8.AVERAGES

Average

The average of a given number of quantities of the same kind is expressed as

$$\text{Average} = \frac{\textit{Sum of the quantities}}{\textit{Number of quantities}}$$

Average is also called the Arithmetic Mean.

Sum of the quantities = Average x Number of quantities

$$\textbf{Number of quantities} = \frac{\textit{Sum of the quantities}}{\textit{Average}}$$

• If all the given quantities have the same value, then the number itself is the average.

• If all the given quantities are not the same, then the average of the given quantities is always greater than the smallest number and less than the largest number

• If each of the given quantities is increased by a constant p, then their average is also increased by P.

• If each of the given quantities is decreased by a constant p, then their average is also decreased by P.

• If each of the given quantities is multiplied by a constant p, then their average is also multiplied by P.

• Whenever the given quantities form an arithmetic sequence and if the given quantities have odd terms, then the average is the middle term in the sequence and if the given quantities have even terms, then the average of the sequence is the average of the middle two terms.

•In order to calculate the weighted average of a set of numbers, multiply each number in the set by the number of times it appears, add all the products and divide by the total number of numbers in the set.

• If the speed of an object from A to B is x km/h and from B to A is y km/h, then the average speed during the whole journey is $\frac{2xy}{x+y}$ km/h

• If the average of N_1 quantities is x and N_2 quantities is y then the average of total $(N_1 + N_2)$ quantities is given by $\frac{N_1 x + N_2 y}{N_1 + N_2}$

Example 1: What is the average of first five even numbers

Solution :. The first five numbers are 2, 4, 6, 8, 10

$$\textbf{Average} = \frac{2+4+6+8+10}{5} = \frac{30}{5} = 6$$

Example 2: The average of five consecutive even numbers is 50. What is the largest of these numbers?

Solution:: Let the numbers be x-4, x -2, x, $x + 2$, $x + 4$.

$$\text{Average} = \frac{\textit{Sum of the quantities}}{\textit{Number of quantities}}$$

$$= \frac{x - 4 + x - 2 + x + x + 2 + x + 4}{5} = 50$$

$$\frac{5x}{5} = 50$$

$$x = 50$$

∴Largest Number $= x + 4 = 50 + 4 = 54$

Example 3: Average weight of 32 students of a class is 30.5 kg. If weight of a teacher is also included then average weight is increased by 500 g. What is the weight of the teacher?

Solution : Total weight of 32 students = 30.5×32 = 976 kg

Average weight of (32 students + 1 teacher) = (30.5 + 0.5) = 31 kg

∴. Total weight of (32 students + 1 teacher) = 31 × 33 = 1023 kg

Weight of teacher = (1023 - 976) kg = 47 kg

Example 4: The average salary per head of all the employees of an institution is Rs.60. The average salary per head of 12 officers is Rs.400 and average salary per head of the rest is Rs.56. Find the total number of employees in the institution.

Solution: Let the total number of employees be x.

Then $\qquad 60 = \dfrac{Total\ salary\ of\ all\ employees}{x}$

$60 = \dfrac{12 \times 400 + (x - 12) \times 56}{X}$

$\Rightarrow \mathbf{60x = 12 \times 400 + (x - 12) \times 56 = 4800 + 56x - 672}$

$\Rightarrow \mathbf{60x - 56x = 4800 - 672}$

$\Rightarrow \mathbf{4x = 4128 \Rightarrow x = 1032}$

Hence, the total number of employees is 1032.

Example 5: If the average of p and q is 58 and the average of q and s is 64, what is the value of s-p?

solution : Given $\dfrac{p+q}{2} = 58 \Rightarrow$ p+q = 116 …(i)

Also $\quad \dfrac{q+s}{2} = 64 \Rightarrow$ q+s = 128 …(ii)

Subtracting Eq. (i) from (ii), we get

(q + s) - (p + q) =128 - 116 => s - p = 12

Example 6: 12 men went to a restaurant. 11 of them spent Rs.5 each and the 12th person spent Rs.11 more than the average expenditure of all. Find the total money spent by them?

