3. SQUARE ROOT & CUBE ROOT

Square Root

The square root of a number is that number the product of which itself gives the given number, i.e. the square root of 400 is 20, the square root of 625 is 25.

The process of finding the square root is called evaluation. The square root of a number is denoted by the symbol **"√"** called the radical sign. The expression "$\sqrt{9}$" is read as "root nine", "radical nine" or "the square root of nine".

How to Find the Square Root of an Integer?

(i) **By the method of Prime Factors:** When a given number is a perfect square, we resolve it into prime factors and take the product of prime factors, choosing one out of every two.

Example 1: Find the square root of 4356.

Solution:

2	4356
2	2178
3	1089
4	363
11	121
	11

$4356 = 2\times 2 \times 3 \times 3 \times 11 \times 11 = 2^2 \times 3^2 \times 11^2$

$\sqrt{4356} = 2 \times 3 \times 11 = 66$

Thus from the above example it is clear that in order to find the complete square root of a given number every prime factor of that number should be repeated twice. Thus, we can make a number which is not a perfect square, a perfect square by multiplying or dividing the number by those factors of it which are not contained in pairs.

Example 2: Find the least number by which 1800 be multiplied or divided to make it a perfect square.

Solution: $1800 = 2\times2\times 2\times3\times3\times5\times5$

The least number by which the given number be multiplied or divided is 2.

To Find the Square Root of a Decimal

Example 3: $\sqrt{1.8225} = \sqrt{\frac{18225}{10000}} = \frac{\sqrt{18225}}{\sqrt{10000}} = \frac{135}{100} = 1.35$

To Find the Square Root of a Fraction

Example 4: Find the square root of $1\frac{13}{36}$.

Solution: $\sqrt{1\frac{13}{36}} = \sqrt{\frac{49}{36}} = \frac{\sqrt{49}}{\sqrt{36}} = \frac{7}{6} = 1\frac{1}{6}$

- The square of a number other than unity is either a multiple of 4 or exceeds a multiple of 4 by 1.
- A perfect square can never end with (a) an odd number of zeroes and (b) 2, 3, 7 and 8.
- The square root of an-intE.g.er is not always an integer i.e., $\sqrt{3}$, $\sqrt{5}$, $\sqrt{11}$ are not integers.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- $\sqrt{a}+\sqrt{b} \neq \sqrt{a+b}$

• $\sqrt{a}-\sqrt{b} \neq \sqrt{a-b}$

Example 5: Find the square root of 0.0016.

Solution: $\sqrt{0.0016} = \sqrt{\frac{16}{10000}} = \frac{\sqrt{16}}{\sqrt{10000}} = \frac{4}{100} = 0.04$

Example 6: Find the value of *x* if $\sqrt{\frac{25.6}{x}} = 8$.

Solution: Given $\sqrt{\frac{25.6}{x}} = 8 \Longrightarrow \frac{25.6}{x} = 64$

$\Longrightarrow 64x = 25.6 \Longrightarrow x=0.4$

Example 7: If $\sqrt{5} = 2.236$, find the value of $\sqrt{245} - \frac{1}{2}\sqrt{80} - \sqrt{20}$ correct to three places of decimal.

Solution: $\sqrt{245} - \frac{1}{2}\sqrt{80} - \sqrt{20} = \sqrt{49 \times 5} - \frac{\sqrt{16\times5}}{2} - \sqrt{4 \times 5}$

$= 7\sqrt{5} - \frac{4\sqrt{5}}{2} - 2\sqrt{5} = 7\sqrt{5} - 2\sqrt{5} - 2\sqrt{5}$

$= 3\sqrt{5} = 3\times2.236 = 6.708$

Example 8: Find the smallest number that must be added to 2400 to make it a perfect square.

Solution:

$48^2 = 2304$

$49^2 = 2401$

Number to be added = $(49)^2-2400 = 2401 - 2400 = 1$

Cube Root

The cube root of a number is that number the cube of which itself gives the given number i.e. the cube root of 64 is 4. The cube root of a number is denoted by the symbol $\sqrt[3]{}$. The expression $\sqrt[3]{8}$ is read as "cube eight" or the "cube root of eight".

To Find the Cube Root of an IntE.g.er:

(i) By the method of prime factors: When a given number is a perfect cube, we resolve it into prime factors and take the product of prime factors, choosing one out of every three.

Example 9: Find the cube root of 74088.

Solution:

2	74088
2	37044
2	18522
7	9261
7	189
3	27
3	9
3	3
	1

$74088 = 2^3 \times 7^3 \times 3^3$

$\sqrt[3]{74088} = 2 \times 7 \times 3 = 42$

To Find the Cube Root of a Decimal:

Example 10: Find the cube root of 19.683.

Solution: $\sqrt[3]{19.683} = \sqrt[3]{\frac{19683}{1000}} = \frac{\sqrt[3]{19683}}{\sqrt[3]{1000}} = \frac{\sqrt[3]{3^9}}{\sqrt[3]{10^3}} = \frac{3^3}{10} = \frac{27}{10} = 2.7$

3	19683
3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

$19683 = 3^9$

To Find the Cube Root of a Fraction:

Example 11: Find the cube root of $1\frac{61}{64}$

Solution: $\sqrt[3]{1\frac{61}{64}} = \sqrt[3]{\frac{125}{64}} = \frac{\sqrt[3]{125}}{\sqrt[3]{64}} = \frac{\sqrt[3]{5^3}}{\sqrt[3]{4^3}} = 1\frac{1}{4}$

(1) $\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$ (2) $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

(3) $\sqrt[3]{a} + \sqrt[3]{b} \neq \sqrt[3]{a+b}$ (4) $\sqrt[3]{a} - \sqrt[3]{b} \neq \sqrt[3]{a-b}$

Example 12: Find the smallest number by which 2400 be divided to make it a perfect cube.

