## 5. HCF \& LCM

## Highest Common Factor (HCF):

HCF of two or more numbers is the greatest number (divisor) that divides all the given numbers exactly. So, HCF is also called the GREATEST COMMON DIVISOR (GCD).

HCF of two or more than two numbers is the product of the least powers of all the prime factors that occur in the numbers.

## Lowest Common Multiple (LCM):

LCM is the least dividend which is exactly divisible by the given numbers.
LCM of two or more than two numbers is the product of the highest powers of all the prime factors that occur in the numbers.
LCM of a number will always be divisible by HCF

## Product of two numbers:

If ' $A$ ' and ' $B$ ' are two numbers and their HCF and LCM are ' $C$ ' and ' $D$ ' respectively. So, the product of those two numbers is the product of HCF and LCM.
HCF of numbers $\times$ LCM of numbers $=$ Product of numbers

| S. No | Type of problem | Approach to the problem |
| :---: | :---: | :---: |
| 1. | Find the GREATEST NUMBER that will exactly divide given numbers. | Required number $=$ HCF of given numbers (greatest divisor) |
| 2. | Find the GREATEST NUMBER that will exactly divide $\mathrm{x}, \mathrm{y}$ and z leaving remainders $\mathrm{a}, \mathrm{b}$ and c respectively. | Required number (greatest divisor) $=\operatorname{HCF}\{(\mathrm{x}-\mathrm{a}),(\mathrm{y}-\mathrm{b})$ and $(\mathrm{z}-\mathrm{c})\}$ |
| 3. | Find the LEAST NUMBER which is exactly divisible by $\mathrm{x}, \mathrm{y}$ and z . | Required number $=\mathrm{LCM}$ of $\mathrm{x}, \mathrm{y}$ and z (least dividend) |
| 4. | Find the LEAST NUMBER which when divided by $x, y$ and $z$ leaves the remainders $\mathrm{a}, \mathrm{b}$ and c respectively. | $\begin{aligned} & \text { It is always observed that }(\mathrm{x}-\mathrm{a})=(\mathrm{y}-\mathrm{b})=(\mathrm{z}-\mathrm{c}) \\ & =\mathrm{k} \text { (say) } \\ & \therefore \text { Required number }=(\mathrm{LCM} \text { of } \mathrm{x}, \mathrm{y} \text { and } \mathrm{z})-\mathrm{k} \end{aligned}$ |
| 5. | Find the LEAST NUMBER which when divided by x , y and z leaves the same remainder ' r ' in each case. | $\begin{aligned} & \text { Required number } \\ & =(\mathrm{LCM} \text { of } \mathrm{x}, \mathrm{y} \text { and } \mathrm{z})+\mathrm{r} \end{aligned}$ |
| 6. | Find the GREATEST NUMBER that will exactly divide $\mathrm{x}, \mathrm{y}$ and z leaves same remainder ' r ' in each case. | $\begin{aligned} & \text { Required number } \\ & =\mathrm{HCF} \text { of (x-r), (y-r) and (z-r) } \end{aligned}$ |
| 7. | HCF of fractions | $\text { HCF of fractions }=\frac{\text { HCF of numerators }}{\text { LCM of denominators }}$ |
| 8. | LCM of fractions | $\text { LCM of fractions }=\frac{\text { LCM of numerators }}{\text { HCF of denominators }}$ |
| 9. | HCF of decimal numbers | Step 1. Find HCF of given numbers without decimals <br> Step 2. In the HCF make decimal point from right to left according to the maximum decimal places among the given numbers. |
| 10. | LCM of decimal numbers | Step 1. Find LCM of given numbers without decimals <br> Step 2. In the LCM make decimal point from right to left according to the maximum decimal places among the given numbers. |

