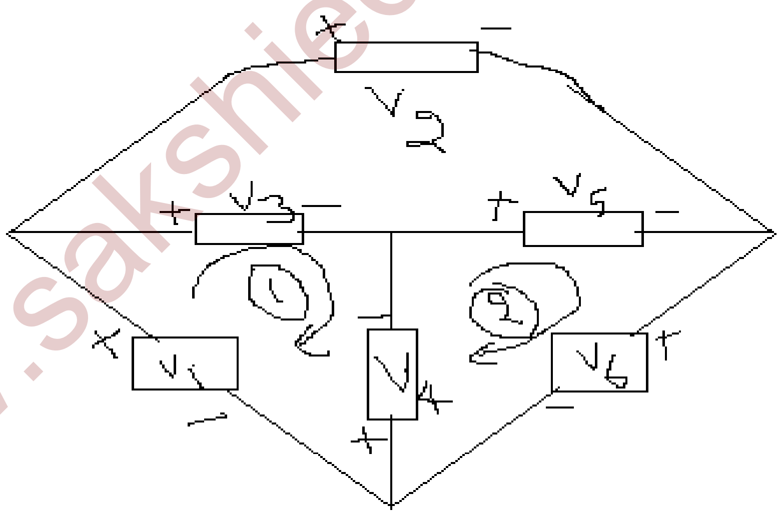
Mesh Nodal Analysis

KIRCHOFF'S LAWS:

Network equations are formed by using two fundamental laws namely Kirchhoff's voltage law (KVL) and Kirchhoff's current law (KCL). The laws are concerned with algebraic sum of currents at a junction or node and the algebraic sum of voltages around a closed loop. To get the algebraic sum we assign specific directions to the current flows and polarities to voltages. A positive sign is assigned to the current leaving the node and a negative sign, if it is entering the node. A positive sign is given to the voltage if the polarities occur in the order of positive to negative as we travel around a loop. A negative sign if it is from negative to positive.

KVL:

Statement: for any lumped electric circuit, for any of its loops, and at any time, the algebraic sum of the branch voltages around the loops zero.



A path is a set of braches, starting at one node and ending at another node. A loop is a set of branches travel starting from one node and ending on the same node. In order to apply KVL, assign a reference direction to the loop or closed paths. Assign suitable signs to branch voltages with reference to reference direction of loop.

By applying KVL for loop1, $V_4 + V_5 + V_6 = 0$

Loop2, $V_1 + V_3 - V_4 = 0$

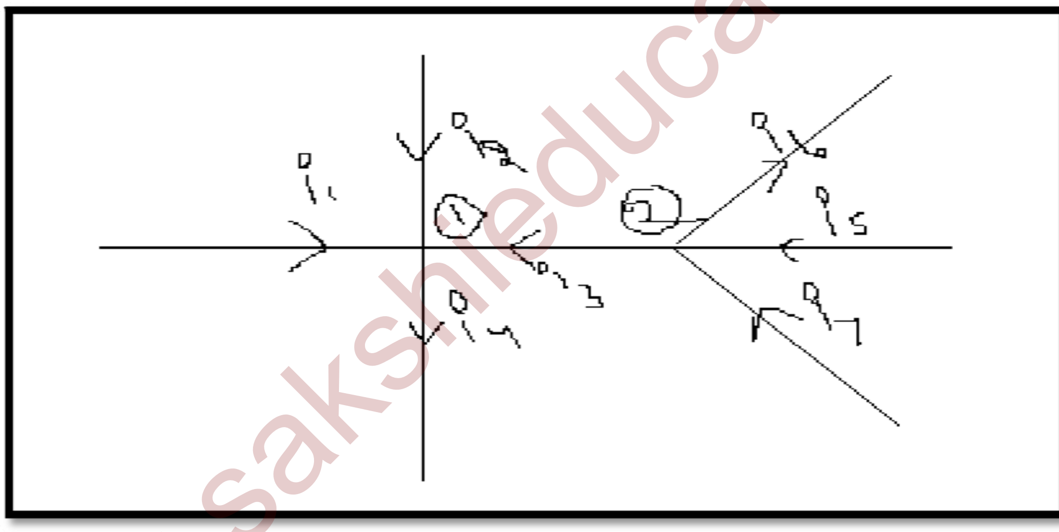
By applying KVL for each of the loops, the equations obtained are called loop equations.

KCL:

Statement: For any lumped electric circuit, for any of its nodes, at any time, the algebraic sum of the currents equal to zero.

(Or)

It also be stated that the sum of currents entering at the junction to sum of currents leaving at same junction.



From the above fig, represents the lumped circuit and also having lumped elements only. They are two nodes in the network. An arrow is used to represent the direction of current. By applying KCL at node 1 we get,

$$i_1 + i_2 + i_3 - i_4 = 0$$

Similarly at node2 we get, $- i_3 + i_5 - i_6 + i_7 = 0$

By applying KCL at each nodes and we get the equations, these are called as nodal equations.

