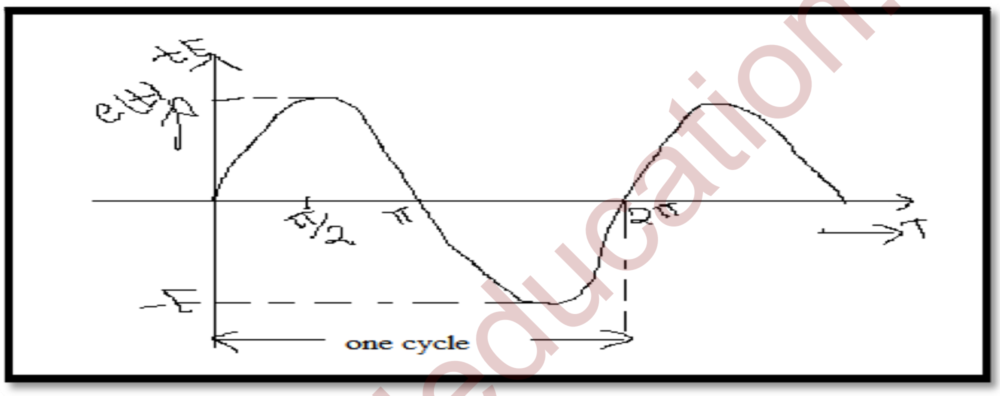
Single Phase Ac Circuits

We have studied the response of the circuit elements to different excitations like DC and periodic quantities using the volt-amp relationship of the passive circuit elements. Out of all these sinusoidal exactions are important, because the generation, transmission and utilization of electrical power is in the form sinusoidal voltages and currents. Hence in this lesson we will study the generation of sinusoidal voltage, their characteristics and their representation by phasors. We will also define RMS, average value of an AC quantity and determine those for different waveforms.

The waveform of a AC quantity with respect time or associated time angle can shown in below figure.



Basic definitions of a periodic waveform:

Cycle: A complete set of values of an alternating is known as cycle.

Time Period: The time taken for one complete cycle is called time period. It is the smallest value of time which separates the recurring values of the alternating quantity. It is expressed in seconds. One angle corresponds 360 degrees or 2π radians.

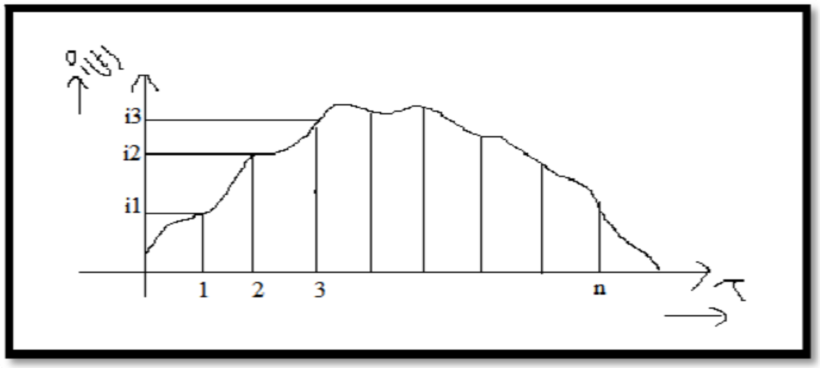
Frequency: It is the number of cycles per second. It is measured in HZ.

$$f=\frac{1}{T}\text{ hz}$$

Peak value or Amplitude: The maximum value (taken from positive to negative) of an alternating quantity in one cycle is known as peak value or amplitude.

Root-Mean-Square (RMS) Value:

The RMS value of an alternating current is that value of steady current, which when flows through a given circuit for a given time produces the same heating effect as produced by the given alternating current flowing through the same circuit for the same time.