## EXERCISE

1. Find the LCM of $2^{3} \times 3^{4} \times 5^{2}, 2^{2} \times 3^{3} \times 7$, $5^{2} \times 7^{2}$.
(a) $2^{3} \times 3^{4} \times 5^{2} \times 7^{2}$
(b) $2^{3} \times 5^{2} \times 72$
(c) $3^{4} \times 5^{2} \times 7^{2}$
(d) $2^{3} \times 7^{2} \times 5$
2. Find the HCF of $2^{2} \times 3^{3} \times 5^{2}, 2^{3} \times 3^{2} \times 5,5^{2} \times 7$.
(a) 5
(b) $2^{3} \times 3^{3} \times 5$
(c) $2^{3} \times 3^{3} \times 7$
(d) $3^{2} \times 5^{2} \times 7$
3. Find the LCM and HCF of $0.25,0.5,0.75$.
(a) $1.5,0.25$
(b) 2,1
(c) $1.5,0.5$
(d) 3,1
4. Find the HCF of $120,150,180$.
(a) 30
(b) 60
(c) 50
(d) 10
5. Find the LCM of $\frac{4}{3}, \frac{8}{9}, \frac{3}{5}$
(a) 20
(b) 24
(c) $\frac{1}{24}$
(d) $\frac{1}{20}$
6. Find the HCF of $\frac{1}{2}$ and $\frac{3}{2}$
(a) $3 / 2$
(b) $1 / 2$
(c) 1
(d) 3
7. Find out the LCM of $4^{5}, 4^{-81}, 4^{12}$ and $4^{7}$.
(a) $4^{5}$
(b) $4^{-81}$
(c) $4^{12}$
(d) $4^{7}$
8. Find the LCM of $5 / 2,8 / 9,11 / 14$.
(a) 290
(b) 380
(c) 420
(d) 440
9. The HCF and LCM of two numbers are 18 and 3780 respectively. If one of them is 540 , then the second one is:
(a) 146
(b) 126
(c) 118
(d) 117
10. The HCF of two numbers is 8 . Which one of the following can never be their LCM?
(a) 32
(b) 24
(c) 48
(d) 60
11. LCM of two numbers is 12 times of their HCF. Sum of LCM and HCF is 195. If one of them is 60 . Find the other.
(a) 48
(b) 45
(c) 52
(d) 36
12. The HCF and LCM of a pair of numbers are
12 and 926 respectively. How many such distinct pairs are possible?
(a) 3
(b) 7
(c) 1
(d) 0
13. Find the greatest number which will divide 321,428 and 535 exactly.
(a) 105
(b) 107
(c) 109
(d) 102
14. Find the greatest number that will divide 640,710 and 1526 so as to leave 11,7 and 9 as remainders respectively.
(a) 36
(b) 37
(c) 42
(d) 29
15. Find the least number which when divided by 16,18 and 20 leaves a remainder 4 in each case, but is completely divisible by 7 .
(a) 465
(b) 3234
(c) 2884
(d) 3234
16. The least number which when divided by $4,6,8,12$ and 16 leaves a remainder of 2 in each case is:
(a) 20
(b) 43
(c) 50
(d) 59
17. Find the least number which when increased by 3 is exactly divisible by $10,12,14$ and 16 .
(a) 1680
(b) 1677
(c) 1697
(d) 1670
18. Find the least number which when decreased by 4 is exactly divisible by $9,12,15$ and 18 .
(a) 188
(b) 182
(c) 186
(d) 184
19. Find the smallest four digit number that is exactly divisible by 8,10 and 12 .
(a) 1080
(b) 1100
(c) 1050
(d) 1120
20. Five bells bE.g.in to toll together and toll at intervals of $24,40,64,72$ and 120 s . After what interval of time will they toll again together ?
(a) 42 min
(b) 36 min
(c) 48 min
(d) 54 min
21. Find the least number which is exactly divisible by $12,15,20$ and 27 .
(a) 650
(b) 520
(c) 600
(d) 540
22. A number when divided by 225 gives a remainder of 32 . What will be the remainder when the same number is divided by 15 ?
(a) 4
(b) 2
(c) 3
(d) 1
23. Five bells bE.g.in to toil together and toll respectively at intervals of $6,7,8,9$ and 12 sec . How many times they will toll together in one hour, excluding the one at the start?
(a) 3
(b) 5
(c) 7
(d) 9
24. What is the smallest whole number that is exactly divisible by $1 \frac{5}{\mathbf{2 8}}, 2 \frac{2}{11}$ and $3 \frac{1}{7}$ ?
(a) 264
(b) 130
(c) 138
(d) 124
25. The least positive intE.g.er which leaves a remainder 2 , when divided by each of the numbers $4,6,8,12$ and 16.
(a) 46
(b) 48
(c) 50
(d) 52
26. HCF of two numbers is 12 and their product is 3600 . How many such pairs of numbers can be formed?
(a) 0
(b) 1
(c) 2
(d) 4
27. The sum of two numbers is 135 and their HCF is 9. How many such pairs of numbers can be formed?
(a) 6
(b) 2
(c) 5
(d) 4
28. The LCM of two numbers is 280 and the ratio of the numbers is $7: 8$. Find the numbers.
(a) 70 and 48
(b) 42 and 48
(c) 35 and 40
(d) 28 and 32
29. A florist has 200 roses and 180 jasmines with him. He was asked to make garlands of flowers with only roses or only jasmines each containing the same number of flowers. What will be the largest number of flowers, he can join together without leaving a single flower?
(a) 16
(b) 17
(c) 20
(d) 19
30. What is the largest number which when divides 1475,3155 and 5255 leaves the same remainder in each case?
(a) 220
(b) 420
(c) 350
(d) 540
31. Find the side of the largest possible square slabs which can be paved on the floor of a room 2 m 50 cm long and 1 m 50 cm broad. Also find the number of such slabs to pave the floor.
(a) 40,18
(b) 30,15
(c) 50,15
(d) 20,25
32. How many numbers less than 10,000 are there which are divisible by 21,35 and 63 ?
(a) 33
(b) 32
(c) 38
(d) 31

