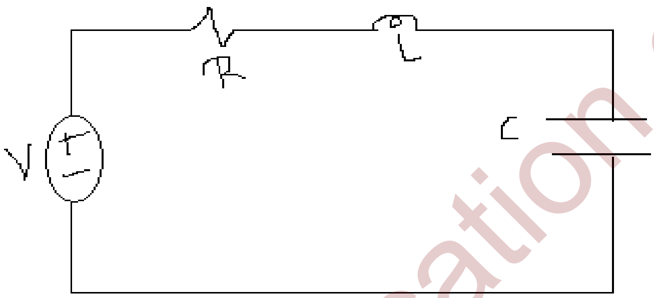
$v_3 + v_4 - v_5 = 0$

Duality and Dual Networks

Duals: Two circuits are said to be dual of each other, if the mesh equations characterize one of them has the same mathematical form as the nodal equations that characterize the other.

Principle of Duality: Identical behavior patterns observed between voltages and currents between two independent circuits illustrate the principle of duality.

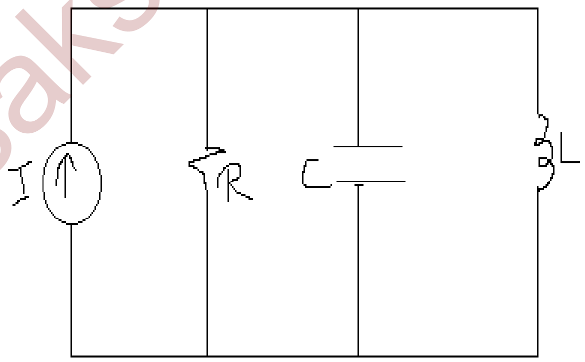
Ex: 1) series R-L-C circuit:



Mesh → KVL → -V + iR + L$\frac{di}{dt}$ + $\frac{1}{C}\int i\,dt$ = 0

V = I R + L$\frac{di}{dt}$ + $\frac{1}{C}\int i\,dt$

2) Parallel G-C-L circuit:



Nodal → KCL→ - I + V G + C $\frac{dv}{dt}$ + $\frac{1}{L}\int v\,dt$

I = V G + C $\frac{dv}{dt}$ + $\frac{1}{L}\int v\,dt$

From that (1) and (2) are mathematically identical, so they are duals.

Some dual elements:

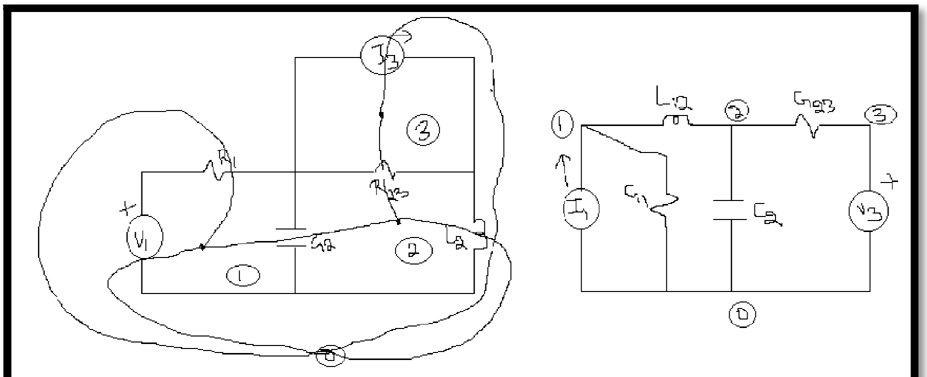
1) Voltage (V) ←→ Current (I)
2) Resistor (R) ← → Conductance (G)
3) Inductor (I) ← → Capacitor (C)
4) KVL ← → KCL
5) V(t) ← → I(t)
6) Mesh ← → nodal
7) Series ← → parallel
8) Vsinwt ← → Icoswt
9) Open circuit ← → short circuit
10) Thevenin ← → Norton
11) Link ← → twig
12) Cut set ← → tie set
13) Tree ← → co-tree
14) Switch in series (getting closed) ← → switching in parallel (getting opened) etc.

Procedure to Obtain a Dual Network:

These rules illustrated below are only for planar or flat networks which do not have any of their branches crossing other branches1.

1) Place a dot in every loop of the network whose dual is obtained and a dot outside the network. Each dot is numbered according to the loop in which it is placed. The outside dot is called the reference node and give number as 0.
2) Connect two dots by a line through each branch. The dots are the nodes of the dual network between two nodes; the element to be connected is the dual of the element crossed by the line.
3) When sources are included, then the line joining the dots should intersect the sources also; between these two nodes the dual of the source is included.
4) The polarity of the source is decided by the fallowing rule. A voltage or current source which drives a current in clockwise in i^{th} loop, then place a positive polarity at i^{th} the dual network. Negative if it is opposite.