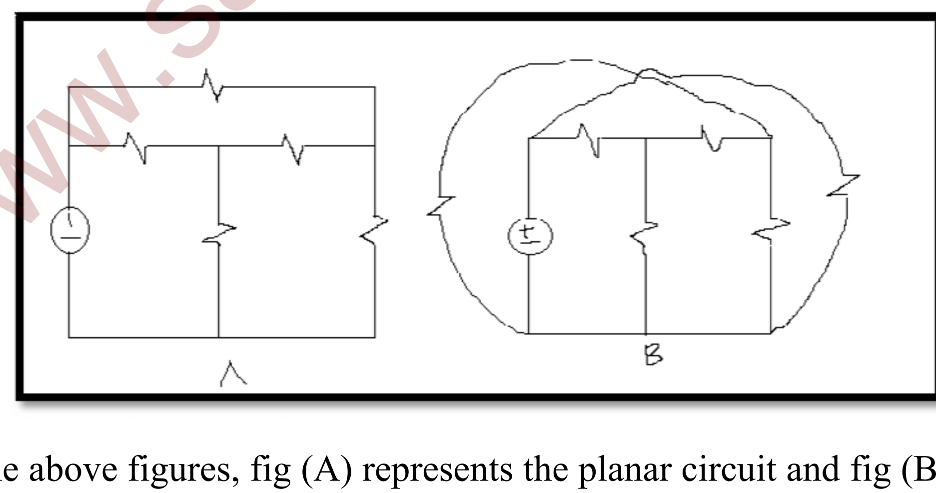
We have seen that using Kirchhoff's laws and Ohm's law we can analyze any circuit to determine the operating conditions (the currents and voltages). The challenge of formal circuit analysis is to derive the smallest set of simultaneous equations that completely define the operating characteristics of a circuit. In this below we will develop two very powerful methods for analyzing any circuit. The node and mesh methods are based on the systematic application of Kirchhoff's laws.

Mesh and Nodal Analysis:

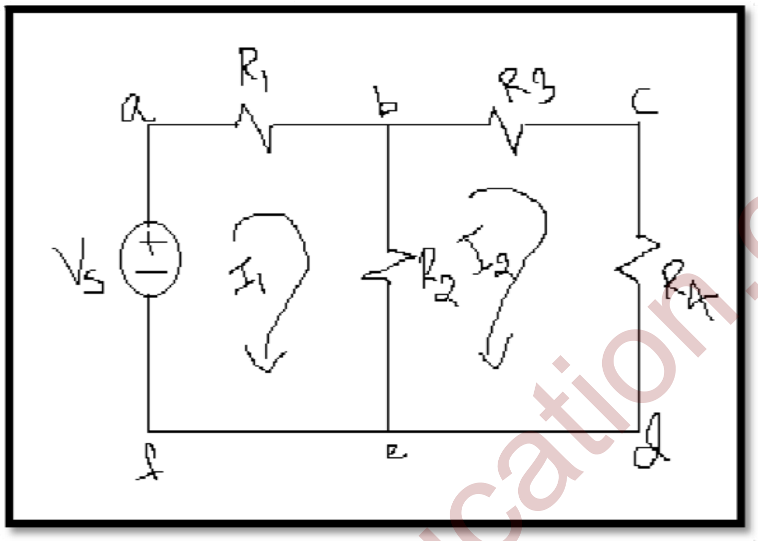
Mesh and nodal analysis are two basic important techniques used in finding the solution for the network. The suitability of either mesh or nodal analysis to a particular problem depends mainly on the number of voltage sources or current sources. If a network has a large number of voltage sources, it is useful to use mesh analysis, as this requires that all the sources in a circuit be voltage sources. Therefore, if there are any current sources in a circuit they are to be converted into equivalent voltage sources, if on the other hand, the network has more current sources, nodal analysis more useful.

Mesh Analysis:

Mesh analysis is applicable only for planar networks. For non-polar circuits mesh analysis is not applicable. A circuit is said to be planar, if it can be drawn on plane surface without crossovers. A non-planar circuit cannot be drawn on a plane surface without crossover.



From the above figures, fig (A) represents the planar circuit and fig (B) represents the non-planar circuit. It has already been discussed that a loop is a closed path. A mesh is defined as a loop which does not contain any other loops within it. To apply mesh analysis, our first step is to check whether the circuit is planar or not and second is to select mesh currents. Finally, writing Kirchhoff's voltage law equations in terms of unknowns and solving those leads to final solution.



Form above fig indicates that there are two loops abefa and bcdeb in the network. Let us assume loops current I_1 is passing throughR_1, (I_1-I_2) is passing throughR_2. By applying Kirchhoff's voltage law, we can write

$$V_S = I_1 R_1 + R_2(I_1\text{-}I_2)$$

$$I_1(R_1 + R_2) - I_2 R_2 = V_S \text{ --------- } (1)$$

Similarly, if we consider the second mesh loop, the current I_2 is passing through R_3 and R_4 and (I_2-I_1) is passing throughR_2. By applying Kirchhoff's voltage law to the second loop

$$R_2\,(I_2\text{-}I_1) + R_3 I_2 + R_4 I_2 = 0$$

$$\text{-}I_1 R_2 + (R_3 + R_2 + R_4)I_2 = 0 \text{ --------- } (2)$$

By solving the above equations, we can find $I_1$and I_2. If we observe the above fig, the circuit consists of five branches and four nodes including the reference node. The number of mesh currents is equal to number of mesh equations.