| Answer Key |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | a | 5 | b | 9 | b | 13 | b | 17 | b | 21 | d | 25 | c | 29 | c |
| 2 | a | 6 | b | 10 | d | 14 | b | 18 | d | 22 | b | 26 | b | 30 | b |
| 3 | a | 7 | c | 11 | b | 15 | c | 19 | a | 23 | c | 27 | d | 31 | c |
| 4 | a | 8 | d | 12 | d | 16 | c | 20 | c | 24 | a | 28 | c | 32 | d |

## SOLUTIONS

1. LCM of given numbers $=2^{3} \times 3^{4} \times 5^{2} \times 7^{2}$
(Take the greatest power of each term)
2. $\mathrm{HCF}=5$ (Take the least powers of common terms).
3. (a) LCM of $0.25,0.5$ and 0.75

The given numbers can be written as of $0.25,0.50$ and 0.75
Now ignoring the decimals we find LCM of 25,50 , and 75

| 5 | $25,50,75$ |
| :--- | :---: |
| 5 | $5,10,15$ |
|  | $1,2,3$ |

$\therefore \mathrm{LCM}=1 \times 2 \times 3 \times 5 \times 5$
$=25 \times 6=150$
$\mathrm{LCM}=150$. Similarly we will find out the HCF for 25,50 and 75.
Since given numbers are not high, we can follow Prime factorization method to get the HCF easily
$25=5^{2}$
$50=2 \times 5^{2}$
$75=3 \times 5^{2}$.
As discussed before $\mathrm{HCF}=5^{2}=25$.
Now we got LCM $=150$ and $\mathrm{HCF}=25$, now after putting decimal places as per given in the question the $\mathrm{LCM}=1.5$, and $\mathrm{HCF}=0.25$.
4. (a) since given numbers are not high, we can follow Prime factorization method to get the HCF easily
$120=2^{3} \times 3 \times 5$
$150=2 \times 3 \times 5^{2}$
$180=2^{2} \times 3^{2} \times 5$
In Prime factorization method after converting the numbers into product of prime factors, take the common factors from all the numbers of the least powers.
Therefore HCF is $\mathbf{2} \times \mathbf{3} \times \mathbf{5}=\mathbf{3 0}$
5. LCM of fraction $=\frac{\text { LCM of Numerators }}{\text { HCF of Denominators }}$

$$
=\frac{L C M[4,8,3]}{H C F[3,9,5]}=24
$$

6. HCF of $\frac{1}{2}, \frac{3}{2}=\frac{\text { HCF of } 1 \text { and } 3}{\text { LCM of } 2 \text { and } 2}=\frac{1}{2}$
7. Clearly, LCM $=4^{12}$
8. $\frac{\operatorname{LCM} \text { of }(5,8,11)}{\text { HCF of }(2,9,14)}=\frac{440}{1}=440$
9. Sol: (b) We know that,

$$
\begin{aligned}
& H C F \times L C M=x \times y \\
& 18 \times 3780=540 \times y \\
& \frac{18 \times 3780}{540}=y \\
& y=126
\end{aligned}
$$

Another number is 126 .
10. Since HCF of any two numbers is also a factor of their LCM, 60 can never be their LCM
since HCF 8 is not a factor of 60 .
11. Let the HCF be $x$

$$
\begin{aligned}
& \text { LCM }=\mathbf{1 2} \boldsymbol{x} \\
& \boldsymbol{x}+\mathbf{1 2 \boldsymbol { x }}=\mathbf{1 9 5} \\
& \mathbf{1 3 x}=\mathbf{1 9 5} \\
& \boldsymbol{x}=\mathbf{1 5} \\
\therefore & \text { HCF }=15 \text { and } \mathrm{LCM}=(12)(15)=180
\end{aligned}
$$

