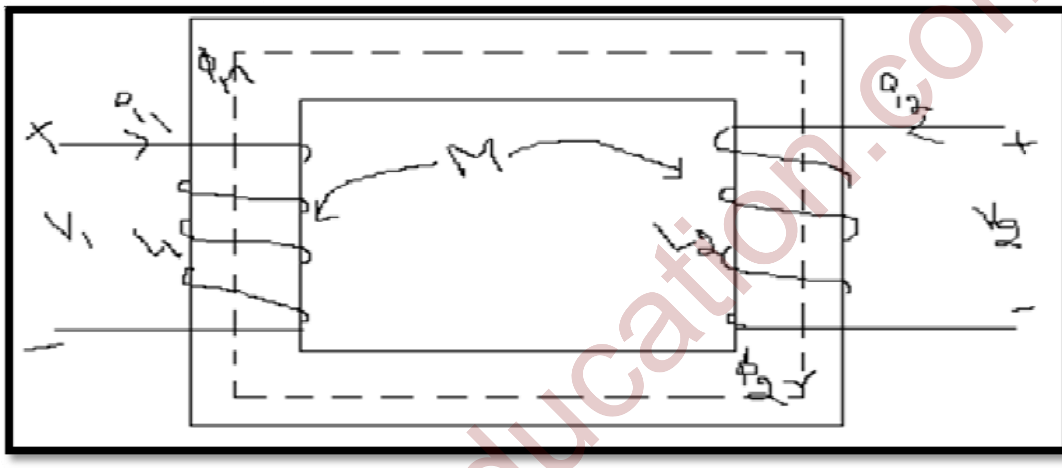
Magnetic Circuit Dot Convention

When two are more coils are inductively coupled, there will be two induced emfs in each coil. The first one is the self induced emf which is because of the time variation of current flowing through it. The second one is the mutual induced emf, which is due to the time variation of current flowing in the other coil. It is necessary to know whether these induced emfs are additive or otherwise. This mainly depends on the sense in which the winding is wound, and directions of currents in the coils.



Let us consider the coils 1 and 2 which are wound on a magnetic core. Let i_1 be the current flowing through the coil1. The direction of magnetic flux produced by this current $\emptyset_1$ is directed upwards (this is according to right hand rule). Now let us consider i_2 flowing in the coil2. The current produces a flux $\emptyset_2$ directed downwards.

In other words the two magnetic fluxes produced by i_1 and i_2 are aiding one another. Hence the two induced voltages in the two coils are having the same sign i.e. positive. Hence

$$V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$V_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \text{ ----------------- (1)}$$

If the flux produced by i_1 and i_2 opposes each other, then the mutual induced emf and self induced emf will have opposite sign (M is negative). Hence

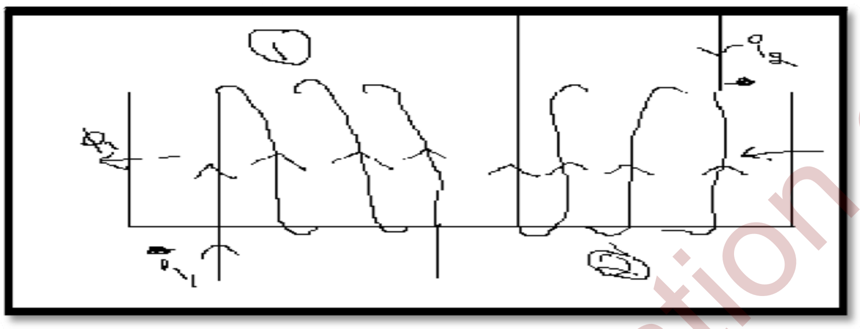
$$V_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$V_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} \text{ ----------------- (2)}$$

The above method of deciding polarity for induced emfs require the representation of the coils as shown in fig, indicating the sense in which the coils are wound. But such a representation is cumbersome to be used in representing the coupled coils in an electrical network. To simplify the representation, we use dot convention to indicate the corresponding terminals of the coils. It facilitates to determine the sign of mutual induced emf.

In case winding direction not given and to give the sign of mutual inductance, this convention is employed.

