Network Topology

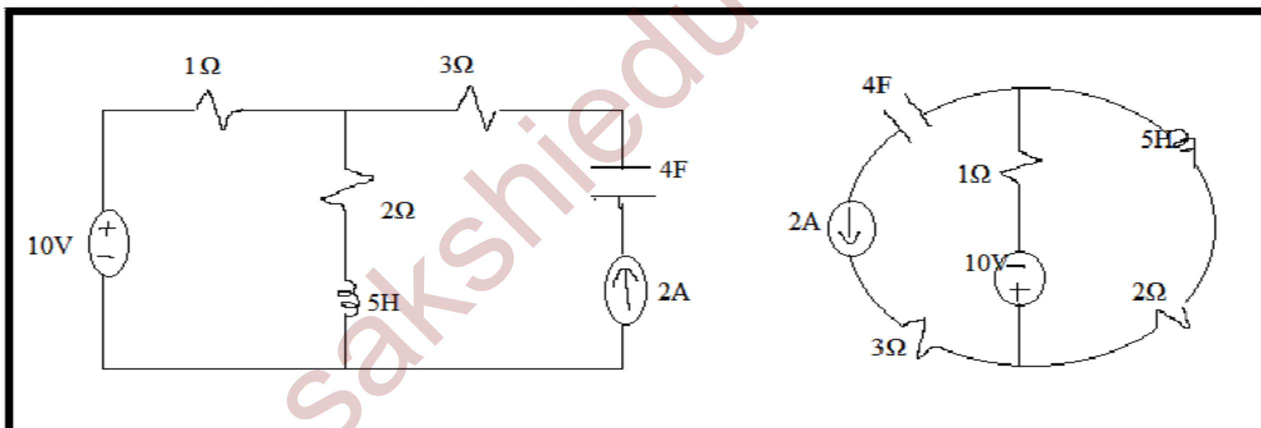
An electric circuit is an interconnection of active and passive elements. Network also a same thing but not a closed path, also a complex circuit which has large number of elements preferred. For analyzing a network, we need a systematic procedure for formulating the network equilibrium equations and solving these equations to determine the current through and voltage across each element of a network. For that purpose we are using KCL and KVL.

Here the problem is to determine the number of independent loop currents or node pair voltages which are to be determined for a given network from the network equations. For that purpose we are using a systematic procedure from the geometry of a network known as network topology.

Important Definitions

Topology: Topology is a branch of geometry applicable to electrical circuits where even by bending, twisting, swapping, stretching and also circuit make up and down will not disturb its circuit property known as topology.

Ex:



Network topology: It is the study of network properties by investigating the interconnections between the branches and nodes of the network. It is concerned with the geometric features of the network.

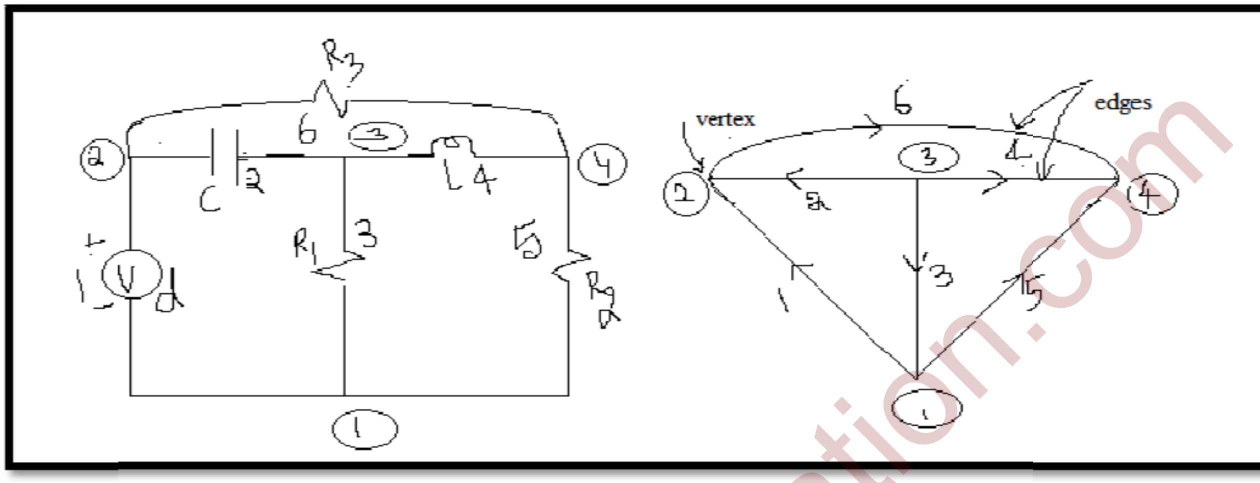
Graph: Graph is a skeleton representation of a circuit or network, where every element is suppressed by its nature and represented as a simple straight line.

- Ideal voltage sources are short circuits.
- Ideal current sources are open circuits.
- In graph theory nodes are numbered like 1, 2, 3, 4, -----
- Branches named like a, b, c, d, ------

- In graph theory, .i.e. l_i = b- (n-1)

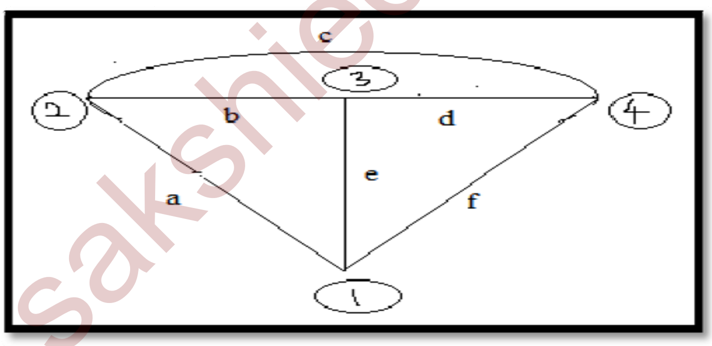
Where l_i= independent loops, n= nodes, b= branch

Ex:



Node or Vertex: It is the junction of two or more elements in a network. Each node or junction in a network is called a vertex in a graph.

Path (P): Path is a traversed from one node to another node with crossing the same node twice.



Ex: no of possible paths from 1 to 3 → 5 P_1 → 1 → 3

P_2 → 1 → 2 → 3

P_3 → 1 → 4 → 3 nodes (n) = 4

P_4 → 1 → 2 → 4 → 3 branches (b) = 6

P_5 → 1 → 4 → 2 → 3 mesh (m) = 3

m_1 → 1 → 2 → 4 → 1

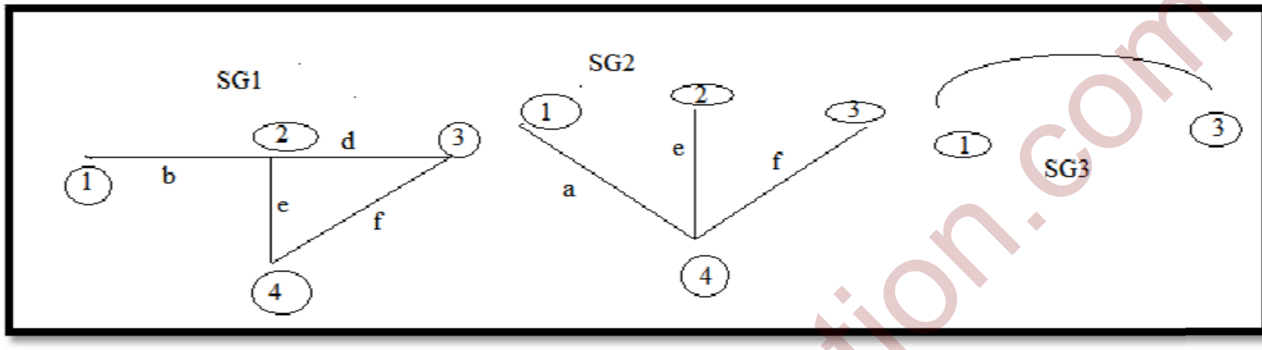
m_2 → 1 → 3 → 2 → 1

$m_3 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 2$

Individual loop (l_i) = 3 + 4 =7

Sub graph: A sub graph consists some of the nodes and branches of the main graph. Even a single branch can be sub graph.

