4. SIMPLIFICATIONS

In simplification of an expression there are certain laws which should be strictly adhered to. These laws are as follows:

'VBODMAS' Rule

This rule gives the correct sequence in which the mathematical operations are to be performed so as to find out the value of a given expression.

Here, 'V' stands for Vinculum (or Bar), 'B' stands for 'Bracket', 'O' stands for 'Of', 'D' stands for 'Division', 'M' stands for 'Multiplication', 'A' stands for 'Addition' and 'S' stand for 'Subtraction'. (a) Here, "VBODMAS' gives the order of simplification. Thus, the order of performing the mathematical operations in a given expression are

First: Vinculum or line bracket or bar
Second: Bracket
Third: Of
Fourth: Division
Fifth: Multiplication
Sixth: Addition &
Seventh: Subtraction
The above order should strictly be followed.
(b) There are four types of brackets.
(i) Square brackets [ ]
(ii) Curly brackets { }
(iii) Circular brackets ()
(iv) Bar or Vinculum -
Thus, in simplifying an expression all the brackets must be removed in the order '-', '( )', '{ }' and '[ ]'.

Modulus of a Real Number

The modulus of a real number x is defined as

$$|x| = \begin{cases} x \ \ if \ x > 0 \\ -x \ if \ x < 0 \end{cases}$$

Basic Formulae:

(i) $(a+b)^2 = a^2+2ab+b^2$
(ii) $(a-b)^2 = a^2-2ab+b^2$
(iii) $(a+b)^2-(a-b)^2 = 4ab$
(iv) $(a+b)^2+(a-b)^2 = 2(a^2+b^2)$
(v) $(a-b)^2 = (a+b)(a-b)$
(vi) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+ bc +ca)$
(vii) $(a^3+b^3) = (a+b)(a^2 - ab+b^2)$
(viii) $(a^3-b^3) = (a-b)(a^2+ab+b^2)$
(ix) $(a^3+b^{3+}c^3-3abc) = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$
(x) $(a^3+b^{3+}c^3) = 3abc$, if $a+b+c=0$

Example 1: Simplify 1005 + 500 - 10 - 80.
Solution: 1005 + 500- 10-80 = 1005 + 490 – 80 = 1495 - 80 = 1415
Example 2: If a* b = 2 (a + b), then what is the value of 5 * 2?
Solution: 5 * 2 = 2(5 + 2) = 2 * 7 = 14

Example 3: 3/5 part of the students in a class are the girls and remaining are the boys. If 2/9 part of the girls and 1/4 part of the boys are absent, then what part of total students is present.

Solution: Let the total number of students be x

Number of girls $= \frac{3}{5}x$

Number of boys $= \frac{2}{5}x$

Number of students absent $= \left(\frac{2}{9} of \frac{3}{5}x\right) + \left(\frac{1}{4} of \frac{2}{5}x\right) = \frac{2}{15}x + \frac{1}{10}x = \frac{7}{30}x$

No. of students Present $= (1 - \frac{7}{30})x = \frac{23}{30}x$

Example 4: Simplify $(2^{10} - 2^9)(2^8 - 2^7)$.

Solution : $(2^{10} - 2^9)(2^8 - 2^7) = 2^9(2-1).\ 2^7(2-1) = 2^{9+7} = 2^{16}$

Example 5: If $\frac{x}{y} = \frac{3}{2}$ find the value of $\frac{x^2+y^2}{x^2-y^2}$.

Solution: **Method-I:** $\frac{x^2+y^2}{x^2-y^2} = \frac{\frac{x^2}{y^2}+1}{\frac{x^2}{y^2}-1}$ (dividing both numerator and denominator by y^2)

$= \frac{\left(\frac{x^2}{y^2}\right)+1}{\left(\frac{x^2}{y^2}\right)-1} = \frac{\left(\frac{3^2}{2^2}\right)+1}{\left(\frac{3^2}{2^2}\right)-1} = \frac{\frac{9}{4}+1}{\frac{9}{4}-1} = \frac{13}{5} = 2\frac{3}{5}$

Method-II: Replace x and y with 3 & 2 respectively .

Example 6: Simplify $\frac{0.3\times0.3+0.03\times0.03-0.6\times0.03}{0.54}$

Solution: $0.3\times 0.3 + 0.03\times 0.03 - 0.6 \times 0.03 = 0.3 \times 0.3 + 0.03\times 0.03 - 2\times0.3\times0.03$ is of the form