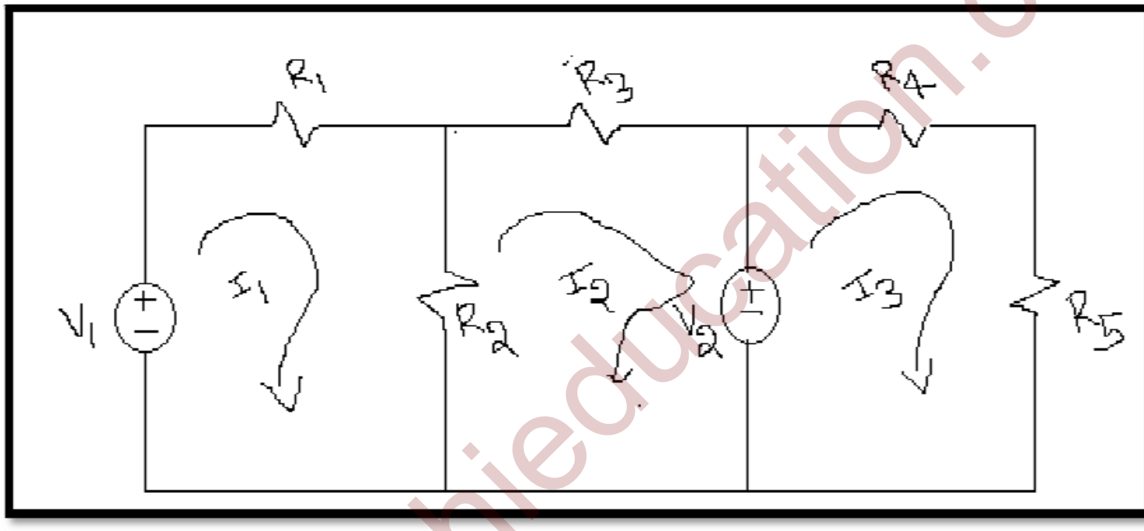
The number of mesh equations= branches-(nodes-1) =the number of mesh currents =5-(4-1) =2.

In general, if we have B number of branches and N number of nodes including the reference node then number of linearly independent mesh equations are

$$M = B - (N - 1)$$

Mesh Equations by inspection method:

The mesh equations for general planar network can be written by inspection without going through the detailed steps. Consider a three mesh networks as shown in fig.



The loop equations are

Loop1, $V_1 = I_1R_1 + R_2(I_1 - I_2)$

$$I_1(R_1 + R_2) - I_2R_2 = V_1 \text{ ------- } (1)$$

Loop2, $R_2(I_2 - I_1) + I_2R_3 = -V_2$

$$-I_1R_2 + I_2(R_3 + R_2) = -V_2 \text{ ------ } (2)$$

Loop3, $I_3R_4 + I_3R_5 = V_2$

$$I_3(R_4 + R_5) = V_2 \text{ ---------- } (3)$$

The general mesh equations for resistive network can be written as,

$$R_{11}I_1 \pm R_{12}I_2 \pm R_{13}I_3 = V_a \text{ ----------- } (4)$$

$\pm R_{21}I_1 \pm R_{22}I_2 \pm R_{23}I_3 = V_b$ ----------- (5)

$\pm R_{31}I_1 \pm R_{32}I_2 + R_{33}I_3 = V_c$ ----------- (6)

By comparing the above equations 1,2&3 with 4,5&6 the fallowing observations taken to be account

1) The self resistance in each mesh.
2) The mutual resistance between all pairs of meshes and
3) The algebraic sum of the voltages in each mesh.

The self resistance of loop1, $R_{11} = (R_1 + R_2)$ is the sum of resistances through which I_1 passes.

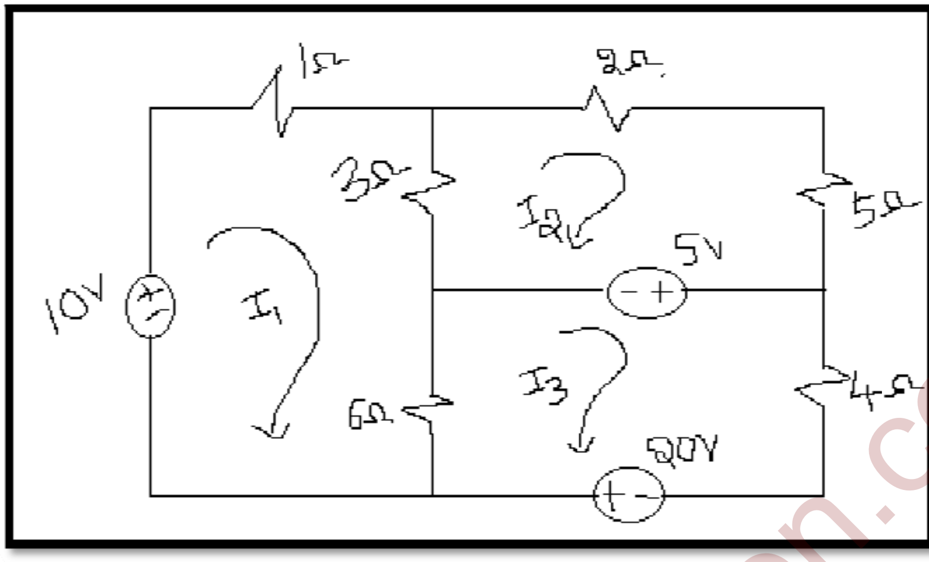
The mutual resistance of loop1, $R_{12} = -R_2$, is the sum of the resistances common to loop currents I_1 and I_2. If the direction of the currents passing through the common resistances is the same, the mutual resistance will have a positive sign and if the direction of the currents passing through the common resistance is opposite then the mutual resistance will have a negative sign.