Dot Convention:



Place a dot at one end of coil1; assume that the current enters at the dotted end of coil, determine the direction of flux produced due to this current. Then place another dot at one of the ends of coil2 such that the current entering at that dotted end in coil2 produces a flux in the same direction. It is illustrated in fig. consider two coils1 and 2 as shown in fig.

1) Place a dot at one end of coil1 and assume that the current enters at that dotted end in coil1.
2) Place another dot at one of the ends of coil2 such that the current entering at that end in coil2 establishes magnetic flux in the same direction.

In order that the flux produced by I_2 flowing in coil2, produces the flux in the same upward direction so that it enters at lower end of coil2. Hence place a dot at that end of coil2. The dotted ends according to the above systems are known as corresponding ends.

To determine the polarity of Mutual Induced Voltage:

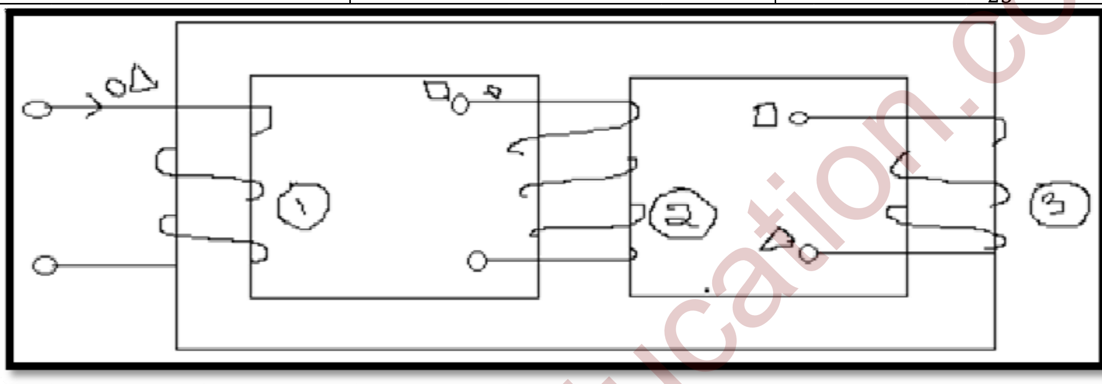
The self induced voltages are always taken as positive and if the mutual induced voltage aids self induced voltage, then it is taken as positive or else negative. Having placed the dots we can determine the sign of mutual induced voltage as follows.

1) Place the dots at the corresponding terminal of coupled coils.
2) Mark the directions of current in the two coils.

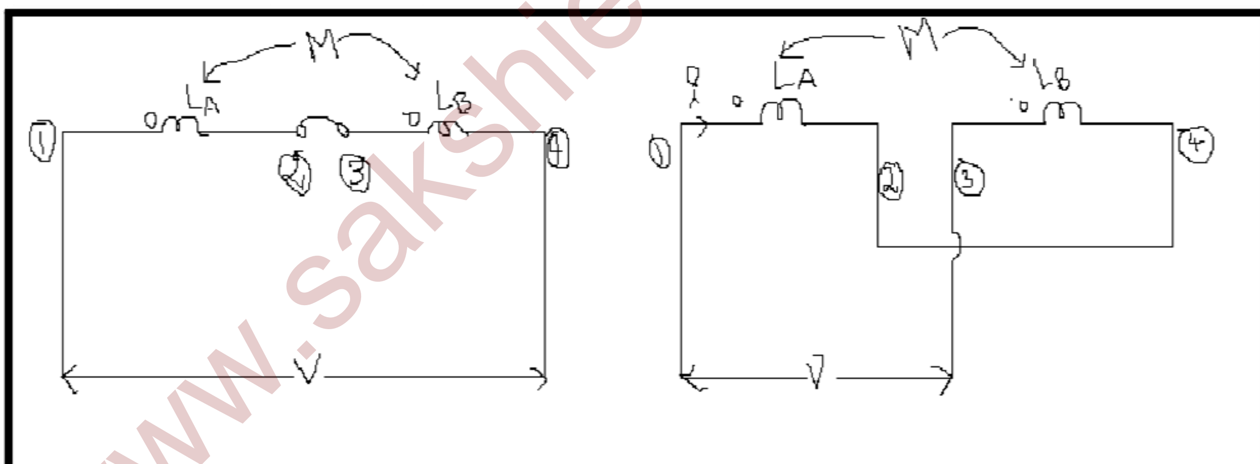
If the currents enter or leave at dotted ends in both the coils, then effect of mutual inductance is additive and sign of M is taken as positive. The voltage expressions are given in equation. Otherwise, if one current enters in and in the other current leaves the dotted terminals, then the sign of M is taken as negative and the effect of M is opposing. The voltage expressions are given in an equation.