The given alternating current wave is divided into n equal number of time intervals. Let the mean value of the current during the intervals bei_1,i_2 - - - - - i_n. Let this current be allowed through a circuit having resistance R. Then the average power dissipated by the resistor in the interval is

$$P = \frac{(i_1{}^2R + i_2{}^2R + - - - - - - + i_n{}^2R)}{n}$$

If a direct current of I amps flows through the same resistance R for the same time, the power dissipated is i^2r. If both the current produces the same amount of heat, then

$$i^2\text{r} = \frac{(i_1{}^2R + i_2{}^2R + - - - - - - + i_n{}^2R)}{n}$$

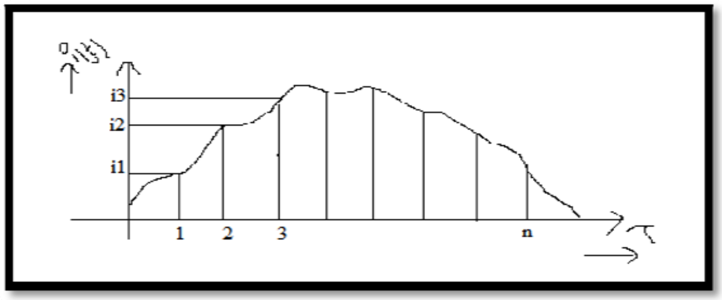
The steady current which produces the same amount of heat as given, alternating current is called RMS value and hence

$$I_{RMS} = \text{I} = \sqrt{\frac{(i_1{}^2 + i_2{}^2 + - - - - - - + i_n{}^2)}{n}}$$

The analytical expression for RMS value of a periodic function y(t) of a time period T is

$$X_{RMS} = \sqrt{\frac{1}{T}\int_0^T x(t)^2 dt}$$

Average Value:



The average value of an alternating current is defined as that value of the steady current which transfers the same amount of charge across any circuit as is transferred by the given AC current in the same circuit for same time. For the alternating current for above fig, the average value is:

$$I_{avg} = \frac{i_1 + i_2 + - - - - - + i_n}{n}$$

For symmetrical waveform the average value over one cycle is zero because of positive and negative half cycles. Hence the average value is determined over one half cycles only.

For unsymmetrical waveform, full cycle is considered.

The analytical expression for average value is,

$$X_{avg} = \frac{1}{T}\int_0^T x(t)\,dt$$

Form factor:

It is defined as the ratio of RMS value to the average value of an alternating quantity.

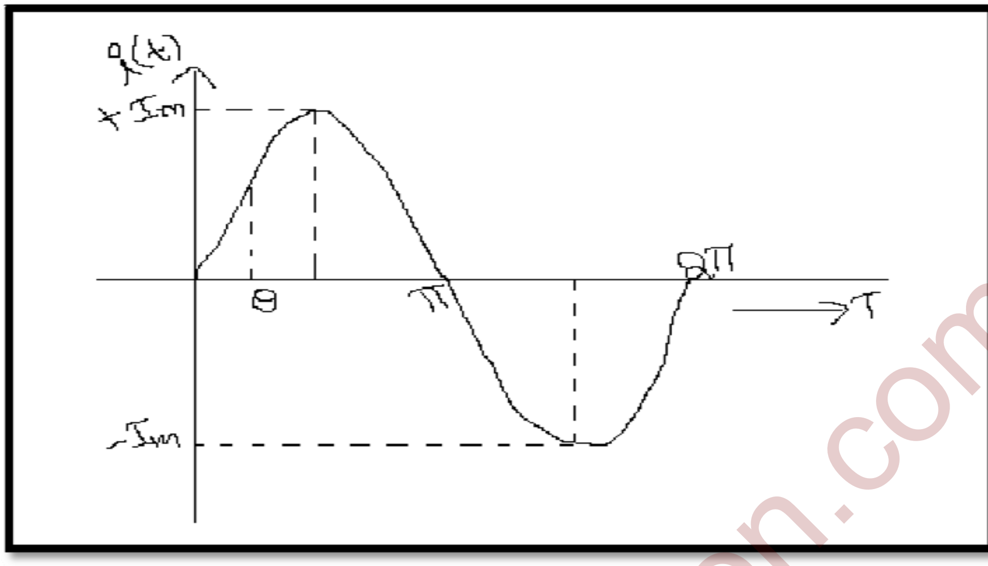
$$\text{Form factor} = \frac{RMS\ Value}{Average\ Value}$$

Peak Factor or Crest Factor:

It is defined as the ratio of peak value to the RMS value of an alternating quantity.

$$\text{Peak factor} = \frac{Peak\ Value}{RMS\ Value}$$

RMS and Average values of a sinusoidal quantity:



RMS Value:

Let the instantaneous value of the sinusoidal quantity is, i= $I_m \sin\theta$

The RMS value of current is, I_{RMS} = $\sqrt{\frac{1}{T}\int_0^T t^2 dt}$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} {I_m}^2 \,(\sin\theta)^2 \,d\theta}$$

$$= \sqrt{\frac{{I_m}^2}{4\pi}\int_0^{2\pi}(1-\cos 2\theta)\,d\theta}$$

$$= I_m\sqrt{\frac{1}{4\pi}(2\pi - 0)}$$

$$= \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

Therefore, RMS value = 0.707 maximum value

Average value:

Sinusoidal variation is symmetrical. The average value is calculated over one and half cycles only. Let the instantaneous value of the sinusoidal quantity is, i= $I_m \sin\theta$. The time period is T seconds or 2π radians. Hence the duration of one half cycles is $\frac{T}{2}$ seconds or π radians.

$$\int_0^{\frac{T}{2}} i(t)dt = \int_0^{\pi} I_m \sin\theta \, d\theta$$

$$= I_m\,[\,-\cos\pi - \cos 0\,]$$

$= 2I_m$

The Average value or mean value of sinusoidal quantity is, $\frac{2I_m}{\pi} = \frac{I_m}{1.57} = 0.636I_m$

Form factor:

Form factor = $\frac{RMS\,Value}{Average\,Value}$

$= \frac{I_{RMS}}{I_{Avg}}$

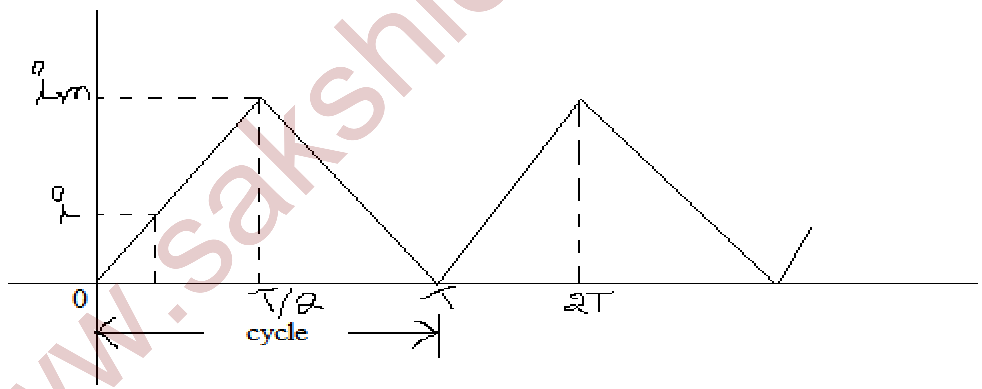
$= \frac{\pi}{2\sqrt{2}}$ =1.11

Peak factor or Crest factor:

Peak factor = $\frac{Peak\,Value}{RMS\,Value}$

= 1.414

Ex: Determine the RMS value, average value and form factor for the given wave form shown in figure.



The instantaneous values over the time period T, in different parts of the variation

$0 \leq t \leq \frac{T}{2}$, i = $\frac{2 * I_m * t}{T}$

$\frac{T}{2} \leq t \leq$, T = $\frac{2 * I_m *(T-t)}{T}$

RMS value:

Square of the instantaneous value

For $0 \leq t \leq \frac{T}{2}$, $i^2 = (\frac{2 * I_m * t}{T})^2 = \frac{4 * I_m{}^2 * t^2}{T^2}$

For $\frac{T}{2} \leq t \leq T$, $i^2 = (\frac{2 * I_m * t}{T})^2 = \frac{4 * I_m{}^2 * (T-t)^2}{T^2}$

Sum of square over one cycle = $\int_0^T i^2 \; dt = \frac{I_m{}^2 * T}{3}$

RMS value = $\frac{I_m}{\sqrt{3}}$

Average value:

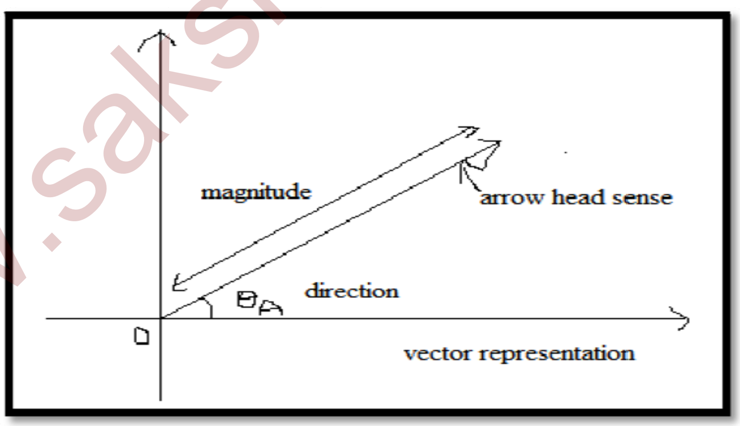
Average value = $\frac{1}{T}\int_0^T i(t) \; dt = \frac{I_m}{2}$