V_a=V_1 is the voltage which drives the loop one. Here, the positive sign is used if the direction of the current is the same as the direction of the source. If the current direction is opposite to the direction of the source, then the negative sign is used.

Similarly, $R_{22} = (R_3 + R_2)$ and $R_{33} = (R_4 + R_5)$ are the self resistances of loops two and three respectively. The mutual resistances R_{13}=0, R_{21}=0, R_{23}=-R_2, R_{31}=0, R_{32}=0 are the sum of the resistances common to the mesh currents indicated in their subscripts.

V_b=-V_2, V_C=V_2 are the sum of the voltages driving their respective loops.

Ex: write the mesh equations for the circuit as shown in fig.



The general mesh equations for resistive network can be written as,

$R_{11}I_1 \pm R_{12}I_2 \pm R_{13}I_3 = V_a$ ----------- (1)

$\pm R_{21}I_1 \pm R_{22}I_2 \pm R_{23}I_3 = V_b$ ----------- (2)

$\pm R_{31}I_1 \pm R_{32}I_2 + R_{33}I_3 = V_c$ ----------- (3)

Consider equation (1).

Self resistance loop1= R_{11} = 1+ 3 +6 = 10Ω

Mutual resistance common to loop1 and loop2= R_{12} =-3Ω (opposite direction)

The mutual resistance common to loop1 and loop3= R_{13} =-6Ω

V_a= +10V, the voltage driving the loop1.

Here, the positive sign indicates the loop current $I_1$is in the same direction as the source element.

Hence we can write equation as, $10I_1$ - $3I_2$ - 6 I_3 = 10V -------- (4)

Consider equation (2).

Self resistance loop2= R_{22} = 2+ 3 +5 = 10Ω

Mutual resistance common to loop1 and loop2= R_{21} =-3Ω (opposite direction)

The mutual resistance common to loop2 and loop3= R_{23} =0

V_b= -5V, the voltage driving the loop2.

Hence we can write equation as, -3I_1 +10I_2= -5V -------- (5)

Consider equation (2).

Self resistance loop3= R_{33} = 6+4 = 10Ω

Mutual resistance common to loop1 and loop3= R_{31} =-6Ω (opposite direction)

The mutual resistance common to loop2 and loop3= R_{32} =0

V_c= 25V, the voltage driving the loop3.

Hence we can write equation as, -6I_1 +10I_3= 25V -------- (6)

Therefore the mesh equations are,

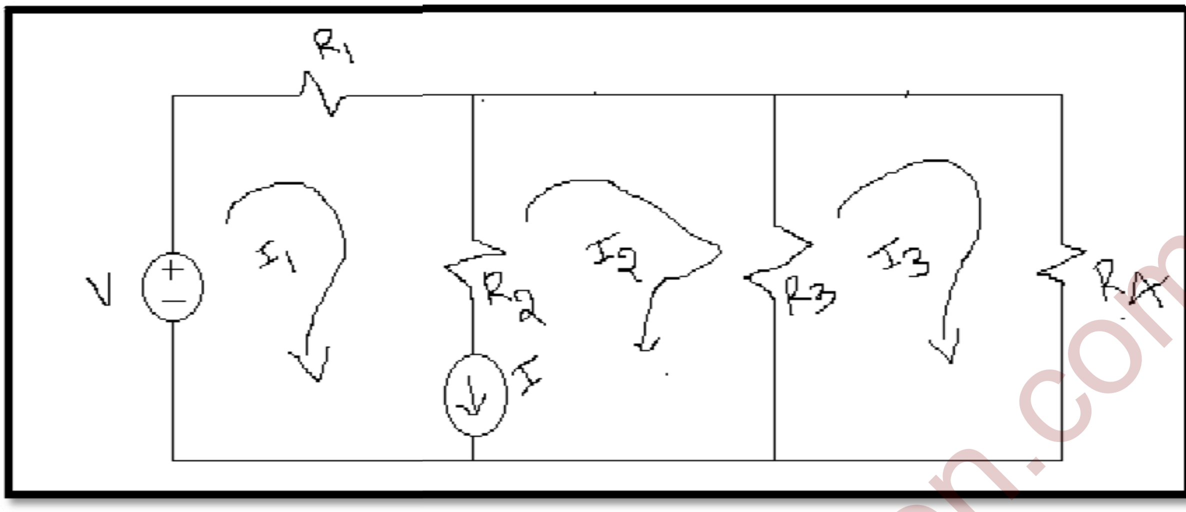
10I_1 - 3I_2 - 6 I_3 = 10V

-3I_1 +10I_2= -5V

-6I_1 +10I_3= 25V

Super Mesh Analysis:

Suppose any of the branches in the network has a current source, then it is slightly difficult to apply mesh analysis straight forward because first we should assume an unknown voltage across the current source and writing mesh equations as before, then relate the source current to the assigned mesh currents. This is generally a difficult approach. One way to overcome this difficulty is by applying the super mesh technique. Here we have to choose the kind of super mesh. A super mesh is constituted by two adjacent loops that have a common current source. As an example illustrated in below fig. here the current source I is in the common boundary for the two meshes 1 and 2. This current source creates a super mesh, which is nothing but a combination of 1 and 2.



$R_1I_1 + R_3(I_2\text{-}I_3)$= V or $R_1I_1 + R_3I_2\text{-}R_3I_3$= V

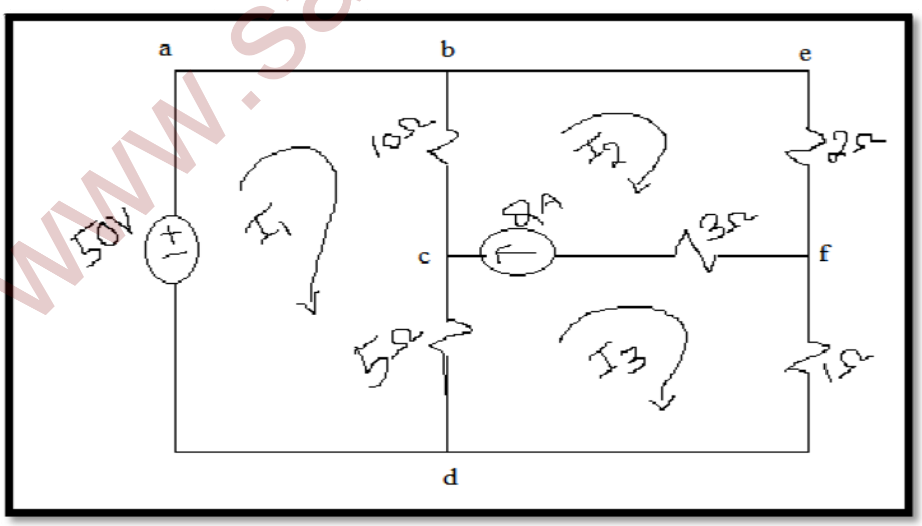
Considering mesh 3 we have,

$R_3(I_3\text{-}I_2) + R_3I_3$= 0

Finally, the current I from current source is equal to the difference between two mesh currents .i.e., $I_1\text{-}I_2$ = I

We have thus formed three mesh equations which we can solve for the three unknown currents in the network.

Ex: determine the current in the 5Ω resistor in the network given below.



From the first mesh abcda we get,

$$50 = 10\ (I_1\text{-}I_2) + 5\ (I_1\text{-}I_3)$$

$$15I_1\text{-}\ 10\ I_2\ \text{-}5I_3 = 50\ \text{-------}\ (1)$$

From the second and third meshes we can form a super mesh,

$$10\ (I_2\text{-}I_1)+ 2I_2+ I_3 + 5\ (I_3\text{-}I_1) =0$$

$$\text{-}15I_1+ 12\ I_2\ +6I_3 = 0\ \text{----------}\ (2)$$

The current source is equal to the difference between 2 & 3 currents,

$$I_2\text{-}I_3 =2\text{A}\ \text{--------}\ (3)$$

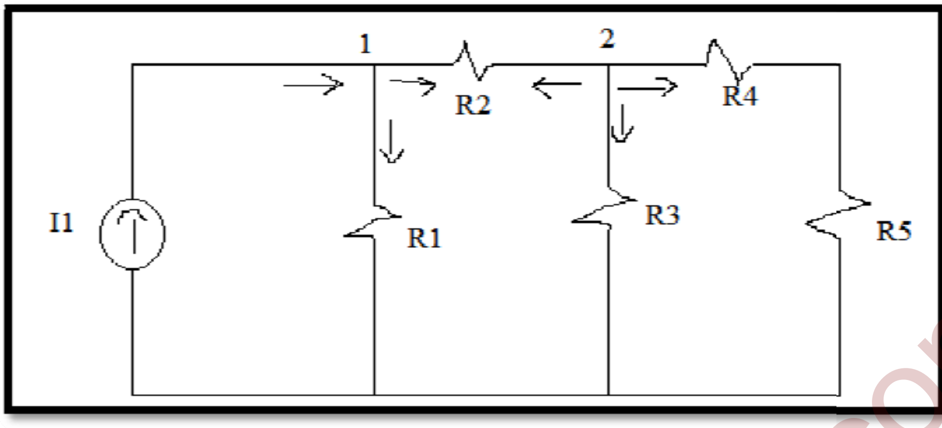
Solving 1, 2&3 we get,

I_1 = 19.99A, I_2 = 17.33A, I_3 = 15.33A

The current in the 5Ω resistor = I_1-I_3 = 4.66A

NODAL ANALYSIS:

