

9. PERCENTAGE

Percentage

'Per cent' means 'per hundred'. It is denoted by the symbol %. Here $x\%$ means x per hundred or $\frac{x}{100}$.

Thus, any percentage can be converted into an equivalent fraction by dividing it by 100.

$$\text{e.g. } 20\% = \frac{20}{100} = \frac{1}{5}; \quad 150\% = \frac{150}{100} = \frac{3}{2}$$

Also, any fraction or decimal can be converted into its equivalent percentage by multiplying with 100.

$$\text{e.g. } \frac{1}{5} = \frac{1}{5} \times 100 = 20\%; \quad \frac{3}{2} = \frac{3}{2} \times 100 = 150\%.$$

Important Formulae

$$1. \text{ Percentage increase} = \frac{\text{Increase}}{\text{Original Value}} \times 100\%$$

$$2. \text{ Percentage decrease} = \frac{\text{Decrease}}{\text{Original Value}} \times 100\%$$

3. If the price of the commodity increases by $r\%$ then the reduction in consumption so as not to

$$\text{increase the expenditure is } \left[\frac{r}{100+r} \times 100 \right] \%$$

4. If the price of the commodity decreases by $r\%$ then the reduction in consumption so as not to

$$\text{increase the expenditure is } \left[\frac{r}{100-r} \times 100 \right] \%$$

5. If A's income is $r\%$ more than B's income then B's income is less than A's income by $\left[\frac{r}{100+r} \times 100 \right] \%$

6. If A's income is $r\%$ less than B's income then B's income is more than A's income by $\left[\frac{r}{100-r} \times 100 \right] \%$

7. Let the population of a town be P and it increases at the rate of $r\%$ per annum, then

$$(a) \text{ Population after- } n \text{ years} = P \left(1 + \frac{r}{100} \right)^n$$

$$(b) \text{ Population } n \text{ years ago} = \frac{P}{\left(1 + \frac{r}{100} \right)^n}$$

8. Let the present value of the machine be P and if it depreciates at the rate of $r\%$ per annum, then

$$(a) \text{ Value of machine after } n \text{ years} = P \left(1 - \frac{r}{100} \right)^n$$

$$(b) \text{ Value of machine } n \text{ years ago} = \frac{P}{\left(1 - \frac{r}{100} \right)^n}$$

Example 1: Express $\frac{3}{2}$ as rate per cent.

$$\text{Solution: } \frac{3}{2} = \left(\frac{3}{2} \times 100 \right) \% = 150\%$$

Example 2: Find 25% of 1000.

$$\text{Solution: } 25\% \text{ of } 1000 = \left(\frac{25}{100} \times 1000 \right) = 250$$

Example 3: What per cent of 6 is 144?

$$\text{Solution: Required percentage} = \left(\frac{144}{6} \times 100 \right) \% = 2400\%$$

Example 4: What per cent of 2.5 kg is 15 g?

$$\text{Solution: Required percentage} = \left(\frac{15}{2.5 \times 1000} \times 100 \right) \% = 0.6\%$$

Example 5: If the price of tea falls by 12%, by how much per cent must a house holder increase its consumption, so as not to decrease its expenditure on tea?

Solution: Increase % in consumption = $\left\{\frac{r}{100-r} \times 100\right\} \% = \left\{\frac{12}{100-12} \times 100\right\} \%$
 $= \left\{\frac{12}{88} \times 100\right\} \% = \frac{150}{11} \% = 13 \frac{7}{11} \%$

Example 6: The value of a machine depreciates at the rate of 10% per annum. If its present value is Rs.162000, what was the value of the machine 2 year ago?

Solution: Value of the machine 2 years ago = Rs. $\left[\frac{162000}{\left(1-\frac{10}{100}\right)^2}\right] = \text{Rs. } (162000 \times \frac{10}{9} \times \frac{10}{9})$
 $= \text{Rs. } 200000$

Example 7: If the price of 1 kg cornflakes is increased by 25%, the increase is Rs. 10. Find the new price of cornflakes per kg.