If more than two coils are mutually coupled the corresponding ends of different pairs of coils are indicated using different symbols. It is illustrated in fig.

Symbols	**Coils**	**Mutual inductance**
dot	1 and 2	M_{12}
Triangle	1 and 3	M_{13}
square	2 and 3	M_{23}



Ex: two coils A and B are connected in series. The coils have self induced L_A and L_B with a mutual inductance M. what is the effective inductance of the series circuit.



The two coils A and B are connected in series as shown in fig

The voltage induced in the circuit (a) = $L_A \frac{di}{dt}$ + M$\frac{di}{dt}$ + $L_B \frac{di}{dt}$ + M$\frac{di}{dt}$

$$= (L_A + L_B + 2M) \frac{di}{dt}$$

Therefore, $L_{eq} = L_A + L_B + 2M$

Voltage induced in the circuit (b) = $L_A\frac{di}{dt}$ - M$\frac{di}{dt}$ + $L_B\frac{di}{dt}$ - M$\frac{di}{dt}$

$$= (L_A + L_B - 2M)\frac{di}{dt}$$

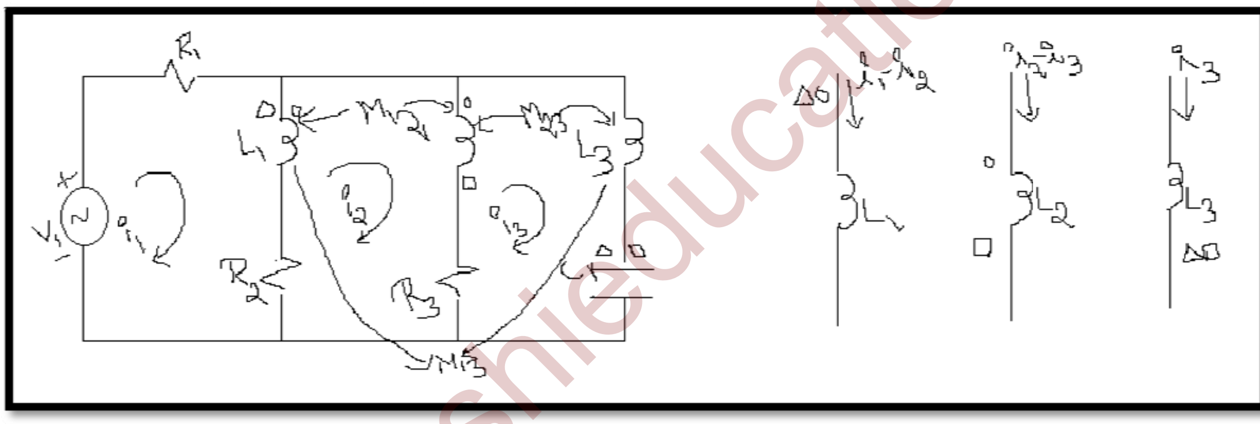
Therefore, $L_{eq} = L_A + L_B - 2M$

Positive sign for aiding connection, negative sign for series opposition.

Loop equations:

It is easier to solve the couple circuit using loop or mesh method. This is illustrated with below example.

Ex: write the loop equations for the circuit shown in fig.



In coils1, we will have self induced emf due to (i_1-i_2) flowing through it and mutual induced emf due to (i_2-i_3) flowing through coil2 and i_3 flowing through coil3.

