5. HCF & LCM

Highest Common Factor (HCF):

HCF of two or more numbers is the greatest number (divisor) that divides all the given numbers exactly. So, HCF is also called the **GREATEST COMMON DIVISOR (GCD)**.

HCF of two or more than two numbers is the product of the least powers of all the prime factors that occur in the numbers.

Lowest Common Multiple (LCM):

LCM is the least dividend which is exactly divisible by the given numbers.

LCM of two or more than two numbers is the product of the highest powers of all the prime factors that occur in the numbers.

LCM of a number will always be divisible by HCF

Product of two numbers:

If 'A' and 'B' are two numbers and their HCF and LCM are 'C' and 'D' respectively. So, the product of those two numbers is the product of HCF and LCM.

HCF of numbers × **LCM of numbers** = **Product of numbers**

S. No	Type of problem	Approach to the problem
1.	Find the GREATEST NUMBER that	Required number = HCF of given numbers
	will exactly divide given numbers.	(greatest divisor)
2.	Find the GREATEST NUMBER that	Required number (greatest divisor)
	will exactly divide x, y and z leaving	$= HCF\{(x-a), (y-b) \text{ and } (z-c)\}$
	remainders a, b and c respectively.	
3.	Find the LEAST NUMBER which is	Required number $=$ LCM of x, y and z (least
	exactly divisible by x, y and z. \longrightarrow	dividend)
4.	Find the LEAST NUMBER which when	It is always observed that $(x-a) = (y-b) = (z-c)$
	divided by x, y and z leaves the	= k (say)
	remainders a, b and c respectively.	\therefore Required number=(LCM of x, y and z) - k
5.	Find the LEAST NUMBER which when	Required number
	divided by x, y and z leaves the same	= (LCM of x, y and z) + r
	remainder 'r' in each case.	
6.	Find the GREATEST NUMBER that	Required number
	will exactly divide x, y and z leaves same	= HCF of (x-r), (y-r) and (z-r)
	remainder 'r' in each case.	
7.	HCF of fractions	HCF of fractions = $\frac{HCF of numerators}{1000 \text{ GeV}}$
0	LCM of fractions	LCM of denominators
0.	LCM of fractions	LCM of fractions = $\frac{1}{HCF of denominators}$
9.	HCF of decimal numbers	Step 1. Find HCF of given numbers without
-		decimals
4		Step 2. In the HCF make decimal point from
		right to left according to the maximum
		decimal places among the given numbers.
10.	LCM of decimal numbers	Step 1. Find LCM of given numbers without
		decimals
		Step 2. In the LCM make decimal point from
		right to left according to the maximum
		decimal places among the given numbers.