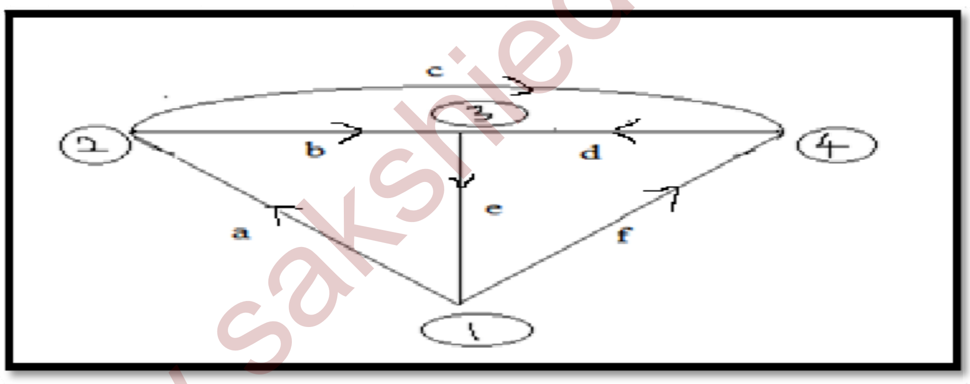
Ex:



Directed or Oriented graph: A graph is said to be directed, if every branch is in given reference direction, which is indicated by placing an arrow on every branch.

- This direction need not necessarily indicate current direction.

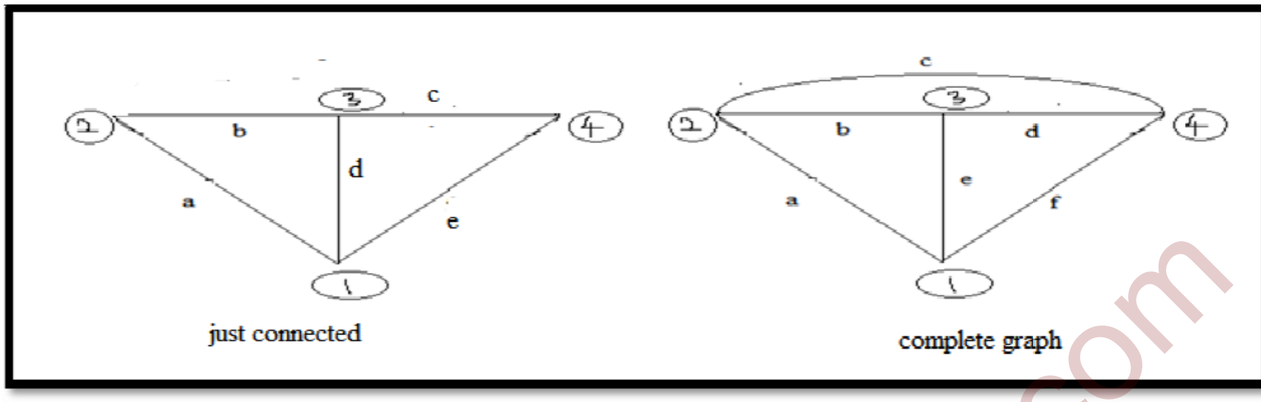
Ex:



Connected Graph: A graph is said to be connected, if there exists at least one path from every node to every other node.

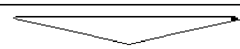
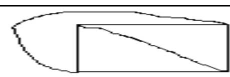
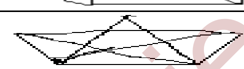
Completely Connected Graph: A graph is said to be complete, if there exists a direct path from every node to every other node.

Ex:



Example problem: The number of edges in a complete graph with ‘n’ nodes is

If ‘n’ nodes then, nc_2 = $\frac{n(n-1)}{2}$

Nodes or vertex	**figure**	**Edges or branches**
2		$\frac{2(2-1)}{2}$=1
3		$\frac{3(3-1)}{2}$=3
4		$\frac{4(4-1)}{2}$=6
5		$\frac{5(5-1)}{2}$=10

Tree: A tree is a sub graph which connects all the nodes without forming closed loops.

- Any tree of a particular graph will have only (n-1) edges.
- A tree with ‘n’ nodes has a rank of (n-1)
- No of trees $\begin{cases} n^{n-2}, n > 2 \rightarrow complete\ graph \\ \left|[A_r]\,[A_r]\right|^t \rightarrow\ any\ graph \end{cases}$

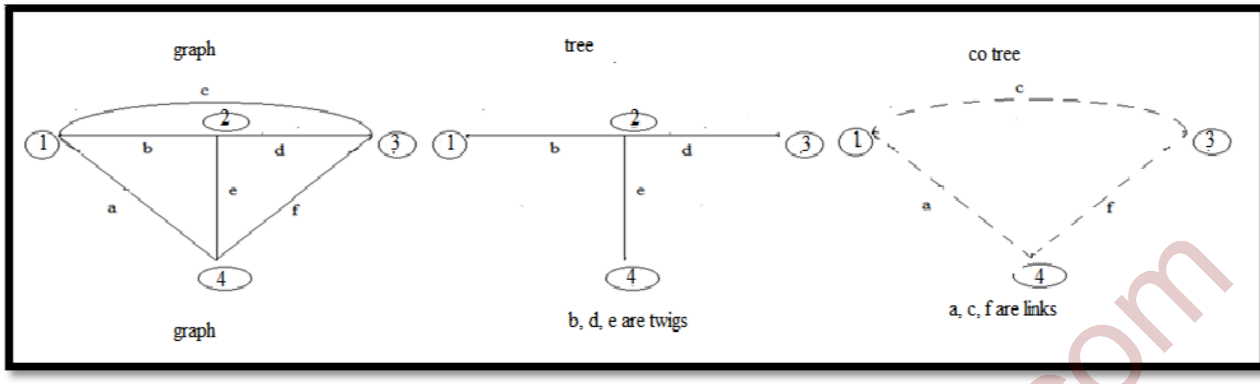
Where $[A_r]$ → reduced incidence matrix.

Twig: The branch of tree specifically called as twig and also indicated by “thick line”. Any tree will have (n-1) twigs.

Co Tree: The set of branches other than tree branches together form a co tree.