In general, in an N nodal circuit, one of the nodes is chosen as reference node then it is possible to write N-1 nodal equations. Each node in circuit can be assigned a number or letter. The node voltage is the voltage of given node with respect to particular node, called the reference node which we assume zero potential. In the circuit shown in fig. node 3 is assumed as the reference node. The voltage at node1 is the voltage at that node with respect to node3. Similarly, the voltage at node2 is the voltage at that node with respect to node3. Applying Kirchhoff's current law at node1, the current entering is equal to the current leaving.



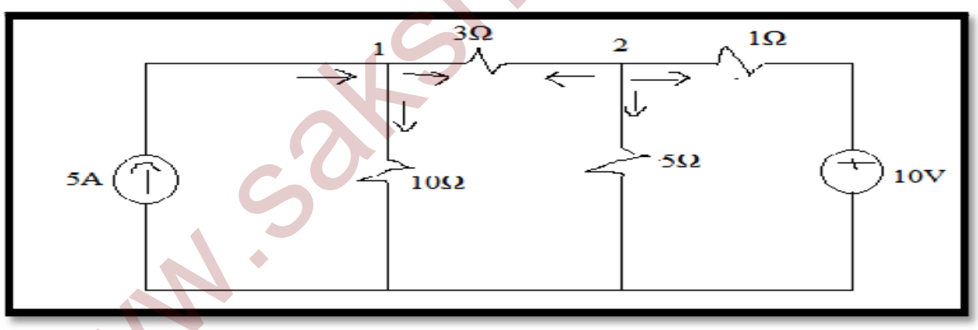
$$I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} \text{ -------- } (1)$$

Where V_1 and V_2 are the voltages at node1 and 2. Similarly, at node2 the current entering is equal to the current leaving as shown in fig.

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_2}{R_4 + R_5} = 0 \text{ ---------- } (2)$$

From the above equations we get the voltages at each node.

Ex: write the node voltage equations and determine the currents in each branch for the network shown in fig.



Applying Kirchhoff's law at node1,

$$5 = \frac{V_1}{10} + \frac{V_1 - V_2}{3}$$

$$V_1[\frac{1}{10} + \frac{1}{3}] - V_2[\frac{1}{3}] = 5 \text{ -------- } (1)$$

Applying Kirchhoff's law at node2,

$$\frac{V_2 - V_1}{3} + \frac{V_2}{5} + \frac{V_2 - 10}{1} = 0$$

$$- V_1[\frac{1}{3}] - V_2[\frac{1}{3}+\frac{1}{5}+1] = 5 \text{ -------- } (2)$$

Solving 1 and 2 we get,

$V_1 = 19.85\text{V}$

$V_2 = 10.9\text{V}$

$I_{10} = \frac{V_1}{10} = 1.985\text{A}$

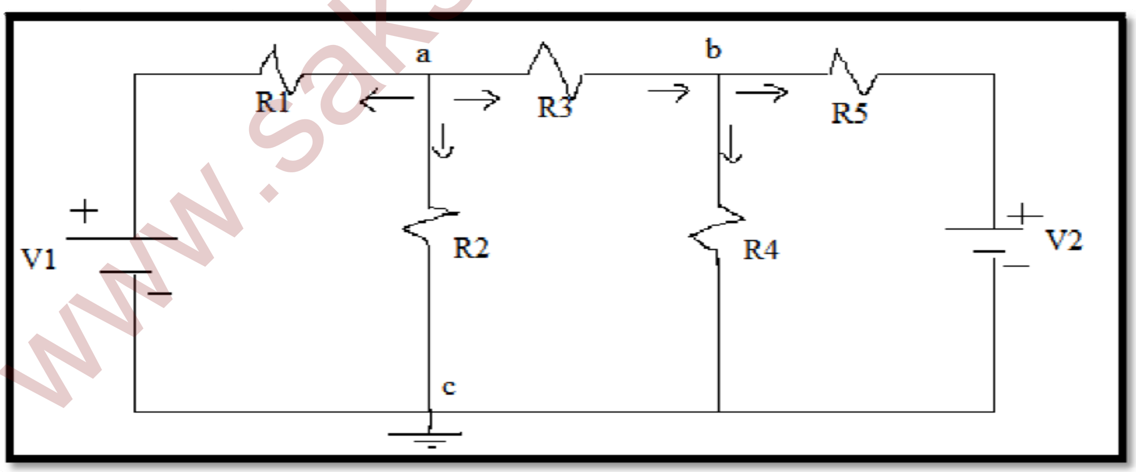
$I_3 = \frac{V_1 - V_2}{3} = 2.98\text{A}$

$I_5 = \frac{V_2}{5} = 2.18\text{A}$

$I_1 = \frac{V_2 - 10}{1} = 0.9\text{A}$

Nodal equations by inspection method:

The nodal equations for a general planar network can also be written by inspection, without going through the detailed steps. Consider a three node resistive network, including the reference node as shown in fig.