Solution: Original Price = $\frac{\text{Difference in price}}{\text{Difference in percent}} \times 100 = \frac{10}{25} \times 100 = \text{Rs. } 40$

New price = $40 \times \frac{125}{100} = \text{Rs. } 50$

EXERCISE

- $0.05 = ?\%$
 (a) 5 (b) 0.05
 (c) 0.5 (d) 50
- $20 \times ? = 25\%$ of 800
 (a) 10 (b) 15
 (c) 8 (d) 12
- What is 40% of 40% equal to?
 (a) 0.16% (b) 16%
 (c) 1.6% (d) 0.016%
- $9 : 4 = ?$
 (a) 125% (b) 200%
 (c) 225% (d) 250%
- A school mini bus brakes from 60 m/h to 40 m/h. What is the percentage decrease in speed?
 (a) 33.3% (b) 66.6%
 (c) 77.7% (d) 45%
- The price of a book is reduced by 25%, what is the ratio of change in price to the old price?
 (a) 1 : 4 (b) 4 : 1
 (c) 5 : 4 (d) 4 : 5
- What percent decrease in salaries would exactly cancel out the 25% increase?
 (a) 25% (b) 20%
 (c) 24% (d) 27%
- A's income is 40% more than B's income. How much per cent is B's income less than A's?
 (a) $28 \frac{2}{7} \%$ (b) $26 \frac{1}{3} \%$
 (c) $28 \frac{4}{7} \%$ (d) $26 \frac{2}{3} \%$
- If A's height is 50% less than that of B, how much percent B's height is more than that of A?
 (a) 100% (b) 80%
 (c) 150% (d) 50%
- A's salary is 20% of B's salary which is 25% of C's salary. What percentage of C's salary is A's salary?
 (a) 8% (b) 5%
 (c) 6% (d) 4%
- The population of a city increases at the rate of 5% per annum. There is additional annual increase of 5% due to influx of job seekers. The percent increase in population after 3 yr is
 (a) 33.1% (b) 33%
 (c) 34% (d) 33.24%
- In an examination, 32% students failed in Mathematics and 40% failed in English. If 22% of the students failed both the subjects then percentage of students who passed in both the subject was
 (a) 30% (b) 40%
 (c) 50% (d) None
- In an examination of n questions, a student replied 15 out of the first 20 questions correctly. Of the remaining questions, he answered one-third correctly. All the questions have the same credit. If the student gets 50% marks, the value of n is:
 (a) 30 (b) 67
 (c) 50 (d) 82
- A's marks in Biology are 20 less than 25% of the total marks obtained by him in Biology, Maths and Drawing. If his marks in Drawing be 50, what are his marks in Maths?
 (a) 60 (b) 47
 (c) 63 (d) cannot be determined
- In an election, a candidate who gets 76% of the votes is elected by a majority of

- 468 votes. What is the total number of votes polled?
 (a) 964 (b) 900
 (c) 1008 (d) 1024
16. 15% of the people eligible to vote are between 18 and 25 yr of age. In an election, 75% of those eligible to vote, who are between 18 and 25, actually voted. In that election, the number of persons between 18 and 25, who actually voted was, what percent of those eligible to vote?
 (a) 12.50% (b) 10.75%
 (c) 11.25% (d) 10.25%
17. A scored 30% marks and failed by 15 marks. B scored 40% marks and obtained 35 marks more than those required to pass. The pass percentage is:
 (a) 33% (b) 40%
 (c) 34% (d) 48%
18. The price of a table is RS. 400 more than that of a chair. If 6 tables and 6 chairs together cost RS. 4800, by what percent is the price of the chair less than that of the table?
 (a) 66.9% (b) 60%
 (c) $66\frac{2}{3}\%$ (d) 44%
19. A salesman is allowed $5\frac{1}{2}\%$ discount on the total sales made by him plus a bonus of $\frac{1}{2}\%$ on the sales over RS. 10,000. If his total earnings were RS. 1990, then his total sales (in RS.) were:
 (a) 60,000 (b) 42,000
 (c) 34,000 (d) 35,000
20. A number x is short of y by 40%. By what per cent is y in excess of x ?
 (a) $33\frac{1}{3}\%$ (b) $66\frac{1}{3}\%$
 (c) $66\frac{2}{3}\%$ (d) $33\frac{1}{6}\%$
21. A man spends 30% of his income on food, 12% on house rent, 28% on miscellaneous. If the savings at the end of a month is Rs. 810, then the man's total income is
 (a) Rs. 2100 (b) Rs. 2400
 (c) Rs. 2600 (d) Rs. 2700
22. The price of a book is first increased by 10% and then decreased by 5%, then the net change in the price will be
 (a) 4.2% increase (b) 4.5% increase
 (c) 4.2% decrease (d) 4.5% decrease
23. A number is decreased by 25% and then increased by 25%. The number so obtained is 8 less than the original number. What was the original number?
 (a) 128 (b) 144
 (c) 136 (d) 114
24. The difference of two numbers is 20% of the larger number. If the smaller number is 20, then the larger number is:
 (a) 25 (b) 46
 (c) 27 (d) 82
25. Two numbers A and B are such that the sum of 5% of A and 4% of B is two-third of the sum of 6% of A and 8% of B. Find the ratio of A : B.
 (a) 1 : 2 (b) 3 : 1
 (c) 3 : 4 (d) 4 : 3
26. A student multiplied a number by $\frac{3}{5}$ instead of $\frac{5}{3}$. What is the percentage error in the calculation?
 (a) 36% (b) 64%
 (c) 55% (d) 35%
27. A tempo is insured to the extent of $\frac{4}{5}$ th of its original value. If the premium on it at the rate of 1.3 percent amounts to RS. 910, the original value of the tempo is:
 (a) Rs. 78,000 (b) Rs. 78,500
 (c) Rs. 80,000 (d) Rs. 87,500
28. When 15% is lost in grinding wheat, a country can export 30 lakh tons of wheat. On the other hand, if 10% is lost in grinding, it can export 40 lakh tons of wheat. The production of wheat in the country is:
 (a) 40 lakh tons (b) 400 lakh tons
 (c) 200 lakh tons (d) 900 lakh tons
29. The sum of the number of boys and girls in a school is 150. If the number of boys is x , then the number of girls becomes $x\%$ of the total number of students. The number of boys is:
 (a) 51 (b) 65
 (c) 60 (d) 95
30. A reduction of 20% in the price of salt enables a person to buy 2.5 kg more for RS. 100. What is the reduced price per kg?
 (a) Rs. 8 (b) Rs. 10
 (c) Rs. 8.50 (d) Rs. 9.50