Link: The branches of co tree are called as links, indicated by dotted lines. Any co tree of a particular graph will have only b – (n-1) links.

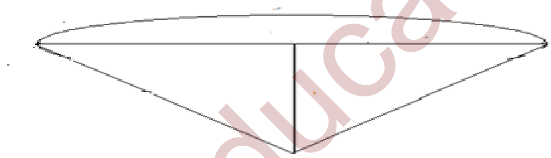
Ex:



From that, graph = tree + co tree

Branches = twigs + links → b = (n-1) + b-(n-1)

Ex: the number of possible trees for the graph below and also draw the trees?



No of trees, n^{n-2} = 4^2 = 16

Incidence Matrix |A|: It is the matrix that gives the relationship between number of nodes and number of branches and the orientation of a particular branch w. r. t node.

- Order of incidence matrix = n * b
- The rank of incidence matrix with 'n' nodes = (n-1)
- The elements of incidence matrix, [A] = $[A_{ij}]_{n*b}$

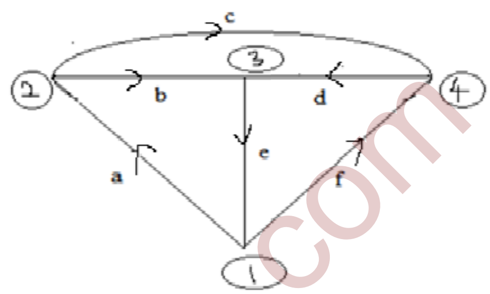
Where A_{ij} = +1, if j^{th} branch is incident on i^{th} node oriented away from it.

A_{ij} = -1, incident and towards

A_{ij} = 0, not incident.

Ex: write the incident matrix for the oriented graph below?

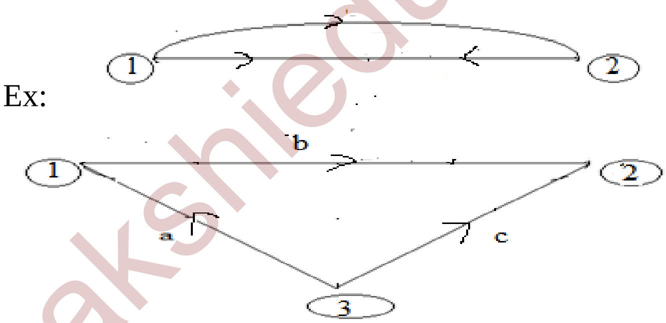
$$[A] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} -1 & +1 & +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & +1 & 0 \\ 0 & 0 & -1 & +1 & 0 & -1 \\ +1 & 0 & 0 & 0 & -1 & +1 \end{bmatrix} \end{array}$$



From that the algebraic sum of the elements every column (vertical) is zero.

Note: the determinant of incident matrix of a closed loop graph is zero (always is a square matrix).

Ex:



$$[A] = \begin{matrix} -1 & +1 & 0 \\ 0 & -1 & -1 \\ +1 & 0 & +1 \end{matrix}$$ determinant of above matrix is zero.

Reduced Incidence Matrix$[A_r]$: If one of the nodes in a graph is considered as a reference and that particular row is neglected while writing the incidence matrix, then it is called reduced incidence matrix.

- The order of reduced incidence matrix is (n-1) * b
- In computer method of electrical system analysis, reduced incidence matrix will be minimized.

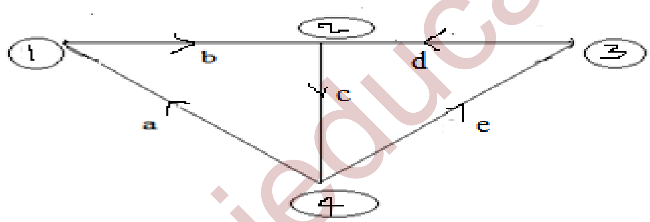
Ex: For the above incidence matrix example, if node 4 is considered as reference and row 4 is neglected, while writing the incidence matrix then the reduced incidence matrix is

$$[A] = \begin{array}{c} \\ ① \\ ② \\ ③ \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} -1 & +1 & +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & +1 & 0 \\ 0 & 0 & -1 & +1 & 0 & -1 \end{bmatrix}_{3\times 6} \end{array}$$

From the above example the algebraic sum of the elements of some the columns is not zero.

Isomorphism: if the incidence matrix of two independent graphs is identical, then they are said to obey the principle of isomorphism.

Ex: The number of possible trees for the fallowing graph and draw them?

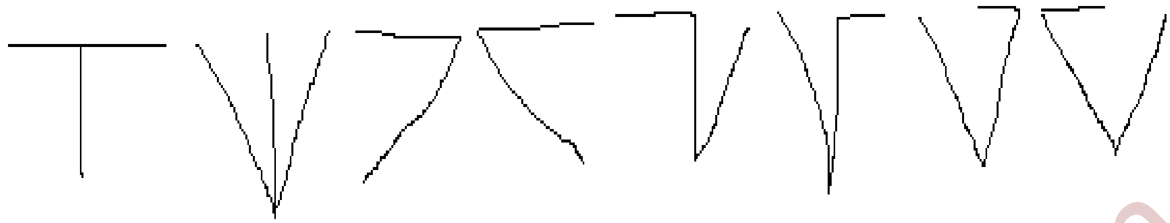


No of trees = $[|[A_r]\ [A_r]|^t]$

$$\begin{array}{c} \\ ① \\ ② \\ ③ \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} -1 & +1 & 0 & 0 & 0 \\ 0 & -1 & +1 & -1 & 0 \\ 0 & 0 & 0 & +1 & -1 \end{bmatrix}_{3 * 5} \end{array} \begin{bmatrix} -1 & 0 & 0 \\ +1 & -1 & 0 \\ 0 & +1 & 0 \\ 0 & -1 & +1 \\ 0 & 0 & -1 \end{bmatrix}_{5 * 3}$$

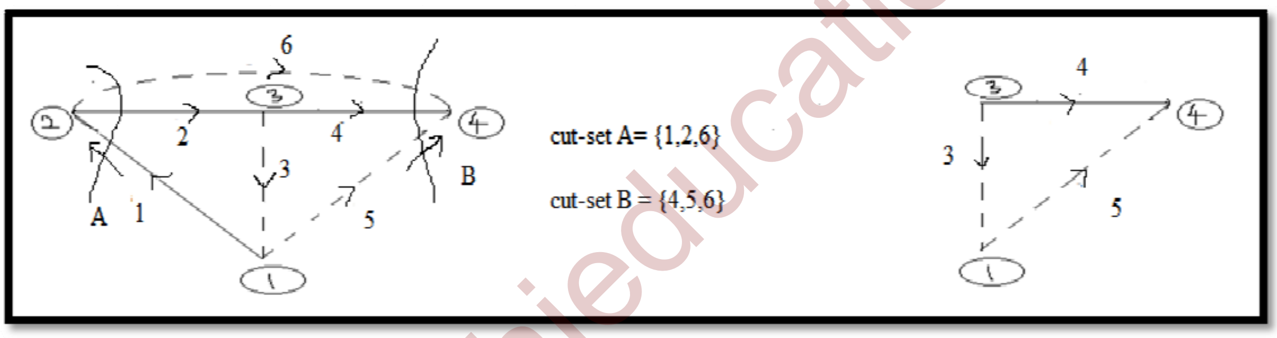
$$[A_r]\ [A_r]|^t = \begin{matrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{matrix}$$

$[|[A_r]\ [A_r]|^t]$ = 2 (6-1) + 1 (-2) + 0 = 8 → trees



Concept of cut-set: A cut-set represents set of branches which when removed can divide the graph in to two parts. A cut-set represents the set of branches which when cut in any direction can divide the graph into two parts, where even by replacing one of these branch will destroy this property and makes the graph connected again.