For loop1,

$$i_1R_1 + L_1\frac{d}{dt}(i_1 - i_2) \quad + M_{12}\frac{d}{dt}(i_2\text{-}i_3) - M_{13}\frac{di_3}{dt} + R_2(i_1\text{-}i_2) = V_1$$

For loop2,

$$R_2(i_2\text{-}i_1) + L_1\frac{d}{dt}(i_2 - i_1) - M_{12}\frac{d}{dt}(i_2\text{-}i_3) + M_{13}\frac{di_3}{dt} + L_2\frac{d}{dt}(i_2 - i_3)$$

$$-M_{21}\frac{d}{dt}(i_2\text{-}i_1) + M_{13}\frac{di_3}{dt} + R_3(i_2\text{-}i_3) = 0$$

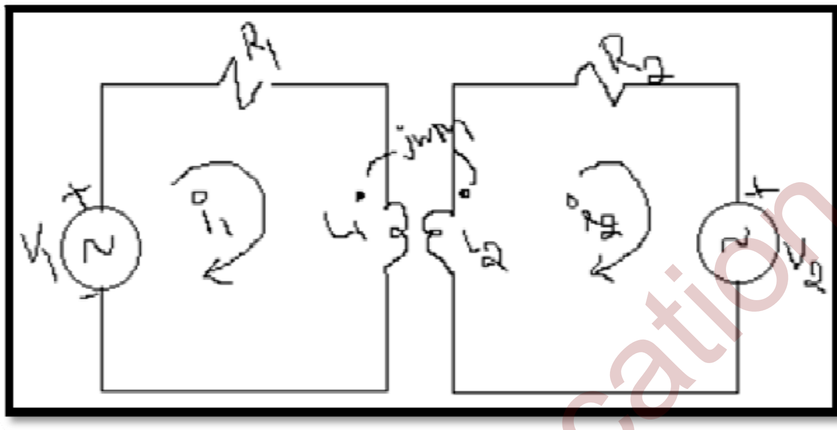
For loop3,

$$R_3(i_3\text{-}i_2) + L_2\frac{d}{dt}(i_3 - i_2) - M_{21}\frac{d}{dt}(i_1\text{-}i_2) - M_{23}\frac{di_3}{dt} + L_3\frac{di_3}{dt} -$$

$$M_{31}\frac{d}{dt}(i_1\text{-}i_2) - M_{32}\frac{d}{dt}(i_3\text{-}i_2) + \frac{1}{C_1}\int i_3\,\text{dt} = 0$$

Conductively coupled equivalent circuits:

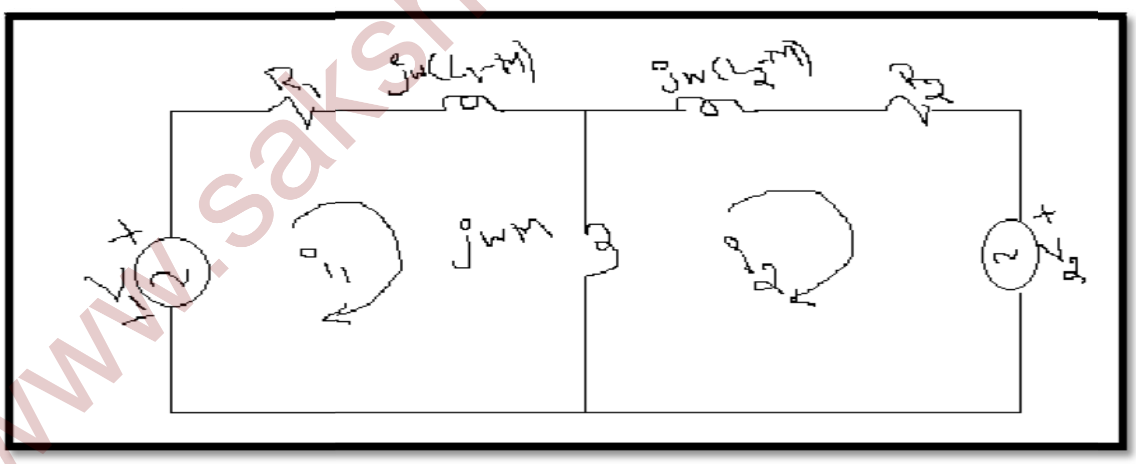
It is possible in analysis to replace a mutual coupled circuit in the fig with a conductively coupled circuit of fig. Let us consider a mutually coupled circuit as shown in fig. let I_1 and I_2 be the loop currents. The loop equations in matrix form for the coupled circuit are:



$(R_1 + j\omega L_1)\, I_1 - j\omega M I_2 = V_1$

$-j\omega M I_1 + (R_2 + j\omega L_2)\, I_2 = -V_2$ ----------- (1)

The loop equations for the network shown in below fig,



For loop1,

$I_1(R_1 + j\omega(L_1\text{-}M)) + (I_1\text{-}I_2)\, j\omega M = V_1$

$I_1(R_1 + j\omega L_1) - I_1 j\omega M + I_1 j\omega M - I_2 j\omega M = V_1$

$I_1(R_1 + j\omega L_1) - I_2 j\omega M = V_1$ ---------- (2)

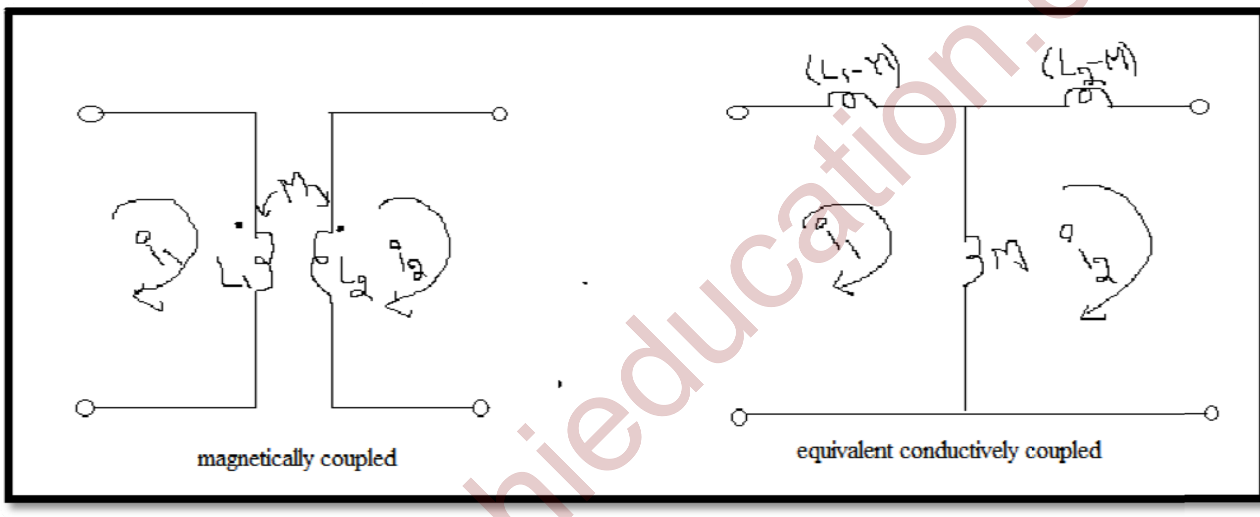
For loop2,

$$(I_2 - I_1)\, j\omega\, M + I_2\, (R_2 + j\omega(L_2 - M)) = - V_2$$

$$I_2\, j\omega\, M - I_1\, j\omega\, M + I_2\, (R_2 + j\omega L_2) - I_2\, j\omega\, M = - V_2$$