Solution: Let the average money spent by the 12 men = Rs. x

Money spent by the 12th man = Rs. (x + 11)

Money spent by the other 11 men = Rs. (11×5) = Rs. 55

Total money spent by 12 men = Rs. (55+x + 11) = Rs. (x + 66)

$\therefore$ x = $\dfrac{x+66}{12} \Rightarrow 12x = x+66 \Rightarrow 11x = 66$

$\Rightarrow x =$Rs.6

Total money spent by 12 men =6×12 = Rs. 72.

EXERCISE

1. Find the average of the first 100 natural numbers
   - (a) 50.5
   - (b) 52
   - (c) 51.5
   - (d) 53

2. Find the mean of the first 25 multiples of 10.
   - (a) 150
   - (b) 125
   - (c) 120
   - (d) 130

3. Find the average weight of 8 students 49 kg,47 kg, 46 kg, 42 kg, 39 kg, 48 kg, 50 kg and 43 kg.
   - (a) 44 kg
   - (b) 42.5 kg
   - (c) 47 kg
   - (d) 45.5 kg

4. The average weight of 20 students in a class is decreased by 500 g, when one boy whose weight is 45 kg is replaced by a new boy. What is the weight of the new boy ?
   - (a) 35 kg
   - (b) 32 kg
   - (c) 36 kg
   - (d) 34 kg

5. A batsman has a certain average of runs for
   16 innings. In the 17th innings, he makes a score of 85 runs , thereby increasing his average by 3 . What is the average after the 17th innings?
   - (a) 60
   - (b) 37

   - (c) 38
   - (d) 27

6. The average marks obtained by 80 students in a certain examination is 28. If the average marks of the students who have passed is 40 and that of those who have failed is 25, how many students failed the examination ?
   - (a) 64
   - (b) 24
   - (c) 16
   - (d) 38

7. The average of 100 numbers is 50. It is found that while calculating the average , two numbers, namely 81 and 66 were wrongly read as 18 and 6. The correct average is
   - (a) 51.23
   - (b) 41.45
   - (c) 36.42
   - (d) 52.46

8. The average age of 25 students in a class is 10 yr. If the teacher's age is included, the average increase by 1 yr. The age of the teacher is
   - (a) 33 yr
   - (b) 42 yr
   - (c) 36 yr
   - (d) 45 yr

9. The average monthly salary of 11 workers and one officer in an organization is 600. When the officer whose salary was rs 1600 per month retired, a new officer was appointed and the average salary of the 12

employees is Rs. 570 per month. The salary of the new officer is?

(a) Rs. 1260   (b) Rs. 1240

(c) Rs. 1220   (d) Rs. 1280

10. A person travels 120 km in 6 h, 130 km in 5 h and 200 km in 4 h. Find his average speed during the whole journey

(a) 36 km/h   (b) 35 km/h

(c) 30 km/h   (d) 32 km/h

11. The average age of students in section *A* of 40 students is 10 yr and the average age of students in section *B* of 30 students is 12 yr. Find the average age of students in both sections taken together.

(a) 10 yr   (b) 11 yr

(c) 10.86 yr   (d) 11.32 yr

12. If average of *a, b, c* is *m* and $ab + bc + ca = 0$, then average of a^2, b^2, c^2 is

(a) m^2 (b) 3 m^2

(c) 6m^2 (d) 9 m^2

13. The average weight of 8 persons increases by 2.5 kg when a new person comes in place of one of them weighing 65 kg. What might be the weight of the new person?

(a) 82 kg   (b) 85 kg

(c) 76.5 kg   (d) 80 kg

14. The average marks fetched by Mohan in History, Geography, Science and Mathematics is 10 more than the marks fetched in Mathematics . If he has got 110 marks aggregate in History and Geography, what will be the aggregate marks fetched in Science and Mathematics?