Let consider the below example the cut-set illustrated as



Cut-set A consists of branches 1, 2, 6 which removed isolate the node (2) from the other part of network when the above elements 1, 2, 6 removed, the graph gets divided into isolated node and the remaining network. Similarly process applies for cut-set B too.

From the concept of a cut-set, KCL can also be defined as for any lumped network, for any of its cut-sets, the algebraic sum of all the branch currents incident to the cut-set is zero.

To apply this law, we assign a reference direction for each cut-set. In order to set the algebraic sum of the currents of branches forming a cut-set, we assign a + sign to the branch of currents whose direction agrees with that of reference direction of cut-set and – sign to opposite direction.

For cut-set A, $+ i_1 - i_2 - i_6 = 0$

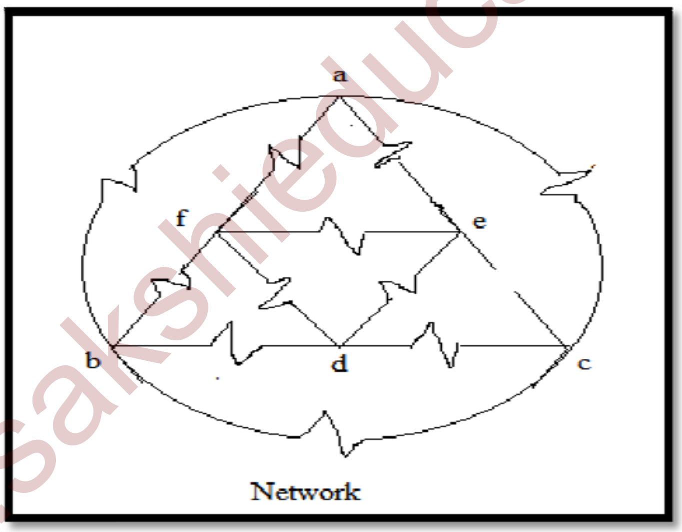
Fundamental or Basic cut-set: A fundamental cut-set is a cut-set which contains one and only one tree branch, where the remaining elements are links.

- A fundamental cut-set cut through a graph which can divide it into two parts, but in the path of cutting it should cut only one twig and rest of them are links.

- The number of fundamental cut-sets for any given graph = twigs = (n-1).
- These fundamental cut-set represents isopotential lines, which are caused as cut-set voltages and the orientation of this fundamental cut-sets is governed by the twig in it.
- It is the matrix that gives the relation between branch voltages and cut-set voltages where every branch voltage cab be expressed in terms of cut-set voltage.
- The order of this matrix is, twigs * branches = (n-1) * b
- The elements of cut-set matrix [C] = $[a_{ij}]_{twigs*branches}$ where

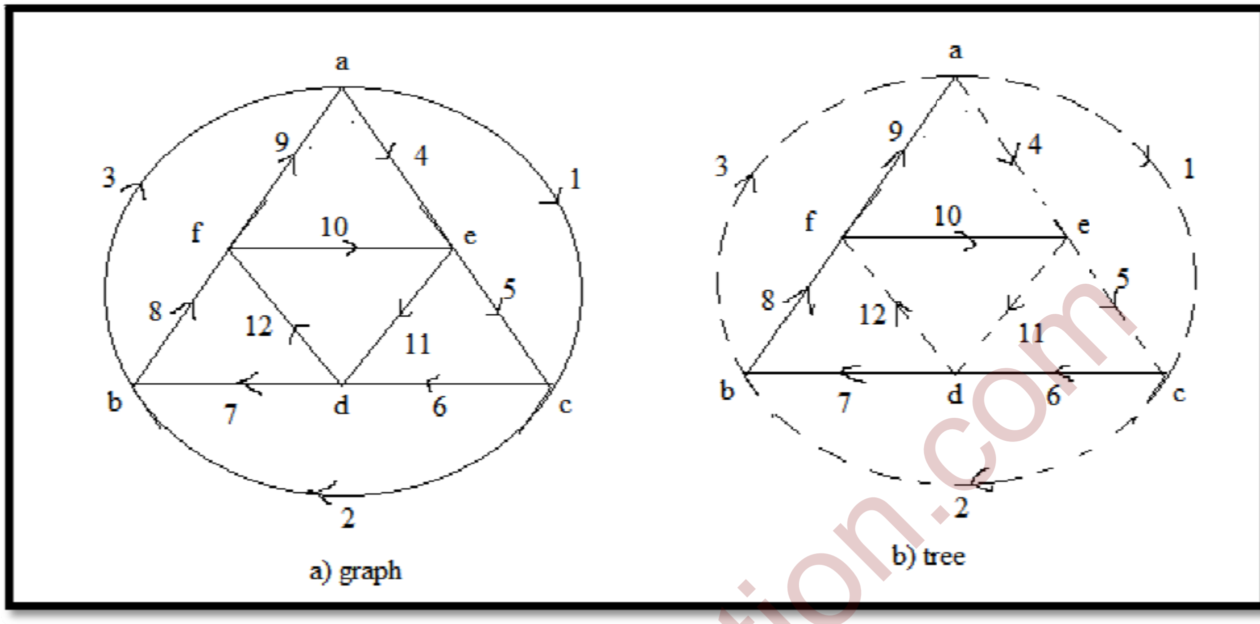
1) a_{ij} = +1, if j^{th} branch voltage is incident to i^{th} cut-set voltage and oriented in same direction.
2) a_{ij} = -1, incident + opposite direction.
3) a_{ij} = 0, not incident.

Ex 2: For the given network draw a graph and a tree. Select suitable tree branch voltage and write the cut-set schedule. Write equation for the branch voltages in terms of tree branch voltages.



