22.PERMUTATIONS AND COMBINATIONS

Permutations: If r objects are to be chosen from n, where n ≥ r and these r objects are to be arranged and the order in which these objects are arranged is important, such an arrangement is called a permutation of n objects taken r a time. Permutation is denoted by ^np_r or p (n, r).

$$^np_r = \frac{n!}{(n-r)!}, r \leq n$$

Combinations: If r objects are to be chosen from n, where n ≥ r and the order of choosing the r objects is not important, then such a choices is called a combination of n objects taken r a time. Combination is denoted by ^nC_r or $C\left(\frac{n}{r}\right)$

$$^nC_r = \frac{n!}{r!(n-r)!}, r \leq n$$

Factorial Notation: Let n be a positive intE.g.er. Then, factorial n, denoted by n! is defined as:

n! = n (n – 1) (n – 2) …. 3.2.1.

EXERCISE

1. If 6 boys and 6 girls have to sit in a round circular music chair. So, that there is a girl between every 2 boys. Find the number of ways they can sit?
(a) 6! × 5!   (b) 6! × 4!
(c) 6! × 3!   (d) 6! × 2!

2. Find the number of ways in which 6gents and 6 ladies be seated in a row. So that all the ladies sit together and all the gents sit together?
(a) 2(5!)^2 (b) 2(6!)^2
(c) 2(7!)^2 (d) 2(8!)^2

3. What is the number of ways in which an ascending A.P, Comprising three numbers can be formed from 1, 2, 3, 4, 5, 6, 7?
(a) 8   (b) 9   (c) 10   (d) 11

4. How many three digit numbers can be formed by using the digits in 755628. If repetition is not allowed?
(a) 100   (b) 120   (c) 150   (d) 180

5. There are 10 students in a batch. In how many ways can the first five ranks be getting in?
(a) 30000   (b) 30240
(c) 30500   (d) 30600

6. Find the number of different words that can be formed from the word 'SUCCEED'
(a) 1390   (b) 1400
(c) 1320   (d) 1260

7. The number of arrangements that can be made with the letters of the word 'MATHS'. So, that Letter M will occupy always the first place is?
(a) 3!   (b) 4!   (c) 5!   (d) 6!

8. How many 3-letter words with or without meaning, can be formed by using all the letters of the word 'LOGARITHMS', if repetition of letters is not allowed?
(a) 700   (b) 720   (c) 750   (d) 760

9. The number of arrangement of the word 'CABLE' So that the vowels always occupy odd positions is?
(a) 12   (b) 18   (c) 24   (d) 36

10. What is the number of words formed from the letters of the word 'JOKE' So that the vowels and consonants alternate?
(a) 4   (b) 8   (c) 12   (d) None of these

11. Find the number of different signals that can be transmitted by arranging 3 yellow flags, 4 red flags and 2 blue flags on a pole. All the flags are used to transmit the signal.
(a) 1200   (b) 1260
(c) 1300   (d) 1350

12. In how many different ways a group of 4 men and 4 women can be formed out of 7 men and 8 women?
(a) 2450   (b) 1050

(c) 117   (d) 232

13. Find the no of parallelograms that can be formed from a set of four parallel lines intersecting from a set of four parallel lines intersecting another set of three parallel lines?

(a) 12   (b) 24   (c) 48   (d) 18

14. How many rectangles can be formed on a chess board?

(a) 1248   (b) 1264

(c) 1296   (d) 1292

15. Find the number of combinations of four things selected out of 8 things?

(a) 50   (b) 60   (c) 70   (d) 80

Answer Key

1	a	6	d	11	b
2	b	7	b	12	a
3	b	8	b	13	d
4	b	9	d	14	c
5	b	10	b	15	c

SOLUTIONS

1. Ans: (a) $6! \times 5!$

Explanation:

Circular permutation = $n! (n – 1)!$

∴ Number of ways = $6! (6 – 1)!$

= $6! \times 5!$

2. Ans: (b) $2 (6!)^2$

Explanation:

The number of ways = 2 (6!)(6!)

= $2 (6!)^2$

3. Ans: (b) 9

Explanation:

The possible A.P types,

For d=1: (1, 2, 3), (2, 3, 4), (3, 4, 5), (4, 5, 6), (5, 6, 7)

For d=2: (1, 3, 5), (2, 4, 6), (3, 5, 7)

For d =3: (1, 4, 7)

The number of required A.P is 9.