Form factor = $\frac{RMS\,Value}{Average\,Value}$

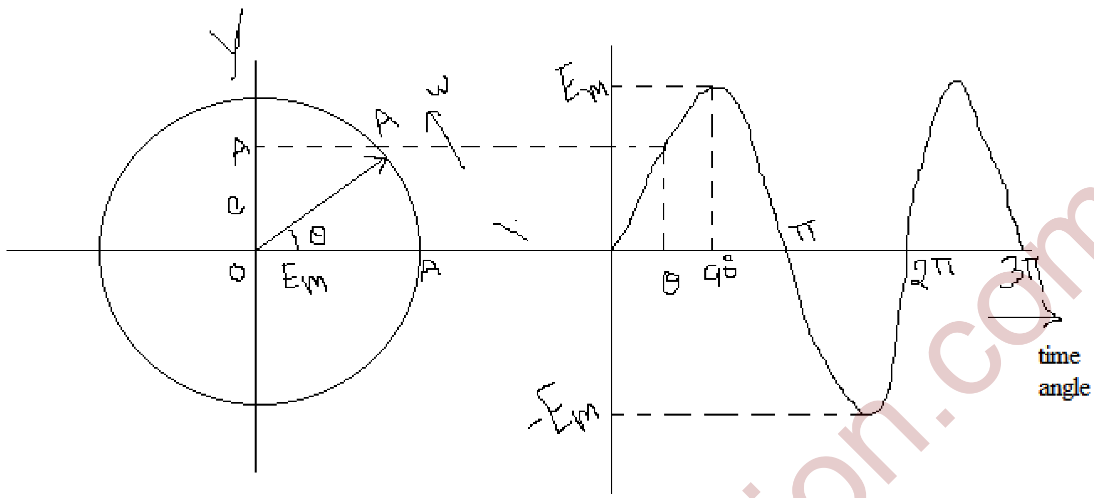
$= \frac{2}{\sqrt{3}}$

Phasor representation of sinusoidal quantities:

Sinusoidal alternating quantities can be represented graphically by a phasor or rotating vector. It is represented by a line of fixed length which is rotating about one of its ends. A vector quantity is one which has a both magnitude and direction. A vector is represented by a direct line of segment, the length which represents the magnitude, the inclinations with respect to a reference axis gives the direction and the arrow head represents the sense in which it acts.



Let us consider a sinusoidal alternating quantity of maximum value E_m and angular frequency ω and whose instantaneous value is e = $E_m \sin \omega t$.



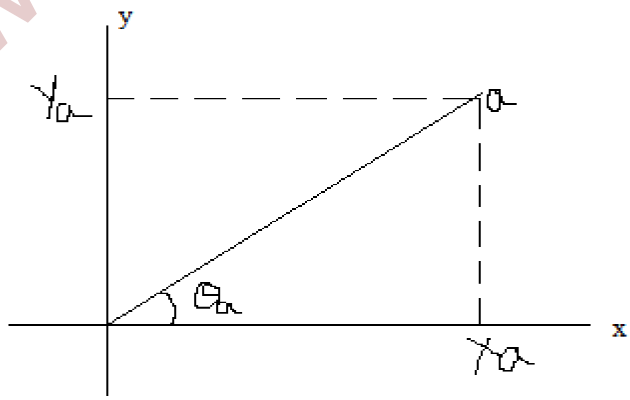
Mathematical representation of Sinusoidal quantities:

We know that the sinusoidal quantities can be geometrically represented by phasor or rotating vectors. The vectors can be represented in many ways they are,

- Complex or Rectangular form
- Trigonometric form
- Exponential form
- Polar form

Complex or rectangular form:

In this the quantity is expressed in terms of two rectangular components i.e. components along two perpendicular axes (x and y components). Consider a phasor having a magnitude OA and it has components X_a and Y_a along x and y axes.