$(a-b)^2 = a^2-2ab+b^2$

$a=0.3$ and $b=0.03=(0.3-0.03)^2 = (0.27)^2$

Given expression $\frac{0.27\times0.27}{0.54} = 0.135$

EXERCISE

1. What is $\frac{1}{6}^{th}$ of 3?

(a) 6 (b) 3 (c) 1/2 (d) 1/3

2. Divide 0.045 by 100.

(a) 0.0045 (b) 0.00045

(c) 0.000045 (d) 0.45

3. Simplify $15.876 - (2.49 + 4.056) \div \frac{1}{2}$.

(a) 2.784 (b) 3.052

(c) 2.984 (d) 3.152

4. Simplify $\frac{0.48\div0.12+0.04\times25}{0.05}$

(a) 100 (b) 110

(c) 90 (d) 105

5. Simplify $\frac{\frac{3}{2}\div\frac{1}{2}\times\frac{3}{2}}{\frac{3}{2}\div\frac{1}{2} of \frac{3}{2}} \div \frac{1}{8}$

(a) 18 (b) 21

(c) 16 (d) 24

6. Simplify $1\div [1+1\div\{1+1\div(1\div1)\}]$.

(a) $\frac{3}{2}$ (b) $\frac{2}{5}$

(c) $\frac{2}{3}$ (d) $\frac{2}{3}$

7. . Find the value of $\frac{(3+3+3+3)\div3}{5+5+5+5\div5}$

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$

(c) **1** (d) $\frac{9}{4}$

8. Simplify $[1 - 2(3- 4)^{-1}]^{-1}$

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{1}{6}$

9. $(16\div 4) \div 4$ is equal to

(a) 4 (b) 1

(c) $\frac{1}{4}$ (d) 16

10. 16 ÷(4 ÷ 4) is equal to

(a) 4 (b) 1

(c) $\frac{1}{4}$ (d) 16

11. Simplify

$$\left(1+\frac{1}{4}\right)\left(1-\frac{1}{4}\right)\left(1+\frac{1}{5}\right)\left(1-\frac{1}{5}\right)\left(1+\frac{1}{6}\right)\left(1-\frac{1}{6}\right)$$

(a) $\frac{3}{8}$ (b) $\frac{5}{8}$

(c) $\frac{7}{8}$ (d) $\frac{1}{8}$

12. Find *P* in the expression, if $\frac{p}{1+\frac{1}{1+\frac{p}{1-p}}}=1$.

(a) 2 (b) $\frac{1}{4}$

(c) 1 (d) $\frac{1}{2}$

13. If x # *y* = *x* + *y*, then find the value of (3 # 4) # 3.

(a) 6 (b) 7

(c) 10 (d) 8

14. If $1^3 + 2^3 + 3^3 + ... + 9^3 = 2025$, then find the value of $(0.1)^3 + (0.2)^3 + ... + (0.9)^3$

(a) 2.025 (b) 202.5

(c) 20.25 (d) 0.2025

15. What least fraction should be added to $\frac{1}{2\times3}+\frac{1}{3\times4}+\frac{1}{4\times5}+.....+\frac{1}{21\times22}$, so that the result is unity.

(a) $\frac{3}{11}$ (b) $\frac{6}{11}$

(c) $\frac{5}{11}$ (d) $\frac{7}{11}$

16. The sum of first 50 positive integers is 1275. What is the sum of the integers from 51 to 100?

(a) 2525 (b) 2550

(c) 3250 (d) 3775

17. 2 tables and 3 chairs cost Rs.3500 while 3 tables and 2 chairs cost Rs.4000. The cost of a table (in rupees) is

(a) 500 (b) 1000

(c) 1200 (d) 1500

18. If *x* + *y* - *z* = *m* and if *x* - *y* + *z* = *n*, then *x* is equal to

(a) (m + n)/2 (b) *m* — *n*

(c) 2*m* + *n* (d) *m* + *n*

19. If the numerator of a fraction is double and the denominator is increased by 3, the new fraction is 3/5. What is the original fraction, if its denominator is more than twice the numerator by 1?

(a) 3/7 (b) 6/13

(c) 1/3 (d) 5/11

20. It is required to change a rupee coin into 2 paisa and 5 paisa coins with the total number of coins equal to 26. Find the number of each type of coins.

(a) 10 and 16 (b) 12 and 14

(c) 10 and20 (d) 10 and 14

21. For what value of x, 8 + $(x - 3)^2$ have the least value?

(a) -3 (b) 0

(c) 3 (d) 5

ANSWER KEY									
1	c	6	c	11	c	16	d	21	c
2	b	7	a	12	c	17	b		
3	a	8	b	13	c	18			
4	a	9	b	14	a	19	a		
5	a	10	d	15	b	20	a		