(a) 90   (b) 70

(c) 75   (d) can't be determined

15. In a journey of 160 km, a train covers the distance 120 km at a speed of 80 km/h and the remaining distance at 40 km/h. The average speed of the train for the whole journey is:

(a) 50 km/h   (b) 64 km/h

(c) 68 km/h   (d) 74 km/h

16. A man travels from destination A to B by car at an average speed of 48 km/h and returns on his bike with an average speed of 16 km/h. Find his average speed for the entire journey

(a) 36 km/h   (b) 24 km/h

(c) 32 km/h   (d) 21 km/h

17. Jittu scores 60, 80, 40, 50 and 90 percent marks in the subjects A, B, C, D and E respectively. However, the weights attached to those subjects are 5, 4, 3, 2 and 1 respectively, Which of the following represents the weighted average of marks scored by Jittu in these five subjects

(a) 75   (b) 78

(c) 62   (d) 79

18. The average of 10 numbers is 7. If each number is multiplied by 12, then the average of new set of numbers is

(a) 72   (b) 86

(c) 79   (d) 84

19. In Arun's opinion his weight is greater than 65 kg but less than 72 kg. His brother does not agree with Arun and he thinks that Arun's weight is greater than 60 kg but less than 70 kg. His mother's view is that his weight cannot be greater than 68 kg. If all of them are correct in their estimation, what is the average of different probable weights of Arun?

(a) 71 kg   (b) 67 kg

(c) 73 kg   (d) 58 kg

20. Find the average weight of four containers, if it is known that the weight of the first container is 100 kg and the total of the second, third and fourth containers' weight is defined by $f(x) = x^2 - [3/4\,(x^2)]$, where $x = 100$.

(a) 650 kg   (b) 880 kg

(c) 760 kg   (d) 460 kg

21. 19 persons went to a hotel for a combined dinner party. 13 of them spent rs.79 each on their dinner and the rest spent Rs4 more than the average expenditure of all the 19. What was the total money spent by them?

(a) rs.1628.4   (b) rs.1536

(c) rs.1492   (d) rs.1632

22. One-half of a certain distance is covered at 40 km/h, one-third of it at 80 km/h and the rest at 120 km/h. Find the average speed for the whole journey.

(a) $51\frac{1}{13}$ km/h   (b) $55\frac{5}{13}$ km/h

(c) $52\frac{3}{13}$ km/h   (d) $56\frac{2}{13}$ km/h

23. What is the average of all number from 1 to 100 that end in 3?

(a) 49   (b) 46

(c) 48   (d) 47

24. Average of two numbers is 14.5 and square root of their product is 10. What are the numbers?
(a) 16 and 9   (b) 25 and 4
(c) 4 and 16   (d) 25 and 9

25. The mean yearly salary of an employee of company was Rs. 20000. The mean yearly salaries of male and female employees were Rs. 20800 and Rs. 16800 respectively. Find the ratio of males to females employed by the company?
(a) 3 :2   (b) 4:1
(c) 2 :1   (d) 5 :3

26. Twelve years ago, the average age of a husband and a wife was 20 yr. The average remains the same today, when they have two children . What is the present age of the youngest child, if they differ in age by 2 yr ?
(a) 12 yr   (b) 9yr
(c) 11 yr   (d) 7 yr

27. A Student on his birthday distributed on an average 5 chocolates per student. If on the arrival of the teacher and the headmaster to whom the student gives 10 and 15 chocolates respectively, the average chocolate distributed per head increase to 5.5, what is the strength of the class?

(a) 28   (b) 30
(c) 32   (d) None of these

ANSWER KEY									
1	a	7	a	13	b	19	b	25	b
2	d	8	c	14	d	20	a	26	d
3	d	9	b	15	b	21	b	27	a
4	a	10	c	16	b	22	b		
5	b	11	c	17	c	23	c		
6	a	12	b	18	d	24	b		

SOLUTIONS

1. Sum of the first n natural numbers
$$= \frac{n(n+1)}{2}$$
Sum of the first 100 natural numbers =
$$\frac{100 \times (100+1)}{2} = 5050$$
Average of first 100 natural numbers
$$= \frac{5050}{100} = 50.5$$