Other number $=\frac{180 \times 15}{60}=45$
12. $\mathrm{HCF}=12$ and $\mathrm{LCM}=926$. If the numbers be of the form $12 a$ and 12 b ; then $\mathrm{LCM}=$ $12 a b$, i.e., LCM is always divisible by HCF Clearly in this question 926 is not divisible by 12 , so no such pair exists.
13. Required number
$=$ HCF of 321, 428 and $535=107$
14. $\mathrm{HCF}[(640-11),(710-7),(1526-9)]=$ $\operatorname{HCF}[629,703,1517]=37$.
15. $\operatorname{LCM}(16,18,20)=720$

The number required is off the $720 k+4$, where $k$ is a natural number. In order to make it divisible by 7 , we put $k=4$. Hence the number is

$$
720 \times 4+4=2884
$$

16. The number $=\operatorname{LCM}(4,6,8,12,16)+2=$ 50
17. Required number

$$
\begin{aligned}
=(\text { LCM of } 10, & 12,14,16)-3 \\
= & 1680-3=1677
\end{aligned}
$$

18. Required number

$$
=(\text { LCM of } 9,12,15,18)+4
$$

$$
=180+4=184
$$

19. The smallest four digit number exactly divisible by 8,10 and 12 should also be divisible by the LCM of 8,10 and 12 .
LCM of 8,10 and $12=120$
So, the required number $=1080$
20. Required time interval
$=\mathrm{LCM}$ of $24,40,64,72$ and 120 s

$$
=2880 \text { seconds }
$$

$$
=48 \mathrm{~min}
$$

21. Required number
$=\mathrm{LCM}$ of $12,15,20$ and $27=540$
22. M-I: Let the number be $225+32=257$, and after dividing 257 with 15 we will get the remainder as 2 .
M-II: The remainder when 32 is divided by 15 is 2
23. The bells would toll together at LCM 6, 7, $8,9,12$, which is 504 .
Number of times, they toll together

$$
=\frac{3600}{504}=7
$$

24. Required number $=\operatorname{LCM}\left[\frac{33}{28}, \frac{\mathbf{2 4}}{21}, \frac{\mathbf{2 2}}{7}\right]=264$
25. The LCM of $4,6,8,12$ and 16 is 48 .

So, required number $=48+2=50$
26. Let the two numbers be $12 x$ and $12 y$.

$$
\begin{gathered}
12 x \times 12 y=3600 \\
x y=25
\end{gathered}
$$

Possible values of $x$ and $y$ are $(1,25)(5,5)$.
But $(5,5)$ are not co primes.
$\therefore$ Only one pair of numbers can be formed.
27. The two numbers are always the multiples of the HCF
$\therefore$ Let the two numbers be 9 x and 9 y .

$$
9 x+9 y=135 \Rightarrow x+y=15
$$

Now, the possible values of $\boldsymbol{x}$ and $\boldsymbol{y}$ are $(1,14),(2,13)(3,12),(4,11),(5,10),(6,9)$, $(7,8)$.
Now, consider only the co prime pairs. These are $(1,14),(2,13),(4,11),(7,8)$.
$\therefore 4$ pairs of numbers can be formed whose sum is 135 and HCF is 9
28. Let the two numbers be $7 x$ and $8 x$ and LCM is $\mathbf{5 6 x}$.
It is given that $\mathrm{LCM}=280$
ie, $56 \boldsymbol{x}=280$ and $\boldsymbol{x}=\mathbf{5}$
ie, numbers are 35 and 40 .
29. $\operatorname{HCF}(200,180)=20$
30. Take the relative differences of the given numbers.
(3155-1475), (5255-3155), (5255-1475)

$$
=1680,2100,3780
$$

$$
\operatorname{HCF}(1680,2100,3780)=420
$$

31. $\operatorname{HCF}(250,150)=50 \mathrm{~cm}$
$\therefore$ The number of slabs $=\frac{\mathbf{2 5 0 \times 1 5 0}}{\mathbf{5 0 \times 5 0}}=15$
32. $\operatorname{LCM}(21,35,63)=315$

The numbers less than 10,000 which are divisible by 315 are given by $\left[\frac{\mathbf{1 0 , 0 0 0}}{315}\right]$ i.e. the integral part when 10,000 is divided by 315 is 31 .
The required answer is 31