$$- I_1\, j\omega\, M + I_2\, (R_2 + j\omega L_2) = - V_2 \text{ ------- } (3)$$

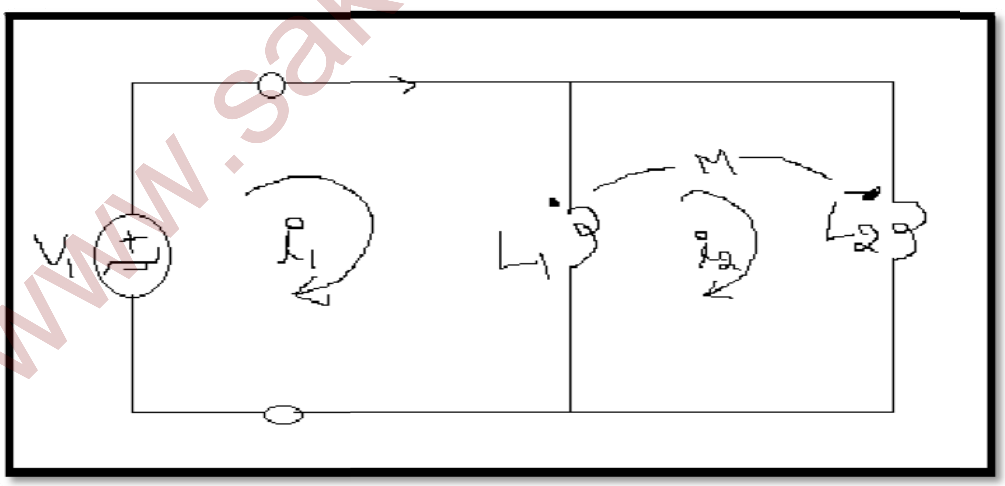
The equations for the networks shown in 2 and 3 are exactly identical. Hence the coupled circuits shown in fig are mutually coupled circuits and can be replaced by a conductively coupled circuit as given as below in fig.



magnetically coupled

equivalent conductively coupled

Ex: obtain the effective inductance of the circuit shown in below



Writing the loop equation values and we get,

$$L_1 \frac{d}{dt}(i_1 - i_2) + M\frac{di_2}{dt} = V_1 \text{ -------- } (1)$$

$L_1 \frac{d}{dt}(i_2 - i_1)$ - M $\frac{di_2}{dt}$ + L_2 $\frac{di_2}{dt}$ - M $\frac{d}{dt}(i_2 - i_1)$ = 0 ------ (2)

Under steady state,

$(I_1 - I_2)$ jωL_1 +I_2 jω M = V_1

I_1 jωL_1 - I_2 jω $(L_1 - M)$ = V_1 ---------------- (3)

$(I_2 - I_1)$ jωL_1 - I_2 jω M + I_2jωL_2 - jω M $(I_2 - I_1)$ = 0

- I_1 j$\omega(L_1 - M) + I_2$jω(L_1 + L_2- 2M) =0 --------- (4)

The loop equations in matrix form,

$$\begin{matrix} j\omega L_1 & -j\omega\,(L_1 - M) \\ -j\omega\,(L_1 - M) & j\omega(\,L_1 + L_2 - 2M\,) \end{matrix} \quad \begin{matrix} I_1 \\ I_2 \end{matrix} = \begin{matrix} V_1 \\ 0 \end{matrix}$$

The effective impedance = $\frac{V_1}{I_1}$

$$I_1 = \frac{\begin{matrix} V_1 & -j\omega\,(L_1 - M) \\ 0 & j\omega(\,L_1 + L_2 - 2M\,) \end{matrix}}{\begin{matrix} j\omega L_1 & -j\omega\,(L_1 - M) \\ -j\omega\,(L_1 - M) & j\omega(\,L_1 + L_2 - 2M\,) \end{matrix}}$$

Solve this we get $I_1 = \frac{j\omega((\,L_1 + L_2 - 2M\,)V_1}{\omega^2(M^2 - L_1 L_2)}$

But $\frac{V_1}{I_1} = \frac{\omega^2(M^2 - L_1 L_2)}{j\omega(\,L_1 + L_2 - 2M\,)}$

$\frac{V_1}{I_1} = \frac{-j\omega(M^2 - L_1 L_2)}{(\,L_1 + L_2 - 2M\,)}$

Therefore $L_{eq} = \frac{L_1 L_2 - M^2}{(\,L_1 + L_2 - 2M\,)}$ (this is for aiding)

Similarly for opposing, $L_{eq} = \frac{L_1 L_2 - M^2}{(\,L_1 + L_2 + 2M\,)}$

Ex: Two coupled coils with L_1 = 0.03H, L_2 = 0.02H and K= 0.6 are connected in four different ways Series, aiding, series opposing and parallel with both arrangement of the winding sense. What are four equivalent inductances?