Network

4)
5) No of nodes of a tree, n_t = 6
6) No of tree branches, n= 6-1 = 5
7) Cut-set schedule:



a) graph b) tree

8)

Tree branch voltages	Branch voltages (v)											
	1	2	3	4	5	6	7	8	9	10	11	12
1	-1	0	1	0	-1	0	0	1	0	0	-1	1
2	-1	0	1	-1	0	0	0	0	1	0	0	0
3	0	0	0	1	-1	0	0	0	0	1	-1	0
4	-1	1	0	0	-1	0	1	0	0	0	-1	1
5	-1	1	0	0	-1	1	0	0	0	0	0	0

9)

Tree Branch	Basic Cut-Set
e_1	1, 3, 5, 8, 11, 12
e_2	1, 4, 3, 9
e_3	4, 5, 10, 11
e_4	1, 2, 5, 7, 11, 12
e_5	1, 2, 5, 6

10)

11) The equations are, $v_1 = -e_1 - e_2 - e_4 - e_5$

12) $v_2 = e_4 + e_5$

13) $v_3 = e_1 + e_2$

14) $v_4 = -e_2 + e_3$

15) $v_5 = -e_1 - e_3 - e_4 - e_5$

16) $v_6 = e_5$

17) $v_7 = e_4$

18) $v_8 = e_1$

19) $v_9 = e_2$

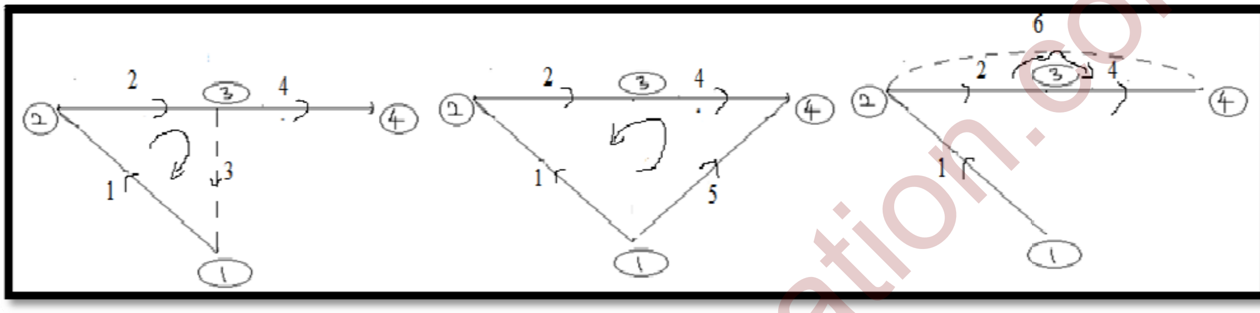
20) $v_{10} = e_3$

21) $v_{11} = - e_1 - e_3 - e_4$

22) $v_{12} = e_1 + e_4$

Tie-Set: Tie-Set is a set of branches which form a closed loop or path. A basic or fundamental tie-set is a closed path for a particular tree and co tree in a graph, which is formed by only one link and rest of them are twigs.

Let's consider the below example,



The branches of 1, 2, and 3 are taken as tree branches 3, 5, 6 branches are link branches. Whenever a link branch is introduced into the tree, it gives raise to one basic loop and it is shown in figure, also its reference direction is same as that of link branch. The other basic loops associated with links 5 and 6 are shown in the figure.

This concept is employed to determine the number of independent loop equations using KVL.

- The number of fundamental loops for any given graph = no of links = b- (n-1).
- These fundamental loop currents are called as tie-set currents and their orientation governed by the link in it.
- The order of this matrix is, links * branches = [b – (n-1) * b].
- The elements of this matrix [M] = $[a_{ij}]_{links*branches}$
  1) a_{ij} = +1, if j^{th} branch currents are incident to i^{th} tie-set currents and oriented in same direction.
  2) a_{ij} = -1, incident + opposite direction.
  3) a_{ij} = 0, not incident.

Choice of independent branch currents and voltages:

The solution of a network involves solving of all branch currents and voltages. We know that the branch current and voltage of every branch is related to each other by its V- I relationship. Hence for branch network if we know b variables (either branch currents or branch voltages), then the other b variables can be uniquely determined. The network equations can be formulated using Kirchhoff's law using branch currents or branch voltages as variables. It can be shown that all the b variables are not independent. We can determine the number of independent branch currents and branch voltages using the concept developed by graph theory.

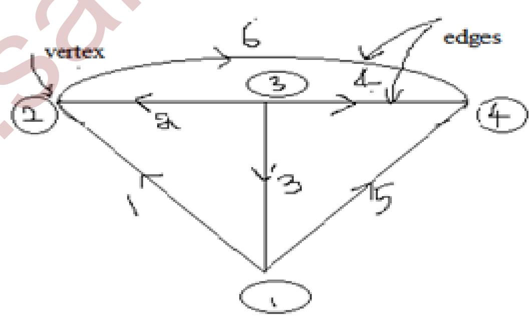
Choice of independent branch currents:

To determine the number of independent branch currents, consider the tree of a given graph which is connected sub graph with no closed path. The addition of each link branch to the tree gives rise to different closed path. Hence the opening or removal of the links destroys all closed paths, which results in forcing all branch currents to zero. Thus, if we set all link branch currents to zero, the currents in all branches of the network automatically to zero. We can conclude that tree branch currents are dependent on link branch currents and can be expressed uniquely in terms of link branch currents. This shows for a given network with b branches and n nodes, the number of independent branch currents equal to number of links is (b- (n-1)) = (b-n+1).

The dependent branch currents or tree branch currents can be expressed in a unique way in terms of link branch currents using the row of the tie-matrix [C].

Hence to solve a given network, we have to determine the independent branch currents, in terms of which other variables can be determined. To determine these independent branch currents we formulate equations by equations by applying KVL to each of the loops, and these equations are called as loop equations and the variables in which equations are formulated are called the loop currents. This method of analysis is called as loop method of analysis and is based on KVL.

To analyze this see the fallowing example, if the branch currents are $i_{1,}i_{2,\_\_\_\_\_\_,}i_8$ then they can be expressed as link branch currents$i_{1,}i_{2,}i_{3,}i_4$ as



$$[i] = [C]\,[i_1] \text{ ------------------------ (1)}$$

Where [i] = column of branch currents

$[i_1]$ = column of link branch currents

[C] = basic tie-set matrix

The tree branch currents for above example are can express as below,

$$\begin{bmatrix} i1 \\ i2 \\ i3 \\ i4 \\ \hdashline i5 \\ i6 \\ i7 \\ i8 \end{bmatrix} = \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{c|c|c|c} +1 & & & \\ & +1 & & \\ & & +1 & \\ & & & +1 \\ +1 & & & -1 \\ -1 & +1 & & \\ & -1 & +1 & \\ & & -1 & +1 \end{array}\right] \end{array} \begin{bmatrix} i1 \\ i2 \\ i3 \\ i4 \end{bmatrix}$$

$i_5 = i_1 - i_4$

$i_6 = -i_1 + i_2$

$i_7 = -i_2 + i_3$

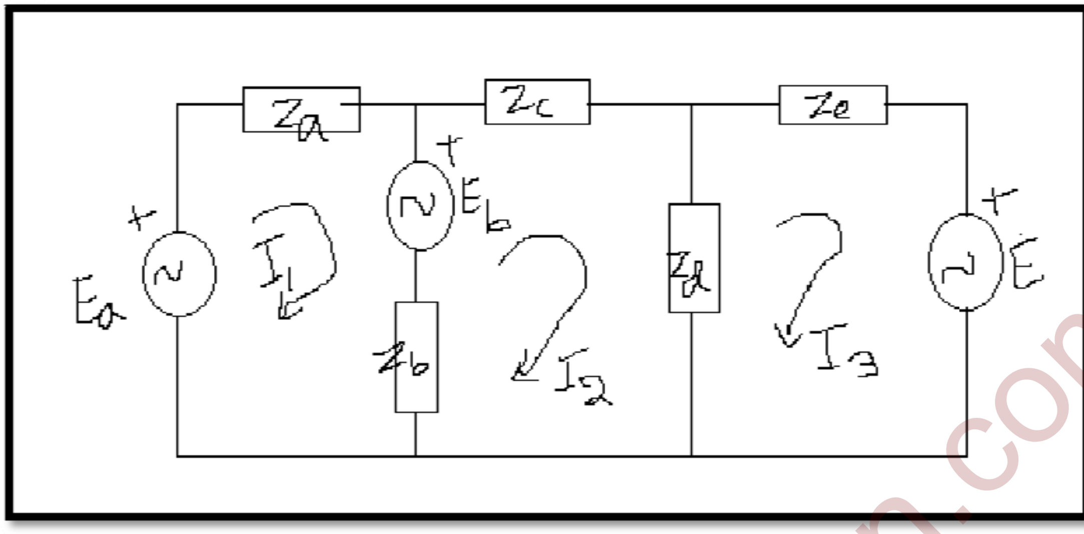
$i_8 = i_3 + i_4$ ----------------- (3)

The loop equations are formulated using equations (1).