Example:



Original Network   Dual Network

Inverse Networks: If two impedances which are duals of each other are expressed in the form of = , where K is a positive number of independent of frequency, then the two impedances are said to be inverse or reciprocal. The inversion is said to be about K.

Ex: = + j (w —) and

= — = + j (w —)

So * = — = —————— =

From the above equation that,

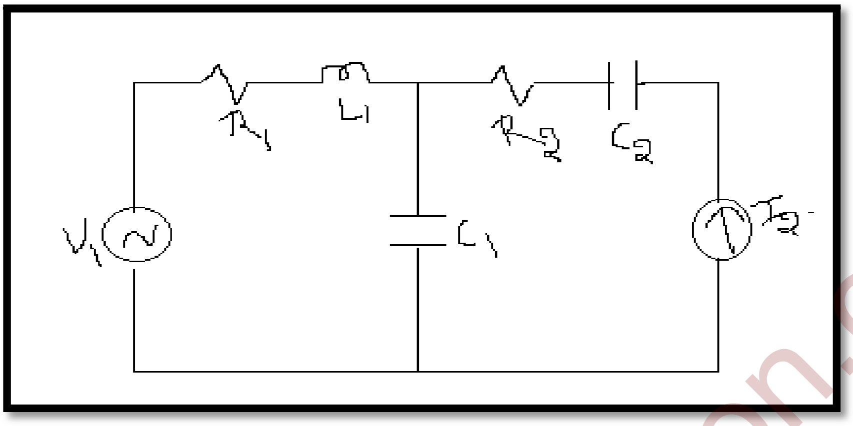
— = — = — =

From that for inverse networks, =

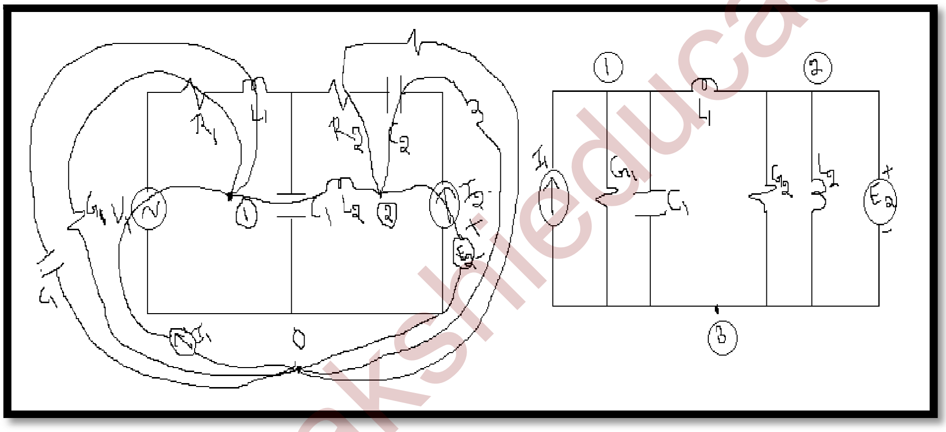
The method of inverting a given network as fallows,

1) Every parallel arrangement of X is replaced in Y by a series arrangement of elements and vice versa.
2) Every inductor in A is replaced by a capacitor = —
3) Every capacitor in A is replaced by an inductor =
4) Every resistor in in A is replaced by a conductance = —

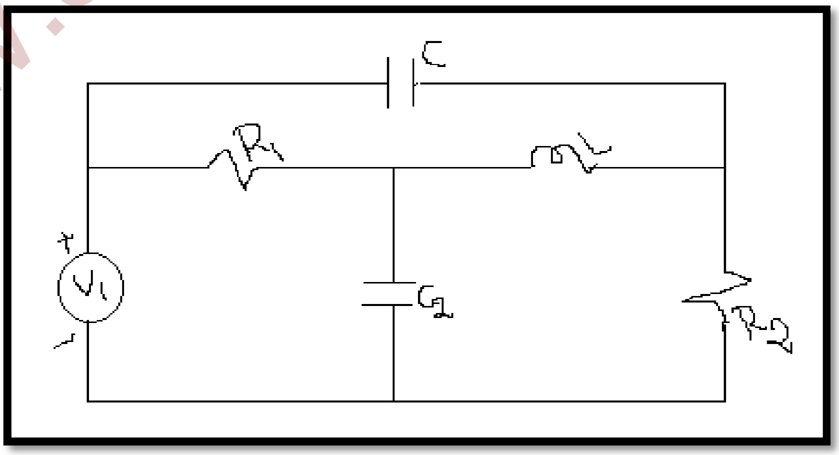
Ex 1: Draw the dual of fallowing network shown in below



The dual of the network is,



Ex 2: Draw the dual of fallowing network shown in below



The dual of the network is,