31. The price per kg of sugar decreases by 20%. By what percentage should the consumption be increased such that expenditure remain the same?
 (a) 18% (b) 30%
 (c) 20% (d) 25%
32. 24% of x + 28% of 500 = 219, find the value of x ?
 (a) 329.17 (b) 330.17
 (c) 392.71 (d) 239.17
33. The value of mobile price depreciates at rate of 3%. The price of mobile in June 2012 is 10000. What could be the rate of mobile in June 2014?

- (a) 9408 (b) 9409
 (c) 9410 (d) 9412

34. The price of a Fan is 2000. After 2 year the price of fan is 500. The value of fan depreciates, at the rate of $R\%$ find $R\%$?
 (a) 33.55 (b) 50
 (c) 43.33 (d) 43.55
35. The value of a machine depreciates at rate of 20% per annum. If its value is Rs. 3200, what was the value of the machine 2 years ago?
 (a) 3240 (b) 1620
 (c) 6800 (d) 5000

ANSWER KEY

1	a	8	c	15	b	22	b	29	c
2	a	9	a	16	c	23	a	30	b
3	a	10	b	17	a	24	a	31	d
4	c	11	a	18	c	25	d	32	a
5	a	12	c	19	c	26	b	33	b
6	a	13	c	20	c	27	d	34	b
7	b	14	d	21	d	28	c	35	d

SOLUTIONS

1. $0.05 = \frac{5}{100} = \left(\frac{5}{100} \times 100\right) = 5\%$
2. Let $20 \times x = 25\%$ of 800
 Then, $x = \left(\frac{25}{100} \times 800 \times \frac{1}{20}\right) = 10$
3. 40% of 40% =
 $\frac{40}{100} \times \frac{40}{100} = \frac{16}{100} = 16\%$
4. $9 : 4 = \left(\frac{9}{4} \times 100\right) \% = 225\%$
5. % change = $\frac{60-40}{60} \times 100\% = 33.3\%$
6. Let the old price be Rs. 100.
 $\therefore \frac{\text{change in price}}{\text{old price}} = \frac{25}{100} = \frac{1}{4}$
7. Let the original salary be Rs. 100.
 New salary = Rs. 125
 Decrease on Salary 125 - 100 = 25
 Decrease on 100 = $\left(\frac{25}{125} \times 100\right) \% = 20\%$
8. Let B's income = Rs. 100
 Then, A's income = Rs. 140
 Required % = $\left(\frac{40}{140} \times 100\right) \% = 28\frac{4}{7}\%$
9. Let B's height = 100 units
 Then, A's height = 50 units

$$\text{Required \%} = \left(\frac{50}{50} \times 100\right) \% = 100\%$$

10. A = 20% of B; B = 25% of C
 $A = \left(\frac{20}{100} \times \frac{25}{100} \times 100\right) \% \text{ of C} = 5\% \text{ of C}$