Another way to find loop currents:

Another way commonly employed in formulating the loop equations using loop currents as the independent variables is illustrated below. We can use the window method to arrive at the number of independent loop currents if it is planar network. This method is illustrated and loop equations are formulated using KVL in terms of loop current variables.

Consider the network shown in fig. the choice of loop currents in loop equations formulated below. Let I_1,I_2 and I_3 are the loop currents in the loops 1, 2 and 3 flowing in the elements forming that loop. The loop equations are obtained by applying KVL for each of the loop in the network.



Loop1: - E_a + I_1 Z_a + E_b + (I_1 -I_2) Z_b = 0 → I_1(Z_a +Z_b) - I_2 Z_b = (E_a -E_b) – (1)

Loop2: (I_2 -I_1) Z_b - E_b + I_2 Z_c + (I_2 -I_3) Z_d = E_b → -I_1 Z_b + I_2 (Z_b+ Z_c+Z_d) - I_3 Z_d = E_b -- (2)

Loop3: (I_3 -I_2) Z_d + I_3 Z_e+ E = 0 → -I_2 Z_d + I_3 (Z_e +Z_d) = E ---- (3)

The equations 1, 2, 3 are the loop equations which are to be solved for the loop currents.

Choice of independent voltages:

In analyzing the network on voltage basis, we have to determine the number of independent branch voltages. We can use graph theory concepts to determine independent voltages.

Consider the tree of a network graph which is connected sub graph with no closed paths. Since it is connected sub graph, there exists a unique path between every pair of nodes only, through tree branches removal of tree branch voltages results in the equal potential of all nodes. Hence of all the tree branch voltages are set equal to zero, where in all the branch voltages of the network will automatically become zero. All the link branch voltages can be expressed in a unique way in terms of tree branch voltages.

Hence for any given network within n nodes and b branches, there will be (n-1) tree branches and hence there will be (n-1) independent branch voltages. The remaining branches voltages can be expressed in terms of these independent branch voltages. To determine these independent branch voltages, we formulate the (n-1) equations by applying KCL to each of the nodes and these equations are nodal equations. This method of analysis is called nodal method of analysis and is based on KCL.

The dependent branch voltages can be expressed in unique way in terms of tree branch voltages using the rows of basic cut-set matrix [B].

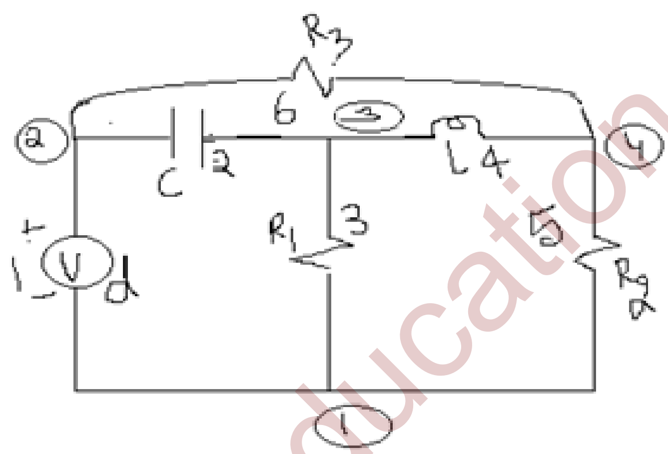
To illustrate, we consider the below example. If the branch voltages are $v_{1,}v_{2,}$------$,v_8$ then they can be expressed in terms of tree branch voltages $[v_{5,}\ v_{6,}\ v_{7,}\ v_8]$ as

$$[v] = [B]\,[v_b] \text{ ----------- } (1)$$

Where [v] = column branch voltages

$[v_b]$ = column of tree branch voltages

[B] = basic cut set matrix



From that graph we can get,

$$\begin{bmatrix} v1 \\ v2 \\ v3 \\ v4 \\ \hline v5 \\ v6 \\ v7 \\ v8 \end{bmatrix} = \begin{array}{c} \begin{array}{cccc} 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} +1 & +1 & & \\ & -1 & +1 & \\ & & -1 & +1 \\ -1 & & & -1 \\ +1 & & & \\ & +1 & & \\ & & +1 & \\ & & & +1 \end{bmatrix} \end{array} \begin{bmatrix} v5 \\ v6 \\ v7 \\ v8 \end{bmatrix}$$

The link branch voltages are expressed in a unique way as

$v_1 = v_5 + v_6$

$v_2 = -v_6 + v_7$

$v_3 = -v_7 + v_8$

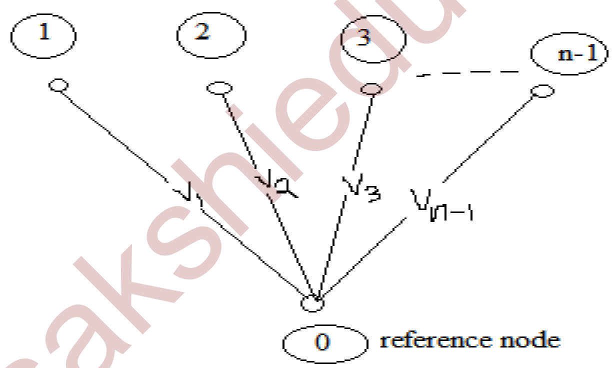
$v_4 = -v_5 + v_8$ --------------- (3)

The nodal equations are formulated using columns of equations (1).

Another way of choosing independent voltages:

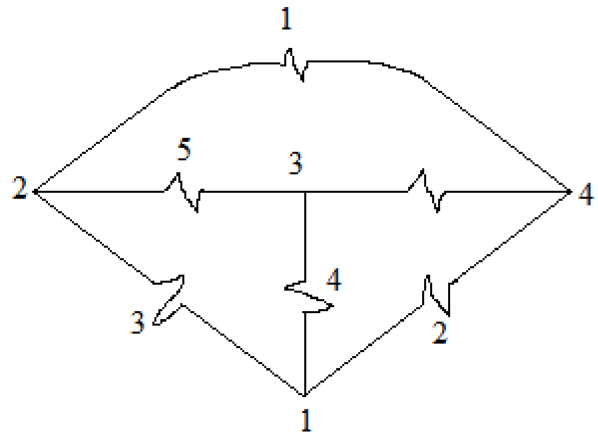
This method is commonly employed in practice instead of using graph theory. In this method, we employ node pair voltages or nodal voltages as the variables in terms of which nodal equations are obtaining KCL at each node.