Solution: $2400 = 2^5 \times 5^2 \times 3$

To make it a perfect cube it must be divided by $2^2 \times 5^2 \times 3 = 300$

EXERCISE

1. Square root of 2025 is
(a) 35 (b) 45
(c) 55 (d) 65

2. Find the square root of 104976
(a) 322 (b) 324
(c) 326 (d) 328

3. Find the value of
$$\sqrt{17 + \sqrt{51 + \sqrt{152 + \sqrt{289}}}}$$
(a) 11 (b) 5
(c) 7 (d) 9

4. $\sqrt{118 + \sqrt{2601}}$ =?
(a) 19 (b) 15
(c) 17 (d) 13

5. Find the value of
$\sqrt{0.09}+\sqrt{0.81}+\sqrt{1.44}+\sqrt{0.0004}$
(a) 2.42 (b) 2.24
(c) 2.28 (d) 2.22

6. If $\sqrt{0.00000576} = 0.0024$, then the square root of 57,60,000 is
(a) $\frac{1}{24}$ (b) 2400
(c) 240 (d) 24

7. Find the square root of $(8 + 2\sqrt{7})(8-2\sqrt{7})$.
(a) 6 (b) 4
(c) 8 (d) 9

8. $\sqrt{27\frac{9}{16}}$ = x. Find x.
(a) 7.15 (b) 8.25
(c) 5.25 (d) 6.15

9. If $\sqrt{3} = 1.732$, then the value of $\frac{1}{\sqrt{3}}$ approx is.
(a) 0.577 (b) 0.477
(c) .0.512 (d) 0.417

10. Find the least six-digit perfect square number.
(a) 100009 (b) 100289
(c) 100441 (d) 100489

11. What is the smallest number by which 1400 be divided to make it a perfect cube?
(a) 130 (b) 145
(c) 160 (d) 175

12. Find the cube root of $4\frac{12}{125}$.
(a) $1\frac{3}{5}$ (b) $1\frac{4}{5}$
(c) $1\frac{2}{5}$ (d) None of these

13. Find the least number by which 234375 be divided to make it a perfect cube.
(a) 20 (b) 8
(c) 15 (d) 10

14. Find the least perfect square number divisible by 2, 3, 4, 5 and 6.
(a) 900 (b) 1600
(c) 2700 (d) 400

Answer Key

1	b	5	a	9	a	13	c
2	b	6	b	10	d	14	d
3	b	7	a	11	d		
4	d	8	c	12	a		

SOLUTIONS

1. $2025 = 5 \times 5 \times 9 \times 9$
$\sqrt{2025} = 45$

2. (b)From the options we can say the square root of 104976 is either 324 or 326, and by inspection method we can say $\sqrt{104976} = 324$

3. Given $\sqrt{17+\sqrt{51+\sqrt{152+\sqrt{289}}}}$
$= \sqrt{17+\sqrt{51+\sqrt{152+17}}}$
$= \sqrt{17+\sqrt{51+\sqrt{169}}}$
$= \sqrt{17+\sqrt{51+13}}$
$= \sqrt{17+\sqrt{64}}$
$= \sqrt{17+8} = \sqrt{25} = 5$

4. $\sqrt{2601} = 51$
$\sqrt{118+\sqrt{2601}} = \sqrt{118+51} =$
$\sqrt{169} = 13$

5. Given expression
$= \sqrt{\frac{9}{100}} + \sqrt{\frac{81}{100}} \div \sqrt{\frac{144}{100}} + \sqrt{\frac{4}{1000}}$
$= \frac{3}{10} + \frac{9}{10} + \frac{12}{10} + \frac{2}{100} = \frac{240}{100} + \frac{2}{100}$
$= \frac{242}{100} = 2.42$

6. $\sqrt{5760000} = \sqrt{0.00000576 \times 10^{12}}$
$\sqrt{0.00000576} \times \sqrt{10^{12}} = 0.0024 \times 10^6$
$= 2400$

7. $\sqrt{(8+2\sqrt{7})(8-2\sqrt{7})}$
$= \sqrt{64-28}$ $= \sqrt{36} = 6.$

8. Given $\sqrt{27\frac{9}{16}} = \sqrt{\frac{441}{16}} = \frac{\sqrt{441}}{\sqrt{16}}$
$= \frac{21}{4} = 5.25$

9. $\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} = \frac{1.732}{3} = 0.577$

10. Least number of 6 digits is 100000.

3	100000	316
	9	
	100	
	61	
626	3900	
	3756	

| 144 |

$$(\mathbf{316})^2 < 100000 < (\mathbf{317})^2$$

Hence required number = $(\mathbf{317})^{\mathbf{2}}$ = 100489

11. $\mathbf{1400 = 2^3 \times 5^2 \times 7}$

To make it a perfect cube, it must be divided by $5^2 \times 7 = 175$

12. $\sqrt[3]{\mathbf{4\frac{12}{125}}} = \sqrt[3]{\mathbf{\frac{512}{125}}} = \mathbf{\frac{\sqrt[3]{512}}{\sqrt[3]{125}}} = \mathbf{\frac{\sqrt[3]{8^3}}{\sqrt[3]{5^3}}} = \mathbf{\frac{8}{5}}$

$=1\mathbf{\frac{3}{5}}$

13. $234375 = 5^7 \times 3$

To make it a perfect cube it must be divided by $5 \times 3 = 15$.

14. L.C.M. of 2, 3 ,4 ,5 and 6 = 60.

Now $60 = 2 \times 2 \times 3 \times 5$.

To make it a perfect square it must be multiplied by 3×5

So required number $= \mathbf{2^2 \times 3^2 \times 5^2 = 900}$

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