11. Total rate of increase in population per annum = 10%
 Initially, let the population be 1000.
 Population after 3 year
 $= 1000 \left(1 + \frac{10}{100}\right)^3 = 1331$
 Increase % = $\left(\frac{1331}{1000} \times 100\right) \% = 33.1\%$

12. $n(A) = 32, n(B) = 40, n(A \cap B) = 22$

So, $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 32 + 40 - 22 = 50$

Percentage failed in either or both the subjects = 50

Hence, percentage of pass
 $= (100 - 50) \% = 50\%$

13. $15 + \frac{1}{3}(n - 20) = 50\% \text{ of } n = \frac{n}{2}$
 $\Leftrightarrow 90 + 2n - 40 = 3n \Leftrightarrow n = 50.$

14. Let $B + M + D = x$. Then, $B = 25\%$ of $x - 20$

$$= \left(\frac{25}{100}x - 20\right) = \left(\frac{x}{4} - 20\right) \text{ and } D = 50.$$

$$\frac{x}{4} - 20 + M + 50 = x \text{ or } M = \left(\frac{3x}{4} - 30\right).$$

So, marks in Maths cannot be determined.

15. Let the total number of votes polled be x .

Thus, votes polled by other candidate = $(100 - 76)\%$ of $x = 24\%$ of x

$$\therefore 76\% \text{ of } x - 24\% \text{ of } x = 468$$

$$\Rightarrow 52\% \text{ of } x = 468 \Rightarrow \frac{52}{100} \times x = 468$$

$$\Rightarrow x = \frac{468 \times 100}{52} = 900$$

16. Let the number of persons eligible to vote be x .

Then, number of eligible persons between 18 and 25 = 15% of x

Number of persons between 18 and 25, who Voted = 75% of 15% of x

$$= \left(\frac{75}{100} \times \frac{15}{100} \times x\right) = \frac{9x}{80}$$

\therefore Required percentage

$$= \left(\frac{9x}{80} \times \frac{1}{x} \times 100\right)\% = 11.25\%$$

17. Let the total marks be x .

Then, $(30\% \text{ of } x) + 15$

$$= (40\% \text{ of } x) - 35$$

$$\Leftrightarrow \frac{30}{100}x + 15 = \frac{40}{100}x - 35$$

$$\Leftrightarrow \frac{1}{10}x = 50 \Leftrightarrow x = 500$$

So, passing marks = $(30\% \text{ of } 500) + 15$

$$= \left(\frac{30}{100} \times 500 + 15\right) = 165.$$

Pass percentage = $\left(\frac{165}{500} \times 100\right)\% = 33\%$

18. Let the price of a chair be Rs. x .

Then, price of a table = Rs. $(x + 400)$.

So, $6(x + 400) + 6x = 4800$

$$\Leftrightarrow 12x = 2400 \Leftrightarrow x = 200$$

Price of a table = Rs. 600

Price of a chair = Rs. 200.

Required percentage =

$$\left(\frac{400}{600} \times 100\right)\% = 66\frac{2}{3}\%$$

19. Let the total sales be Rs. x .

Then, $5\frac{1}{2}\%$ of $x + \frac{1}{2}\%$ of $(x - 10000)$

$$= 1990$$

$$\Leftrightarrow \frac{11}{2} \times \frac{1}{100} \times x + \frac{1}{2} \times \frac{1}{100} \times (x - 10000)$$

$$= 1990$$

$$\Leftrightarrow 12x - 10000 = 398000$$

$$\Leftrightarrow 12x = 408000$$

$$\Leftrightarrow x = 34000.$$

$$20. \% \text{ excess} = \frac{40}{100-40} \times 100\% = 66\frac{2}{3}\%$$

$$21. \text{ Saving} = [100 - (30 + 12 + 28)]\% = 30\%$$

$$\therefore 30\% \text{ of } x = 810$$

$$\Rightarrow \frac{30x}{100} = 810$$

$$\Rightarrow x = \frac{810 \times 100}{30} = \text{Rs. } 2700$$

22. Let the original price be Rs. 100.

New final price = 95% of $(110\%$ of Rs. 100)

$$= \text{Rs. } \frac{95}{100} \times \frac{110}{100} \times 100$$

$$= \text{Rs. } 104.5$$

\therefore Increase in price = 4.5%

23. Let the original number be x .

Final number obtained = 125% of $(75\%$ of x)

$$= \frac{125}{100} \times \frac{75}{100} \times x = \frac{15x}{16}$$

$$= x - \frac{15x}{16} = 8$$

$$= x = 128$$

24. Let the larger number be x ,

$$\text{Then, } x - 20 = \frac{20}{100}x \Leftrightarrow x - \frac{1}{5}x = 20$$

$$\Leftrightarrow \frac{4}{5}x = 20 \Leftrightarrow x = \left\{20 \times \frac{5}{4}\right\} = 25$$