I

	EXER	CISE			
1.	Find the LCM of $\overline{2^3 \times 3^4 \times 5^2}$, $2^2 \times 3^3 \times 7$,		(a) 3		(b) 7
	$5^2 \times 7^2$.		(c) 1		(d) 0
	(a) $2^3 \times 3^4 \times 5^2 \times 7^2$ (b) $2^3 \times 5^2 \times 72$	13.	Find the g	reatest 1	number which will divide
	(c) $3^4 \times 5^2 \times 7^2$ (d) $2^3 \times 7^2 \times 5$		321,428 at	nd 535 e	exactly.
2.	Find the HCF of $2^2 \times 3^3 \times 5^2$, $2^3 \times 3^2 \times 5$, $5^2 \times 7$.		(a) 105		(b) 107
	(a) 5 (b) $2^3 \times 3^3 \times 5$		(c) 109		(d) 102
	(c) $2^3 \times 3^3 \times 7$ (d) $3^2 \times 5^2 \times 7$	14.	Find the	greatest	number that will divide
3.	Find the LCM and HCF of 0.25, 0.5, 0.75.		640,710 a	nd 1526	5 so as to leave 11, 7 and
	(a) 1.5, 0.25 (b) 2, 1		9 as remai	inders re	espectively.
	(c) 1.5, 0.5 (d) 3, 1		(a) 36		(b) 37
4.	Find the HCF of 120, 150,180.		(c) 42		(d) 29
	(a) 30 (b) 60	15.	Find the le	east nun	nber which when divided
	(c) 50 (d) 10		by 16,18	and 20	leaves a remainder 4 in
5.	Find the LCM of $\frac{4}{3}$, $\frac{8}{9}$, $\frac{3}{5}$		each case,	but is c	ompletely divisible by 7.
	(a) 20 (b) 24 (c) $\frac{1}{2}$ (d) $\frac{1}{2}$		(a) 465 🍙		(b) 3234
	(a) 20 (b) 24 (c) 24 (a) 20 1 3		(c) 2884		(d) 3234
6.	Find the HCF of $\frac{1}{2}$ and $\frac{3}{2}$	16.	The least	number	which when divided by
	(a) 3/2 (b) 1/2		4, 6, 8, 1	2 and 1	6 leaves a remainder of 2
	(c) 1 (d) 3	A	in each ca	se is:	
7.	Find out the LCM of 4^5 , 4^{-81} , 4^{12} and 4^7 .		(a) 20		(b) 43
	(a) 4^5 (b) 4^{-81}		(c) 50	1 /	(d) 59
	(c) 4^{12} (d) 4^7	17.	Find the	least	number which when
8.	Find the LCM of 5/2,8/9,11/14.	9	increased	by 3	is exactly divisible by
	(a) 290 (b) 380		10,12,14	and 16.	(h) 1(77)
	(c) 420 (d) 440		(a) 1000		(0) 1077 (d) 1670
9.	The HCF and LCM of two numbers are 18	18	(C) 1097 Find the	laast	(u) 1070 number which when
	and 3780 respectively. If one of them is	10.	decreased	by A	is exactly divisible by
	540, then the second one is:		9 12 15 ar	od 18	is exactly divisible by
	(a) 146 (b) 126		(a) 188	iu 10.	(b) 182
10	(c) 118 (d) 117		(a) 100		(d) 184
10.	The HCF of two numbers is 8. Which one	19.	Find the s	mallest	four digit number that is
	(a) 22 (b) 24	•	exactly div	visible t	by 8,10 and 12.
	(a) 52 (b) 24 (d) 60		(a) 1080		(b) 1100
11	(C) 48 (d) 00		(c) 1050		(d) 1120
11,	HCE Sum of LCM and HCE is 195. If one	20.	Five bells	bE.g.in	to toll together and toll
	of them is 60. Find the other		at interval	ls of 24	4, 40, 64, 72 and 120 s.
	(a) 48 (b) 45		After what	at interv	al of time will they toll
	(a) + 3 $(b) + 3(c) 52 (d) - 36$		again toge	ether?	
12	The HCE and I CM of a pair of numbers		(a) 42min		(b) 36min
14,	are		(c) 48min	1	(d) 54min
	12 and 926 respectively. How many such	21.	Find the	least n	umber which is exactly
	distinct pairs are possible?		divisible b	oy 12,15	,20 and 27.
			(a) 650		(b) 520

(c) 600 (d) 540

- 22. A number when divided by 225 gives a remainder of 32. What will be the remainder when the same number is divided by 15?
 - (a) 4 (b) 2
 - (c) 3 (d) 1
- 23. Five bells bE.g.in to toil together and toll respectively at intervals of 6, 7, 8, 9 and 12 sec. How many times they will toll together in one hour, excluding the one at the start?
 - (a) 3 (b) 5 (c) 7(d) 9
- 24. What is the smallest whole number that is exactly divisible by $1\frac{5}{28}$, $2\frac{2}{11}$ and $3\frac{1}{7}$? (b) 130 (a) 264
 - (c) 138 (d) 124
- 25. The least positive intE.g.er which leaves a remainder 2, when divided by each of the numbers 4,6,8,12 and 16.
 - (a) 46 (b) 48
 - (c) 50 (d) 52
- 26. HCF of two numbers is 12 and their product is 3600. How many such pairs of numbers can be formed?
 - (a) 0(b) 1 (d) 4 (c) 2
- 27. The sum of two numbers is 135 and their HCF is 9. How many such pairs of numbers can be formed? (b) 2