In the fig, the nodes a, b are the actual nodes and c is reference node.

At node a, $I_1 + I_2 + I_3 = 0$

$$\frac{V_a - V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a - V_b}{R_3} = 0$$

$$V_a\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - \left(\frac{1}{R_3}\right) V_b = 0 \text{ ------------ } (1)$$

At node b, $I_4 + I_5 = I_3$

$$\frac{V_b - V_a}{R_3} + \frac{V_b}{R_4} + \frac{V_b - V_2}{R_5}$$

$$- V_a\left(\frac{1}{R_3}\right) + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) V_b = \frac{V_2}{R_5} \text{ ---------- } (2)$$

The general equations can be written as,

$$G_{aa}\, V_a + G_{ab}\, V_b = I_1 \text{ --------- } (3)$$

$$G_{ba}\, V_a + G_{bb}\, V_b = I_2 \text{ --------- } (4)$$

By comparing above equations, we have

Self conductance at node a, $G_{aa} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$ is the sum of conductance's connected to node a.

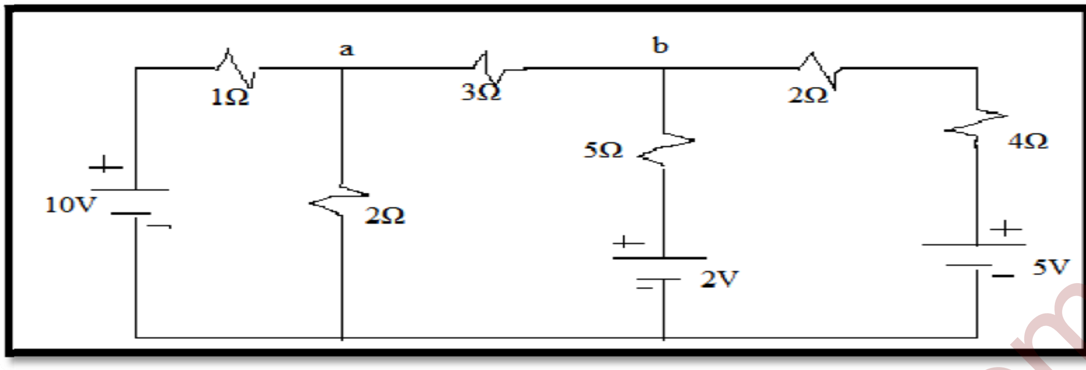
Similarly, Self conductance at node b, $G_{bb} = \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)$ is the sum of conductance's connected to node b.

$G_{ab} = -\left(\frac{1}{R_3}\right)$, is the sum of mutual conductance's connected to node a and node b (here all the mutual inductances have negative signs).

Similarly, $G_{ba} = -\left(\frac{1}{R_3}\right)$, is the sum of mutual conductance's connected to node b and node a.

I_1 and I_2 are the source currents at node a and node b respectively. The current which drives into the nodes has positive sign, while the current that drives away from the node has negative sign.

Ex: for the circuit shown in fig write the node equations by inspection method.



The general equations are,

$$G_{aa}\ V_a + G_{ab}\ V_b = I_1 \text{ --------- } (1)$$

$$G_{ba}\ V_a + G_{bb}\ V_b = I_2 \text{ --------- } (2)$$

Self conductance at node a, $G_{aa} = (\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}) = (1 + \frac{1}{2} + \frac{1}{3})$ is the sum of conductance's connected to node a.

Similarly, Self conductance at node b, $G_{bb} = (\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}) = (\frac{1}{6} + \frac{1}{5} + \frac{1}{3})$ is the sum of conductance's connected to node b.

$G_{ab} = -(\frac{1}{3})$, is the sum of mutual conductance's connected to node a and node b (here all the mutual inductances have negative signs).

Similarly, $G_{ba} = -(\frac{1}{3})$, is the sum of mutual conductance's connected to node b and node a.

$I_1 = 10/1 = 10$ A, source current at node a.

$I_2 = \frac{2}{5} + \frac{5}{6} = 1.23$A, source current at node b.