25. 5% of $A + 4\%$ of $B = \frac{2}{3}$ (6% of $A + 8\%$ of B)

$$\Leftrightarrow \frac{5}{100}A + \frac{4}{100}B = \frac{2}{3} \left(\frac{6}{100}A + \frac{8}{100}B\right)$$

$$\Leftrightarrow \frac{1}{20}A + \frac{1}{25}B = \frac{1}{25}A + \frac{4}{75}B$$

$$\Leftrightarrow \frac{1}{100}A = \frac{1}{75}B \Leftrightarrow \frac{A}{B} = \frac{100}{75} = \frac{4}{3}$$

26. Let the number be x . Then, Error

$$= \frac{5}{3}x - \frac{3}{5}x = \frac{16}{15}x$$

$$\text{Error \%} = \left(\frac{16x}{15} \times \frac{3}{5x} \times 100\right)\% = 64\%$$

27. Let the original value of the tempo be Rs. x .

Then, 1.3% of $\frac{4}{5}$ of $x = 910$

$$\Leftrightarrow \frac{13}{10} \times \frac{1}{100} \times \frac{4}{5} \times x = 910$$

$$\Leftrightarrow x = \left(\frac{910 \times 100 \times 100 \times 5}{13 \times 4}\right) = 87500.$$

28. Let the total production be x lakh tons.

Then,

15% of $x - 10\%$ of $x = (40 - 30)$ lakh tons

$$\Leftrightarrow 5\% \text{ of } x = 10 \text{ lakh tons}$$

$$\Leftrightarrow x = \left(\frac{10 \times 100}{5}\right) = 200 \text{ lakh tons.}$$

29. We have : $x + x\%$ of $150 = 150$

$$\Leftrightarrow x + \frac{x}{100} \times 150 = 150$$

$$\Leftrightarrow \frac{5}{2}x = 150$$

$$\Leftrightarrow x = \left(\frac{150 \times 2}{5}\right) = 60$$

30. Let the original price be RS. x per kg

Reduced price = RS. $\frac{4x}{5}$ per kg

$$\therefore \frac{100}{\frac{4x}{5}} - \frac{100}{x} = 2.5$$

$$\Rightarrow \frac{500}{4x} - \frac{100}{x} = \frac{25}{10}$$

$$\Rightarrow \frac{100}{4x} = \frac{25}{10}$$

$$\Rightarrow x = \frac{100 \times 10}{4 \times 25} = 10$$

Reduced price = RS. $\left(\frac{4 \times 10}{5}\right)$ per kg = 8

31. Increase % = $\left\{\frac{r}{100-r} \times 100\right\}\%$

$$= \left\{\frac{20}{100-20} \times 100\right\}\%$$

$$= \left\{\frac{20}{80} \times 100\right\}\% = 25\%$$

32. (a) 329.17

$$0.24 \times x + (0.28 \times 500) = 219$$

$$X = \frac{219 - (0.28 \times 500)}{0.24} = 329.17$$

33. (b) 9409

Value of mobile after 2 years

$$= 10,000 \times \left(1 - \left(\frac{3}{100}\right)\right)^2$$

$$= 10000 \times \left(\frac{97}{100}\right)^2$$

$$= 9409$$

34. Ans: (b) 50

It depreciates at the rate of R% per annum then value of the fan after n years =

$$P \left[1 - \left(\frac{R}{100}\right)\right]^n$$

$$2000 = \frac{500}{\left(1 - \frac{r}{100}\right)^2}$$

$$\left(1 - \frac{r}{100}\right)^2 = \frac{1}{4}$$

$$\left(1 - \frac{r}{100}\right) = \frac{1}{2}$$

$$\frac{100-r}{100} = \frac{1}{2}$$

$$R = 50\% \text{ (depreciation)}$$

35. Let the present value of a machine be P. suppose it depreciates at the rate of R% per annum.

Then, Value of the machines n years ago

$$= \left(\frac{P}{1 - \left(\frac{R}{100}\right)^n}\right)$$

R%=20%; present value=3200; n=2

Value of a machine 2 years ago

$$= \text{Rs} \left[\frac{3200}{1 - \left(\frac{20}{100}\right)^2} \right] = \frac{3200}{\left(\frac{80}{100}\right)^2} = 5000$$