4. Ans: (b) 120

Explanation:

Number of digits n = 5

r = 3

So, $^5P_3 = \dfrac{5!}{(5-3)!} = \dfrac{6 \times 5 \times 4 \times 3 \times 2!}{2!} = 6 \times 5$

$\times 4 \times 3 = 360$

360 three digit numbers can be formed by using the digits in 755628

5. Ans: (b) 30240

Explanation:

Number of students = 10

First 10 ranks have to be get = r = 5

$^nP_r = \dfrac{n!}{(n-r)!}$

$^{10}P_5 = \dfrac{10!}{(10-5)!} = \dfrac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5!}$

= $10 \times 9 \times 8 \times 7 \times 6$

= 30240

Value of $^{10}P_5 = 30240$

6. Ans: (d) 1260

Explanation:

Number of letters = 7 = n

Number of C = 2 = p

Number of E = 2 = q

$\dfrac{n!}{p! \, q!} = \dfrac{7!}{2!2!} = \dfrac{7 \times 6 \times 5 \times 4 \times 3 \times 2!}{2! \times 2 \times 1}$

= $7 \times 6 \times 5 \times 2 \times 3 = 1260$

7. Ans: (b) 4!

Explanation:

M	A	T	H	S

Always M is in first place

M				

∴ Number of ways = 4!

8. Ans: (b) 720

Explanation:

In 'LOGARTHMS'

Total number of letters = 10 No's

Out of 10 letters, 3 letter can be formed = $^{10}P_3 = \dfrac{10 \times 9 \times 8 \times 7!}{7!}$ =720

9. Ans: (d) 36

Explanation:

No of letters in 'CABLE' = 5 No's

No of vowels = (A, E) = 2 No's

No of arranging vowels at any two odd places out of three = $^3P_2$

Since 5 letters word has 3 odd places.

And number of arranging 3 consonants on the remaining 3 places =3!

$\therefore$ required no of arrangements $= {}^3P_2 \times 3! = \frac{3!}{2!} \times 3! = 6 \times 6 = 36$ arrangements

10. Ans: (b) 8

Explanation:

Word name: 'JOKE'

Vowels: O, E

Consonants: J, K

$\therefore$ Possible arrangement

Beginning with consonant: JOKE, KOJE, JEKO, KEJO = 4 Numbers

Beginning with vowel: OJEK, OKEJ, EJOK, EKOJ = 4 Numbers

Required number = 4+4 = 8 numbers.

11. Ans: (b) 1260

Explanation:

Total number of flags = 9

Number of different signals which can be transmitted $= \frac{9!}{3! \, 4! \, 2!}$

$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 \times 2 \times 1 \times 4!}$

$= 9 \times 4 \times 7 \times 5 = 1260$

12. Ans: (a) 2450

Explanation:

4 men out of 7 men and 4 women out of 8 women can be formed in ${}^7C_4 \times {}^8C_4$ ways

That is, $\frac{7 \times 6 \times 5 \times 4}{1 \times 2 \times 3 \times 4} \times \left(\frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} \right) = 35 \times 70 = \mathbf{2450} \; \mathbf{ways}$

13. Ans: (d) 18

Explanation:

Required number $= {}^4C_2 \times {}^3C_2$

${}^4C_2 = \frac{4!}{2!(4-2)!} = \frac{4 \times 3 \times 2 \times 1}{2! \times 2!} = 6$

${}^3C_2 = \frac{3!}{1!(2!)} = 3$

${}^4C_2 \times {}^3C_2 = 6 \times 3 = 18$

14. Ans: (c) 1296

Explanation:

There are 9 horizontal and 9 vertical lines on a chess board. Hence,

The no of rectangles $= {}^9C_2 \times {}^9C_2$

${}^nC_r = \frac{n!}{r!(n-r)!}$

${}^9C_2 = \frac{9!}{2!(9-2)!} = \frac{9!}{2! \times 7!} = \frac{9 \times 8 \times 7!}{2! \times 7!} = 36$

$\therefore {}^9C_2 \times {}^9C_2 = 36 \times 36 = 36^2 = 1296$

15. Ans: (c) 70

Explanation:

Out of 8 things 4 things selected $= {}^8C_4$

${}^8C_4 = \frac{8!}{4!(8-4)!}$

$= \frac{8 \times 7 \times 6 \times 5 \times 4!}{4! \times 4 \times 3 \times 2 \times 1} = 70$