Consider a network having nodes and choose one of the nodes as the reference node. Theoretically any one of the nodes can be chosen as reference node and the potentials of all other nodes are measured with respect to node. A node to which more number of elements is connected is generally taken as reference node. Consider below example



The reference node is numbered zero and assumed to be at zero potential. The other nodes are numbered as 1, 2, -----, n-1and the potentials of these nodes with reference to node (0) are v_1,v_2, -----, v_{n-1} respectively. Thus there will be (n-1) node pair voltages which are independent voltages. All other voltages can be expressed in terms of these node pair voltages. The nodal equations are formulated in terms of these variables, by applying KCL at each node but expect at reference node. This method is known as nodal method of analysis.

Ex 1: For the given network draw the graph and choose a possible tree. Construct the basic tie-set schedule. Write the equation for the branch currents and in terms of the link currents and write separately the independent equations.

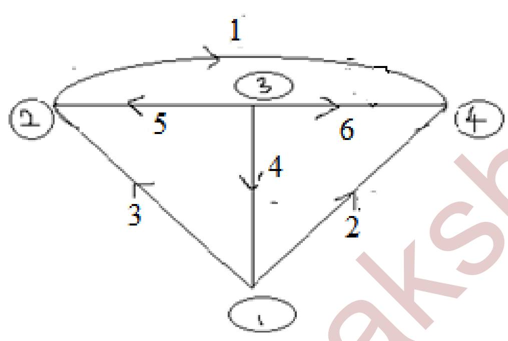


No of nodes of a tree, n_t = 4

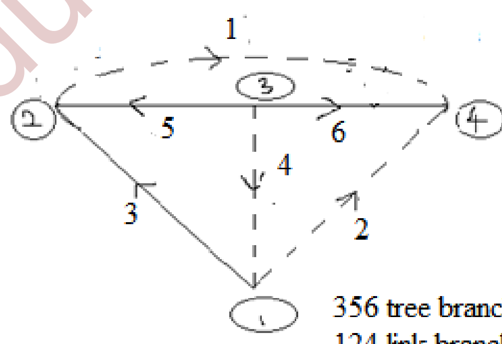
No of tree branches, n= 4-1 = 3

Total no of branches, b = 6

No of links, l = no of independent loop currents = b – n = 6 – 3 = 3



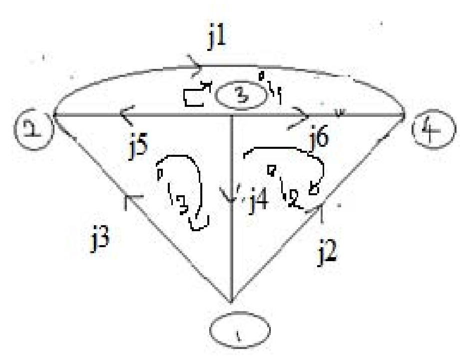
a



356 tree branches
124 link branches

b

Link No (i)	Branch No (j)					
	1	2	3	4	5	6
1	+1	0	0	0	+1	-1
2	0	+1	+1	0	-1	+1
3	0	0	+1	+1	-1	0



Branch currents in terms of independent link currents from fig c

$j_1 = i_1$

$j_2 = i_2$

$j_4 = i_3$ identities

$j_3 = i_2 + i_3$

$j_3 = i_1 - (i_2 + i_3)$

$j_6 = i_2 - i_1$ independent equations

The loop equations from the rows of tie-set matrix are,

$v_1 + v_5 - v_6 = 0$

$v_2 + v_3 - v_5 + v_6 = 0$

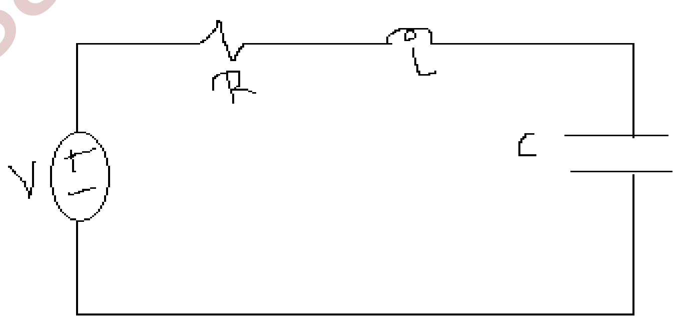
$v_3 + v_4 - v_5 = 0$

Duality and Dual Networks

Duals: Two circuits are said to be dual of each other, if the mesh equations characterize one of them has the same mathematical form as the nodal equations that characterize the other.

Principle of Duality: Identical behavior patterns observed between voltages and currents between two independent circuits illustrate the principle of duality.

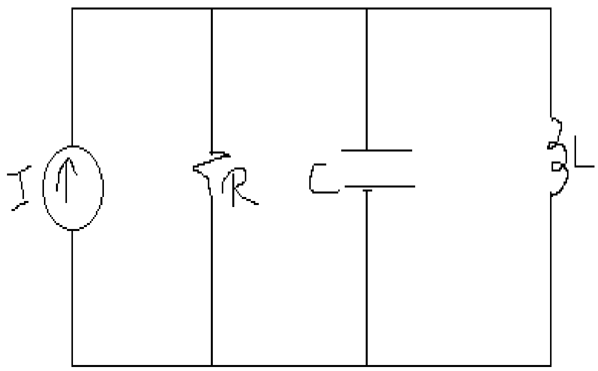
Ex: 1) series R-L-C circuit:



Mesh → KVL → $-V + iR + L\frac{di}{dt} + \frac{1}{C}\int i\, dt = 0$

$$V = I R + L\frac{di}{dt} + \frac{1}{C}\int i\, dt$$

2) Parallel G-C-L circuit:



Nodal → KCL→ - I + V G + C $\frac{dv}{dt} + \frac{1}{L}\int v\, dt$

$$I = V G + C \frac{dv}{dt} + \frac{1}{L}\int v\, dt$$

From that (1) and (2) are mathematically identical, so they are duals.

Some dual elements:

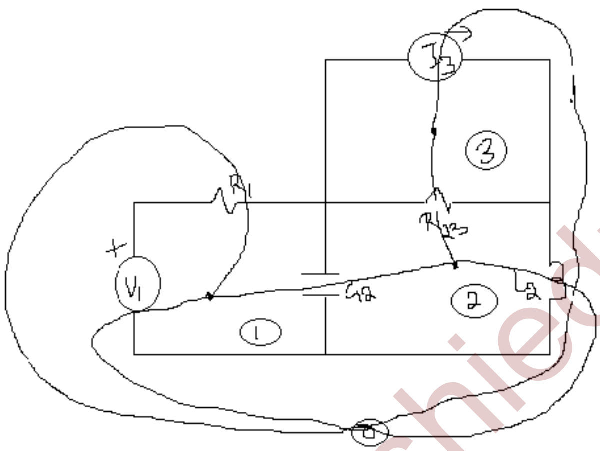
1) Voltage (V) ←→ Current (I)
2) Resistor (R) ← → Conductance (G)
3) Inductor (I) ← → Capacitor (C)
4) KVL ← → KCL
5) V(t) ← → I(t)
6) Mesh ← → nodal
7) Series ← → parallel
8) Vsinwt ← → Icoswt
9) Open circuit ← → short circuit
10) Thevenin ← → Norton
11) Link ← → twig
12) Cut set ← → tie set
13) Tree ← → co-tree
14) Switch in series (getting closed) ← → switching in parallel (getting opened) etc.