(a) 6

(c) 5

28. The LCM of two numbers is 280 and the ratio of the numbers is 7 : 8. Find the numbers.

(d) 4

- (a) 70 and 48 (b) 42 and 48
- (c) 35 and 40 (d) 28 and 32
- **29.** A florist has 200 roses and 180 jasmines with him. He was asked to make garlands of flowers with only roses or only jasmines each containing the same number of flowers. What will be the largest number of flowers, he can join together without leaving a single flower?
 - (a) 16 (b) 17
 - (d) 19 (c) 20
- **30.** What is the largest number which when divides 1475, 3155 and 5255 leaves the same remainder in each case?
 - (a) 220 (b) 420 (c) 350 (d) 540
- **31.** Find the side of the largest possible square slabs which can be paved on the floor of a room 2m 50 cm long and 1 m 50 cm broad. Also find the number of such slabs to pave the floor.
 - (a) 40,18 (b) 30,15
 - (c) 50,15 (d) 20,25
- 32. How many numbers less than 10,000 are there which are divisible by 21,35 and 63?
 - (a) 33 (b) 32 (c) 38 (d) 31

Answer Key															
1	a	5	b	9	b	13	b	17	b	21	d	25	с	29	c
2	a	6	b	10	d	14	b	18	d	22	b	26	b	30	b
3	a	7	с	11	b	15	с	19	a	23	c	27	d	31	c
4	a	8	d	12	d	16	c	20	c	24	a	28	с	32	d

SOLUTIONS

- 1. LCM of given numbers = $2^3 \times 3^4 \times 5^2 \times 7^2$ (Take the greatest power of each term)
- **2.** HCF = 5 (Take the least powers of common terms).
- **3.** (a) LCM of 0.25, 0.5 and 0.75 The given numbers can be written as of 0.25, 0.50 and 0.75

Now ignoring the decimals we find LCM of 25, 50, and 75

5

5	25, 50, 75				
5	5, 10, 15				
	1, 2, 3				
$\therefore LCM = 1 \times 2 \times 3 \times 5 \times 5$					
$= 25 \times 6 = 150$					

LCM = 150. Similarly we will find out the HCF for 25, 50 and 75.

Since given numbers are not high, we can follow Prime factorization method to get the HCF easily

$$25 = 5^2$$

- $50 = 2 \times 5^2$
- $75 = 3 \times 5^2$.

As discussed before HCF = $5^2 = 25$. Now we got LCM = 150 and HCF = 25. now after putting decimal places as per given in the question the LCM = 1.5, and HCF = 0.25.

- 4. (a) since given numbers are not high, we follow can Prime factorization method to get *k* the HCF easily
 - $120 = 2^3 \times 3 \times 5$

$$150 = 2 \times 3 \times 5^2$$

$$180 = 2^2 \times 3^2 \times 5$$

In Prime factorization method after converting the numbers into product of prime factors, take the common factors from all the numbers of the least powers.

- Therefore HCF is $2 \times 3 \times 5 = 30$
- 5. LCM of fraction = $\frac{LCM \text{ of } Numerators}{UCE}$ HCF of Denominators 6. HCF of $\frac{1}{2}$, $\frac{3}{2} = \frac{HCF \text{ of } 1 \text{ and } 3}{LCM \text{ of } 2 \text{ and } 2} =$

- **7.** Clearly, $LCM = 4^{12}$
- *LCM of* (5,8,11) 8. = 440HCF of (2,9,14)
- 9. Sol: (b) We know that, $HCF \times LCM = x \times y$ $18 \times 3780 = 540 \times v$ $\frac{18\times3780}{540} = y$ 540 v = 126

Another number is 126.