Given data,

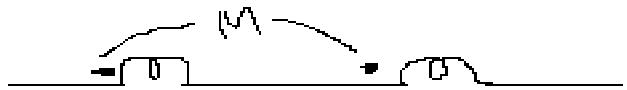
L_1 = 0.03H

L_2 = 0.02H

K= 0.6

From the formula, M = K$\sqrt{L_1 L_2}$ = 0.6 $\sqrt{0.03 * 0.02}$ = 0.0146H

Series adding:



L_{eq} = L_1 + L_2 + 2M

= 0.03 + 0.02 + 2 (0.0146)

= 0.0792H

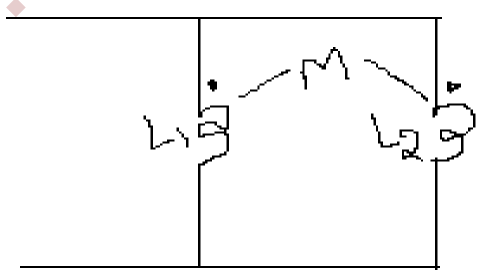
Series opposing:

L_{eq} = L_1 + L_2 - 2M

= 0.03 + 0.02 - 2 (0.0146)

= 0.0208H

Parallel aiding:

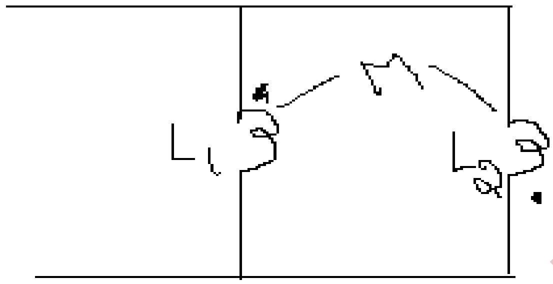


$$L_{eq} = \frac{L_1 L_2 - M^2}{(L_1 + L_2 - 2M)}$$

$$= \frac{(0.03)(0.02)-(0.0146)(0.0146)}{0.03+0.02-2(0.0146)}$$

$= 0.018598$H

Parallel opposing:



$$L_{eq} = \frac{L_1 L_2 - M^2}{(L_1 + L_2 \mp 2M)}$$

$$= \frac{(0.03)(0.02)-(0.0146)(0.0146)}{0.03+0.02+2(0.0146)}$$

$= 0.004884$H

Ex: two coils with inductances in the ratio of 5:1 having coefficient of coupling, k = 0.5. When these coils are connected in series aiding, the equivalent inductance is 44.4mH. FindL_1, L_2, M

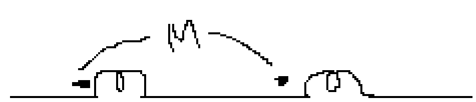
Given data,

$L_1 : L_2 = 5:1 \rightarrow 5L_1 = L_2$

K = 0.5

M = K$\sqrt{L_1 L_2}$ = 0.5$\sqrt{L_1 (5L_1)}$

M = 1.118L_1



In series aiding, $L_{eq} = L_1 + L_2 + 2M$

$0.0444 = L_1 + 5L_1 + 2(1.118L_1)$

$L_1 = 0.00539$H

$5L_1 = L_2$ = 5 *0.00539 = 0.02695

M = $1.118L_1$ = 1.118 * 0.00539 = 0.006026H

Ex: Two coils connected in series having an equivalent inductance of 0.5H, when connected in aiding and equivalent inductance 0.3H, when connected in opposing. Calculate the mutual inductance of the coil.

Given data,

When coils connected in aiding,

$L_{eq} = L_1 + L_2 + 2M = 0.5H$ --------- (1)

When coils connected in opposing,

$L_{eq} = L_1 + L_2 - 2M = 0.3H$ ---------- (2)

Performing (1) – (2) we get

4M= 0.2 → M = 0.05H