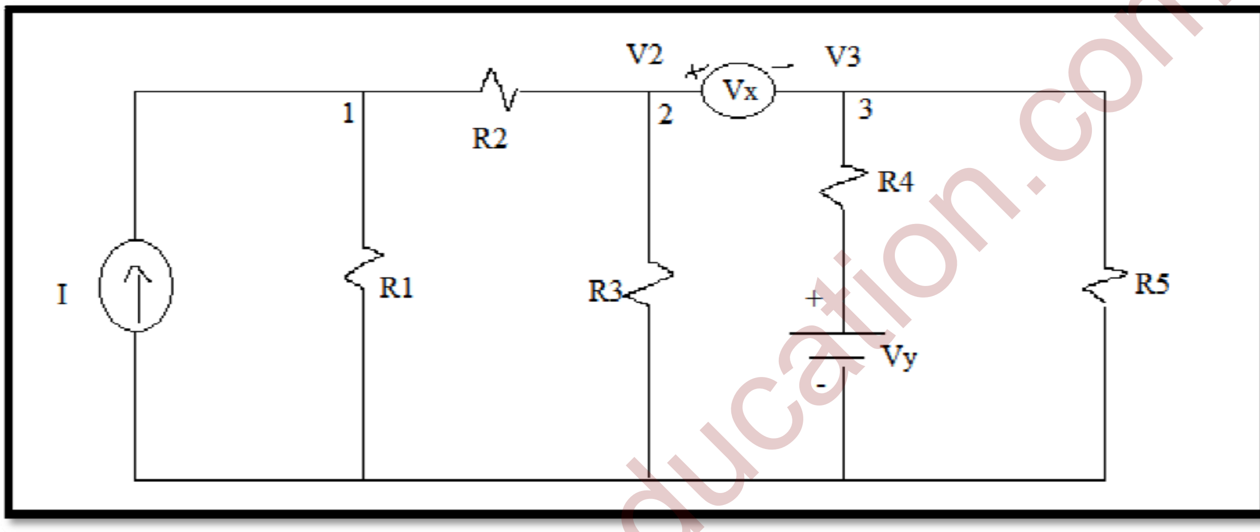
Therefore the nodal equations are,

$$1.83\ V_a - 0.33\ V_b = 10$$

$$-0.33\ V_a + 0.7\ V_b = 1.23$$

Super Node Analysis:

Suppose any of the branches in the network has a voltage source, then it is slightly difficult to apply nodal analysis. One way to overcome this difficulty is to apply the super node technique. In this method, the two adjacent nodes that are connected by a voltage source are reduced to a single node and then the equations are formed by applying Kirchhoff's current law as usual. This is illustrated using below fig.



It is clear from fig that node 4 is the reference node. Applying Kirchhoff's current law at node1, we get

$$\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} = \mathrm{I} \text{ -------- } (1)$$

Due to the presence of voltage source V_x in the between node 2 and 3, it is slightly difficult to find out the current. The super node technique can be conveniently applied in this case.

Accordingly we can write the combined equation for nodes2 and 3

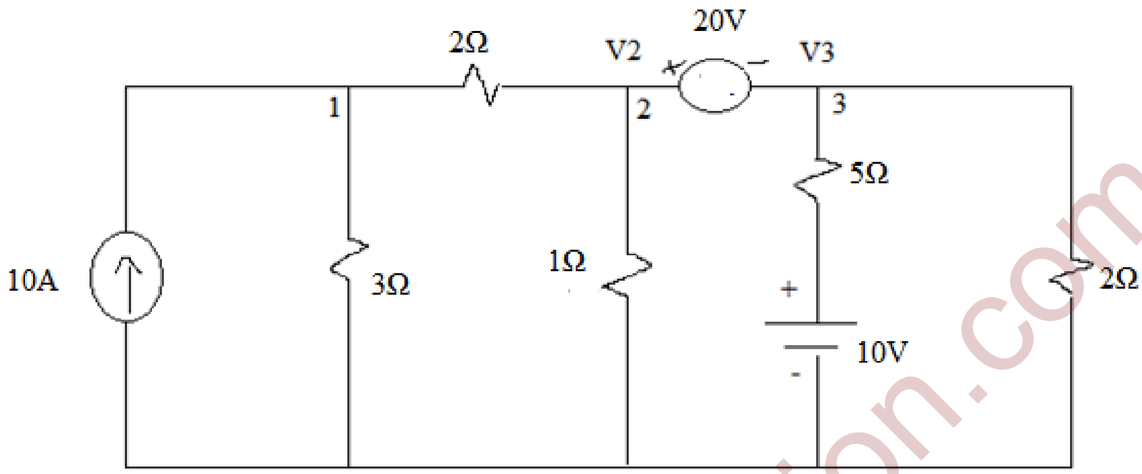
$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_3 - V_y}{R_4} + \frac{V_3}{R_5} = 0 \text{ --------- } (2)$$

The other equation is

$$V_2 - V_3 = V_x \text{ --------- } (3)$$

From the above three equations we can find the three unknown voltages.

Ex: determine the current through 5Ω resistor for the circuit shown in fig.



At node1,

$$\frac{V_1}{3} + \frac{V_1 - V_2}{2} = 10$$

0.83 V_1- 0.5 V_2 -10 =0 --------- (1)

At nodes2 and 3

$$\frac{V_2 - V_1}{2} + \frac{V_2}{1} + \frac{V_3 - 10}{5} + \frac{V_3}{2} = 0$$

-0.5 V_1+1.5 V_2+0.7 V_3 -2 =0 --------- (2)

The other equation is

V_2 - V_3 = 20 --------- (3)

The current in the 5Ω resistor is, $I_5 = \frac{V_3 - 10}{5}$

Solving the above equations we obtain V_3 = -8.42V

Current, I_5 = -3.68A (the current flowing through node 3)