10. Since HCF of any two numbers is also a factor of their LCM, 60 can never be their LCM

since HCF 8 is not a factor of 60.

11. Let the HCF be x

$$LCM = 12x$$

 $x + 12x = 195$

$$x + 12x - 13$$

 $13x = 195$

$$x = 15$$

: HCF = 15 and LCM =
$$(12)(15) = 180$$

- Other number = $\frac{180 \times 15}{60} = 45$
- **12.** HCF = 12 and LCM = 926. If the numbers be of the form 12a and 12b; then LCM = 12ab, i.e., LCM is always divisible by HCF Clearly in this question 926 is not divisible by 12, so no such pair exists.
- **13.** Required number

= HCF of 321, 428 and 535 = 107

- **14.** HCF [(640 11), (710 7), (1526 9)] =HCF [629, 703, 1517] = 37.
- **15.** LCM (16, 18, 20) = 720

The number required is off the 720 k + 4. where k is a natural number. In order to make it divisible by 7, we put k = 4. Hence the number is

$720 \times 4 + 4 = 2884.$

- **16.** The number = LCM (4, 6, 8, 12, 16) + 2 =50
- **17.** Required number

$$=$$
 (LCM of 10, 12, 14, 16) $-$ 3

$$= 1680 - 3 = 1677$$

18. Required number

$$=$$
 (LCM of 9, 12, 15, 18) + 4

= 180 + 4 = 184**19.** The smallest four digit number exactly divisible by 8, 10 and 12 should also be divisible by the LCM of 8,10 and 12. LCM of 8, 10 and 12= 120 So, the required number = 1080**20.** Required time interval = LCM of 24, 40, 64, 72 and 120 s = 2880 seconds = 48 min **21.** Required number = LCM of 12, 15, 20 and 27 = 540 **22.** M-I: Let the number be 225 + 32 = 257, and after dividing 257 with 15 we will get the remainder as 2. **M-II:** The remainder when 32 is divided by 15 is 2 23. The bells would toll together at LCM 6, 7, 8, 9, 12, which is 504. Number of times, they toll together $=\frac{3600}{504}=7$ **24.** Required number = $LCM[\frac{33}{28}, \frac{24}{21}, \frac{22}{7}] = 264$ 25. The LCM of 4, 6, 8, 12 and 16 is 48. So, required number = 48 + 2 = 50**26.** Let the two numbers be 12x and 12y. $12x \times 12y = 3600$ xy = 25Possible values of x and y are (1,25) (5,5). But (5,5) are not co primes. \therefore Only one pair of numbers can be formed. 27. The two numbers are always the multiples of the HCF \therefore Let the two numbers be 9x and 9y. $9x + 9y = 135 \implies x + y = 15$ Now, the possible values of x and y are (1,14), (2,13), (3,12), (4,11), (5,10), (6,9),(7,8).Now, consider only the co prime pairs. These are (1,14), (2,13), (4,11), (7,8). \therefore 4 pairs of numbers can be formed whose sum is 135 and HCF is 9

28. Let the two numbers be 7x and 8x and LCM is **56***x*. It is given that LCM = 280ie, 56x = 280 and x = 5ie, numbers are 35 and 40. **29.** HCF (200, 180) = 20 **30.** Take the relative differences of the given numbers. (3155-1475), (5255 - 3155), (5255 - 1475) = 1680, 2100, 3780HCF (1680, 2100, 3780) = 420**31.** HCF (250, 150) = 50 cm $\therefore \text{ The number of slabs} = \frac{250 \times 150}{50 \times 50} = 15$ **32.** LCM (21, 35, 63) = 315 The numbers less than 10,000 which are divisible by 315 are given by $\left[\frac{10,000}{315}\right]$ i.e. the integral part when 10,000 is divided by 315 is 31. The required answer is 31

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