2. Sum of the first 25 multiples of 10
$$= 10 + 20 + 30 + ...250$$
$$= 10 [1 + 2 + 3 + ... +25]$$
$$= \frac{10 \times 25 \times (25+1)}{2} = \frac{10 \times 25 \times 26}{2} = 3250$$
Required average $= \frac{3250}{25} = 130$

3. Average weight
$$= \frac{49+47+46+42+39+48+50+43}{8}$$
$$= \frac{364}{8} = 45.5 \text{ kg}$$

4. Total decrease $= (20 \times \frac{1}{2})$ kg $= 10$kg
Weight of new man $= (45-10)$ kg $= 35$kg

5. Let the average score be x runs, then by given condition total score of 16 innings $= 16x$.
$$\frac{16x+85}{17} = (x + 3) => x = 34$$
∴ Average after 17th innings
$$= 34 + 3 = 37 .$$

6. Let the number of students who passed
$$= x$$
Then $40x + (80 - x) \times 25 = 80 \times 25$
$$\Rightarrow 40x - 25x + 25 \times 80 = 80 \times 28$$
$$\Rightarrow x = \frac{2240-2000}{15} = \frac{240}{15} = 16$$
Hence the number of students who failed
$$= (80 - 16) = 64$$

7. Sum of the numbers
$$= (100 \times 50 - 18 - 6 + 81 + 66)$$
$$= 5123$$
$\Rightarrow$Correct average $= \frac{5126}{100} = 51.23$

8. Sum of the ages of 25 students
$$= 25 \times 10 = 250 \text{ years}$$

Let the age of the teacher = x yrs

Then, $(10+1) = \frac{(250+x)}{(25+1)}$

$\Rightarrow 250 + x = (10 + 1)(25 + 1)$

$= 11 \times 26$

$\Rightarrow x = 11 \times 26 - 250$

$= 36\ years$

9. Let the average monthly salary of each worker = $Rs.\ x$

Total salary of 11 workers and 1 officer

$= 11x + 1600$

Then, $600 = \frac{11x + 1600}{12}$

$\Rightarrow 11x = 600 \times 12 - 1600$

$= 5600$

Let the salary of the new officer be Rs. y.

Then, $570 = \frac{11x + y}{12}$

$\Rightarrow 11x + y = 570 \times 12$

$\Rightarrow y = 570 \times 12 - 11x$

$\Rightarrow y = 570 \times 12 - 5600 = 1240$

$\therefore$ Hence the salary of the new officer

$= Rs.\ 1240.$

10. Total distance travelled = 120 + 130 + 200= 450 km

Total time of travel = 6 + 5 + 4 = 15 h

Average speed = $\frac{450}{15}$ = 30 km/ph

11. Required average = $\frac{40 \times 10 + 30 \times 12}{40 + 30}$ = 10.86 years

12.

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$= a^2 + b^2 + c^2$

But $\frac{a+b+c}{3} = m \Rightarrow a+b+c = 3m$

$\therefore \frac{a^2+b^2+c^2}{3} = \frac{(a+b+c)^2}{3} = \frac{9}{3} m^2 = 3m^2$

13. By shortcut: The weight of the new person

$= 8 \times 2.5 + 65 = 85\ kg$

14. By given condition,

$\frac{History + Geography + Science + Maths}{4}$

$=10 + Maths$

$\Rightarrow$ (History + Geography) + Science - 3 Maths = 40

$\Rightarrow 110 + Science - 3\ Maths = 40$

$\Rightarrow 3\ Maths - Science = 70$

Since there is no relation given between Maths and Science the question cannot be solved.