Procedure to Obtain a Dual Network:

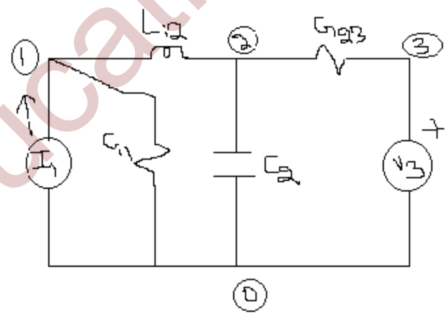
These rules illustrated below are only for planar or flat networks which do not have any of their branches crossing other branches1.

1) Place a dot in every loop of the network whose dual is obtained and a dot outside the network. Each dot is numbered according to the loop in which it is placed. The outside dot is called the reference node and give number as 0.
2) Connect two dots by a line through each branch. The dots are the nodes of the dual network between two nodes; the element to be connected is the dual of the element crossed by the line.
3) When sources are included, then the line joining the dots should intersect the sources also; between these two nodes the dual of the source is included.
4) The polarity of the source is decided by the fallowing rule. A voltage or current source which drives a current in clockwise in i^{th} loop, then place a positive polarity at i^{th} the dual network. Negative if it is opposite.

Example:



Original Network



Dual Network

Inverse Networks: If two impedances Z_a, Z_b which are duals of each other are expressed in the form of $Z_a * Z_b = K^2$, where K is a positive number of independent of frequency, then the two impedances are said to be inverse or reciprocal. The inversion is said to be about K.

Ex: $Z_a = R_a + \mathrm{j}\,(\mathrm{w}L_a - \frac{1}{wC_a})$ and

$$Y_b = \frac{1}{Z_b} = G_b + \mathrm{j}\,(\mathrm{w}C_b - \frac{1}{wL_b})$$

So $Z_a * Z_b = \frac{Z_a}{Y_b} = \frac{R_a + \mathrm{j}\,(\mathrm{w}L_a - \frac{1}{wC_a})}{G_b + \mathrm{j}\,(\mathrm{w}C_b - \frac{1}{wL_b})} = K^2$

From the above equation that,

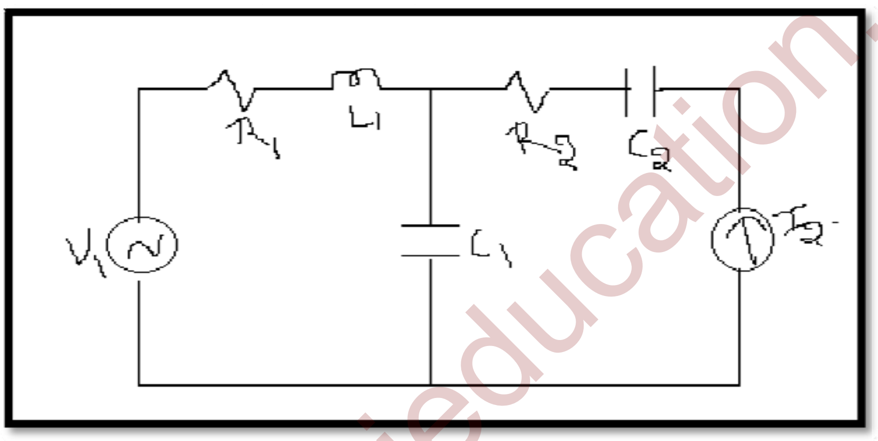
$$\frac{R_a}{G_b} = \frac{L_a}{C_b} = \frac{L_b}{C_a} = K^2$$

From that for inverse networks, $Z_{jk} = K^2\ Y_{jk}$

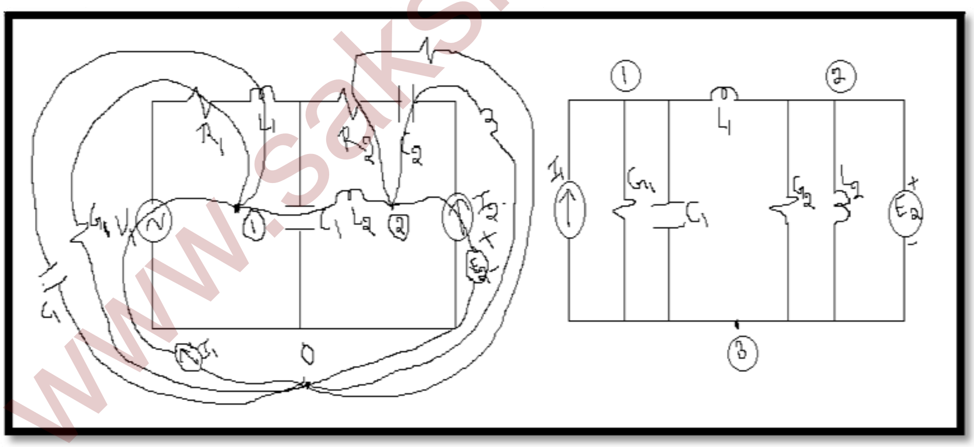
The method of inverting a given network as fallows,

1) Every parallel arrangement of X is replaced in Y by a series arrangement of elements and vice versa.
2) Every inductor L_a in A is replaced by a capacitor $C_b = \frac{L_a}{K^2}$
3) Every capacitor C_a in A is replaced by an inductor $L_b = K^2\ C_a$
4) Every resistor in R_a in A is replaced by a conductance $G_a = \frac{R_a}{K^2}$

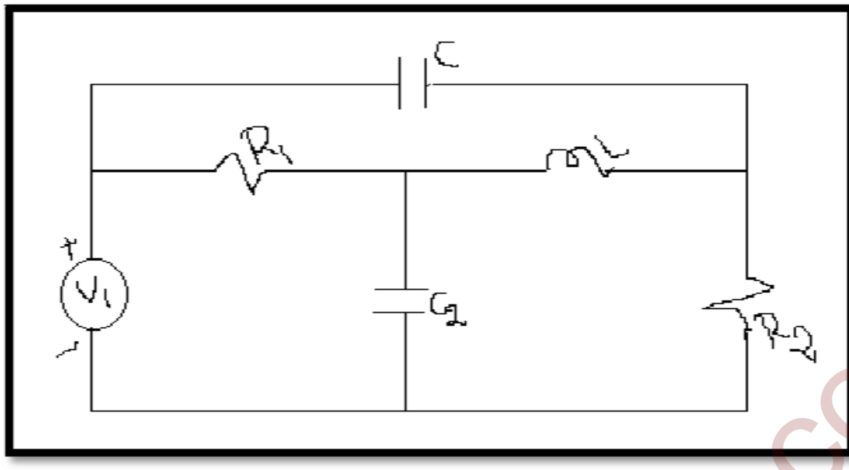
Ex 1: Draw the dual of fallowing network shown in below



The dual of the network is,



Ex 2: Draw the dual of fallowing network shown in below



The dual of the network is,