Magnitude = $\sqrt{X_a + Y_a}$

Direction of OA = tan θ_a = $\frac{Y_a}{X_a}$

These complex forms are convenient for addition and subtraction.

Ex: A = 4 + j3 and B = 5 +j4

A + B = 9 + j7

A – B = -1 – j1

Trigonometric form:

A = X_A + j Y_A

X_A = |A| cos θ_A

Y_A = |A| sin θ_A

A = |A| (cos θ_A +sin θ_A)

Exponential form:

$e^{j\theta}$ = cos θ + j sin θ

A = |A|$e^{j\theta_A}$

Polar form:

The polar form representation is, A = A|_θ_A

Where θ_A is in degrees and is positive if measured in anti clockwise direction from the reference axis.

The conjugate of complex form is, A = X_A + j Y_A → A = A|_θ_A

A = X_A - j Y_A → A = A|_-θ_A

These polar forms are convenient for multiplication and division.

Ex: A = 10|_30 and B = 5|_-60

AB = 50|_-30

$\frac{A}{B}$ = 2|_90

j- notation:

The mathematics used in Electrical Engineering to add together resistances, currents or DC voltages uses what are called “real numbers”. Real numbers alone are not used in dealing with frequency dependent sinusoidal sources and vectors, rather normal or real numbers, Complex Numbers were introduced to allow complex equations to be solved with numbers that are the square roots of negative number $\sqrt{-1}$.

In electrical engineering this type of number is called an “imaginary number” and to distinguish an imaginary number from a real number the letter “j” known commonly in electrical engineering as the j-operator, is used. The letter j is placed in front of a real number to signify its imaginary number operation.

Examples of imaginary numbers are j3, j12, j100 etc. Then a complex number consists of two distinct, but very much related parts: a Real Number plus an Imaginary Number.

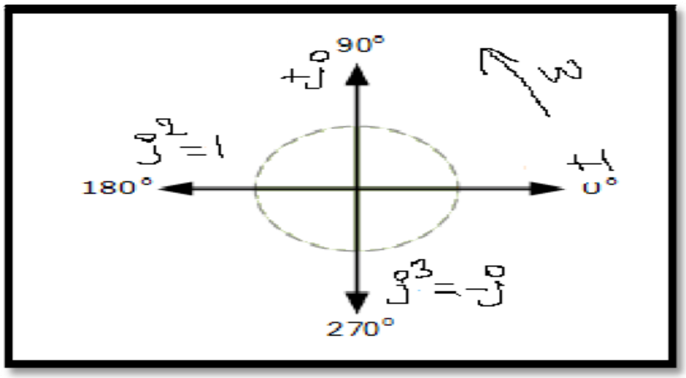
Complex Numbers represent points in a two dimensional complex or s-plane that are referenced to two distinct axes. The horizontal axis is called the “real axis”, while the vertical axis is called the “imaginary axis”. The real and imaginary parts of a complex number, Z are abbreviated as Re(z) and Im(z), respectively.

Complex numbers that are made up of real (the active component) and imaginary (the reactive component) numbers can be added, subtracted and used in exactly the same way as elementary algebra is used to analyze DC Circuits.

The rules and laws used in mathematics for the addition or subtraction of imaginary numbers are the same as for real numbers, j2 + j4 = j6 etc. The only difference is in multiplication because two imaginary numbers multiplied together becomes a positive real number, as two negatives make a positive. Real numbers can also be thought of as a complex number but with a zero imaginary part labeled j0.

The j-operator has a value exactly equal to $\sqrt{-1}$, so successive multiplication of “j “, (j x j) will result in j having the following values of, -1, -j and +1. As the j-operator is commonly used to indicate the anticlockwise rotation of a vector, each successive multiplication or power of “j “, j^2, j^3 etc, will force the vector to rotate through an angle of 90^o anticlockwise as shown below. Likewise, if the multiplication of the vector results in a -j operator, then the phase shift will be -90^o, i.e. a clockwise rotation.

Vector Rotation of the j-operator:



So by multiplying an imaginary number by j^2 will rotate the vector by 180^o anticlockwise, multiplying by j^3 rotates it 270^o and by j^4 rotates it 360^o or back to its original position. Multiplication by j^{10} or by j^{30} will cause the vector to rotate anticlockwise by the appropriate amount. In each successive rotation, the magnitude of the vector always remains the same.