SOLUTIONS

1. $\frac{1}{6}$ th of 3 = $\frac{1}{6}\times 3 = \frac{1}{2}$

2. $\frac{0.045}{100} = 0.00045$

3. The given expression is

$15.876-(2.49 + 4.056)\div\frac{1}{2}$

$= 15.876 -(6.546) \div \frac{1}{2}$

$= 15.876-(6.546 \times 2)$

$= 15.876- 13.092 = 2.784$

4. $0.48 \div 0.12 + 0.04 \times 25$

$= \frac{0.48}{0.12} + 0.04 \times 25$

$= 4 + 1 = 5$

Given expression = $\frac{5}{0.05} = 100$

5. Given expression $\frac{\frac{3}{2}\div\frac{1}{2}\times\frac{3}{2}}{\frac{3}{2}\div\frac{1}{2}of\frac{3}{2}}\div\frac{1}{8} = \frac{\frac{3}{2}\times\frac{2}{1}\times\frac{3}{2}}{\frac{3}{2}\div\frac{3}{4}}\div\frac{1}{8}$

$= \frac{\frac{9}{2}}{\frac{3}{2}\times\frac{4}{3}}\div\frac{1}{8} = \frac{\frac{9}{2}}{2}\div\frac{1}{8} = \frac{9}{4}\times\frac{8}{1} = 18$

6. Given expression

$= 1 \div [1 + 1 \div \{ 1 + 1 \div (1 \div 1) \}]$

$=1\div[1 + 1\div \{1 + 1\div1\}]$

$= 1\div [1 + 1\div\{1 + 1\}]$

$= 1\div[1 +1\div2] = 1\div[1+\frac{1}{2}] = 1\div\frac{3}{2} = \frac{2}{3}$

7. Given expression = $\frac{(12)\div3}{5+5+5+1} = \frac{4}{16} = \frac{1}{4}$

8. Given expression $=[1 - 2 (3- 4)^{-1}]^{-1}$

$= [1-2(-1)^{-1}]^{-1} = [1+2]^{-1} = 3^{-1} = \frac{1}{3}$

9. $(16 \div 4) \div 4 = 4 \div 4= 1$

10. $16 \div (4 \div 4) = 16 \div 1 = 16$

11. Given expression is

$\left(1+\frac{1}{4}\right)\left(1-\frac{1}{4}\right)\left(1+\frac{1}{5}\right)\left(1-\frac{1}{5}\right)\left(1+\frac{1}{6}\right)\left(1-\frac{1}{6}\right) = \frac{5}{4}\times\frac{3}{4}\times\frac{6}{5}\times\frac{4}{5}\times\frac{7}{6}\times\frac{5}{6} = \frac{7}{8}$

12. Given, $\frac{p}{1+\frac{1}{\frac{(1-p)+p}{1-p}}} = 1 \Rightarrow \frac{p}{1+\frac{1}{\frac{1}{1-p}}} = 1$

$\Rightarrow \frac{p}{1+(1-p)} = 1 \Rightarrow \frac{p}{2-p} = 1$

$\Rightarrow$ p=2-p or p=1

13. Given $x \# y = x + y$

Then, $(3 \# 4) \# 3 = (3 + 4) \# 3 = 7 \# 3= 10$

14. Given $1^3 + 2^3 + 3^3 + ... + 9^3 = 2.025$

Now, $(0.1)^3 + (0.2)^3 + ... + (0.9)^3$

$= (0.1)^3 [1^3 + 2^3 + ... + 9^3]$

$= (0.1)^3 (2025) = 0.001 \times 2025 = 2.025$

15. Given expression is

$\frac{1}{2\times3} + \frac{1}{3\times4} + \frac{1}{4\times5} + \cdots .. + \frac{1}{21\times22}$

$= \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + ... + \left(\frac{1}{21} - \frac{1}{22}\right) = \frac{1}{2} - \frac{1}{22} = \frac{10}{22} = \frac{5}{11}$

Thus, least fraction to be added = $1-\frac{5}{11} = \frac{6}{11}$

16. Sum of 1 to 100 = $\frac{100(100+1)}{2} = 5050$

$\therefore$ Sum of 51 to 100 = sum of 1 to 100 - sum of 1 to 50 = 5050- 1275 = 3775

17. 2t + 3c =3500 …….(i)

3t+ 2c =4000 …….(ii)

On solving t = Rs.1000

18. $x + y - z = m$ and $x - y + z = n$

Adding the above two we have,

$2x = m + n$ or $x = (m + n)/2$.

19. M-I: Let the fraction be $\frac{x}{2x+1}$

Then, $\frac{2x}{2x+1+3} = \frac{3}{5} \Rightarrow 10x = 6x + 12$

$\Rightarrow 10x - 6x = 12 \Rightarrow x = 3$

So, the fraction is $\frac{3}{7}$

M-II: 0TP

20. M-I: Let *x* be the number of 5 paisa coins, then (26 - x) will be the number of 2 paisa coins.

So, $5x + (26 - x) \times 2 = 100$

$5x + 52 - 2x = 100$

$3x = 48 \Rightarrow x = 16$

Number of 5 paisa coins = 16

Number of 2 paisa coins = 26 — 16 = 10

M-II: 0TP

21. Clearly for $x = 3$, given expression has least value.