15. Average speed =

$$\frac{Total\ distance\ travelled}{Total\ time\ taken}$$

$= \frac{160}{\frac{120}{80} + \frac{40}{40}} = \frac{16}{2.5} = 160 \times \frac{2}{5}$

$= 64\ km/h$

16. Average speed $= (\frac{2xy}{x+y})$km/hr

$= (\frac{2 \times 48 \times 16}{48 + 16})$km/hr

$= 24$ km/hr

17. Weighted average =

$\frac{(60 \times 5) + (80 \times 4) + (40 \times 3) + (50 \times 2) + (90 \times 1)}{5 + 4 + 3 + 2 + 1}$

$= \frac{930}{15} = 62$ marks

18. Required Answer = $12 \times 7 = 84$

19. According to Arun, his weigh x kg is such that

$65 < x < 72$

According to his brother,

$60 < x < 70$

According to his mother, x $\leq$ 68

From (i), (ii) and (iii), we get

$65 < x \leq 68$ i.e.,

$x = 66, 67\ or\ 68$

Average $= \frac{66+67+68}{3} = 67$ kg

20. Sum $= 100 + (100)^3 - \frac{3}{4}(100)^2 =$

$100 + (100)^2[1 - \frac{3}{4}]$

$= 100 + 10000 \times \frac{1}{4} = 2600$

Avg. $= \frac{2600}{4} = 650$ kg

21. Let the average expenditure of all be Rs. x, then by given condition

$19x = 13\ x\ 79 + (x + 4)6$

$\Rightarrow 13x = 1051 \Rightarrow x = \frac{1051}{13}$

$= 80.846$

$\therefore$ Total money spent by them

$= 19 \times 80.846$

$= Rs.\ 1536\ (approx)$

22. Let total journey = x km

Total, time taken

$= (\frac{x}{2 \times 40} \div \frac{x}{3 \times 80} + \frac{x}{6 \times 120})$h

$= (\frac{x}{40} + \frac{x}{240} + \frac{x}{720})$h $= (\frac{13x}{720})$ h

Average speed

$= x \times \frac{720}{13x} = \frac{720}{13} = 55\frac{5}{13}$ km/h

23. Numbers between 1 and 100 which end in 3 are 3, 13, 23, 33, 43, 53, 63, 73, 83, and 93.

Sum of the numbers = 3 + 13 + 23 + ... + 93

This is an *AP* with $a = 3$, $l = 93$ and

d =10

$$S_n = \frac{n}{2}[\,a + l\,] = \frac{10}{2}[\,3 + 93\,] = 480$$

Hence, required average $= \frac{480}{10} = 48$

24. Let the number be a and b. Then,

$$\frac{a + b}{2} = 14.5$$
$$\Rightarrow a + b = 29$$
$$\Rightarrow \sqrt{ab} = 10$$
$$\Rightarrow ab = 100$$
$$(a - b)^2 = (a + b)^2 - 4ab$$
$$= (29)^2 - (4 \times 100) = 441$$
$$\Rightarrow (a - b) = 21$$

On solving $a + b = 29$ and $a - b = 21$, we get $a = 25$ and b = 4.

25. Let the number of males be x and the number of females be y.

Sum of the salaries of the men
$$= 20800x$$

Sum of the salaries of the females
$$= 16800y$$

Now, $20000 = \frac{20800x + 16800y}{x+y}$

$\Rightarrow 20000\,(x + y)$
$$= 20800x + 16800y$$

$$\Rightarrow 800x = 3200y$$
$$\Rightarrow \frac{x}{y} = \frac{4}{1} \Rightarrow x:y = 4:1$$

Method: II

Options to question.

26. Sum of the ages of husband and wife 12 year ago $= (2 \times 20)$ year = 40 years

Present sum of the ages of husband and wife $= (40 + 2 \times 12)$ year = 64 years

Let the age o f the two children be x and $x + 2$

Now, $20 = \frac{64+x+x+2}{4} = \frac{66+2x}{4}$

$$\Rightarrow 66 + 2x = 80 \Rightarrow 2x = 14$$
$$\Rightarrow x = 7$$

Hence, the age of the younger child is 7 years.

27. Let x be the total strength of class.

Old average chocolate per student = 5, Total chocolate $= 5x$

By given condition:

$$\frac{5x+2(12.5)}{x+2} = 5.5$$
$$(5x + 25) = 5.5x + 11$$
$$0.5x = 14$$
$$x = 28$$