21. PROBABILITY

The probability is the chance of occurring of a certain event when expressed quantitatively, i.e. probability is a quantitative measure of the certainty.

Event:

The possible outcomes of a trial are called events.

Sample space:

The set of all possible outcomes of an experiment is called a sample space. We generally denote it by S.

Algebra of events:

If A and B are two events associated with sample space S, then

(i) A∪B is the event that either A or B or both occur.

(ii) A∩B is the event that A and B both occur.

Mathematical definition of probability:

Probability of an event A, denoted as P(A), is defined as

$$P(A)=\frac{Number\ of\ cases\ favourable\ to\ A}{Number\ of\ possible\ outcomes}$$

The probability of the happening of a certain event is denoted by p and that of not happening by q.

Here, p, q are non-negative and cannot exceed unity, i.e. $0\le p\le 1$ and $0\le q\le 1$.

For any two events A and B, $\boldsymbol{P(A\cup B)=P(A)+P(B)-P(A\cap B)}$.

If A and B are two independent events, then $\boldsymbol{P(A\ and\ B)=P(A).P(B)}$

EXERCISE

1. In a simultaneous toss of two coins, find the probability of two heads?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{5}$

2. In a simultaneous toss of two coins, find the probability of exactly one tail?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{5}$

3. In a simultaneous toss of three coins find the probability of all heads

(a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{5}{8}$ (d) $\frac{7}{9}$

4. If four coins are tossed, find the probability to get at least 1 head?

(a) $\frac{3}{5}$ (b) $\frac{1}{6}$ (c) $\frac{15}{16}$ (d) $\frac{5}{6}$

5. A Dice is thrown. Find the probability that the number showing on the dice is divisible by 2

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$ (e) $\frac{1}{52}$

6. In a single throw of two dice, find the probability of getting doublet?

(a) $\frac{2}{6}$ (b) $\frac{3}{5}$ (c) $\frac{6}{7}$ (d) $\frac{1}{6}$

7. Two dice are tossed. Find the probability that the total is a prime number?

(a) $\frac{5}{12}$ (b) $\frac{7}{12}$ (c) $\frac{12}{16}$ (d) $\frac{6}{16}$

8. In simultaneous throw of a pair of a dice, find the probability that the sum of numbers shown on the two faces divisible by 5 or 6

(a) $\frac{13}{36}$ (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $\frac{1}{36}$

9. What is the probability of one card drawn at random from the pack of playing cards may be either a queen or an ace?

(a) $\frac{2}{13}$ (b) $\frac{3}{13}$ (c) $\frac{7}{13}$ (d) $\frac{11}{13}$

10. One card is drawn from pack of playing cards. Obtain the probability that it is a letter card or a heart

(a) $\frac{8}{663}$ (b) $\frac{19}{66}$ (c) $\frac{11}{26}$

(d) $\frac{25}{52}$ (e) $\frac{37}{66}$

11. A card is drawn from a pack of 52 cars. Find the probability that it is a diamond.

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$
(d) $\frac{1}{8}$ (e) $\frac{1}{52}$

12. The probability of getting a king and a queen when two cards are drawn from a pack of 52 cards is:
(a) $\frac{8}{663}$ (b) $\frac{19}{66}$ (c) 63
(d) $\frac{4}{663}$ (e) $\frac{37}{66}$

13. If 3 cards are drawn simultaneously form a pack of well shuffled cards, find the probability of then being all Queen.
(a) $\frac{26}{221}$ (b) $\frac{143}{11050}$ (c) $\frac{4}{13}$
(d) $\frac{1}{2}$ (e) $\frac{1}{5525}$

14. A card is drawn from a pack of 100 cards numbered 1 to 100. Find the probability of drawing a number which is square?
(a) $\frac{2}{5}$ (b) $\frac{3}{15}$ (c) $\frac{7}{10}$ (d) $\frac{1}{10}$

15. A box contains 49 tickets numbered 1 to 49. One ticket drawn at randomly, find the probability that number on the ticket is either divisible by 3 or is a perfect square?
(a) $\frac{21}{49}$ (b) $\frac{12}{49}$ (c) $\frac{2}{49}$ (d) $\frac{37}{49}$

16. One bag contains 4 white 2 black balls. Another contains 3 white and 5 black balls. One ball is drawn from each bag. Find the probability that both are white?
(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) $\frac{3}{5}$

17. A can solve 80% of the problems given in an exam and B can solve 70%. What is the probability that exactly one of them will solve a problem selected at random from the exam?
(a) $\frac{19}{50}$ (b) $\frac{28}{50}$ (c) $\frac{19}{79}$ (d) $\frac{28}{79}$

18. In an arrangement of 'SHIP'. Find the probability that 'S' letter occupies the first place?
(a) $\frac{2}{3}$ (b) $\frac{1}{4}$ (c) $\frac{2}{4}$ (d) $\frac{3}{4}$

19. A bag contains 5 red, 8 blue balls and also contains 4 green 7 black balls. If a ball is drawn, find the probability to that is not green?
(a) $\frac{1}{4}$ (b) $\frac{3}{5}$ (c) $\frac{5}{6}$ (d) $\frac{2}{4}$

20. If events A and B are independent and P(A) = 0.15 $P(A \cup B) = 0.45$ then P(B)=?
(a) $\frac{1}{4}$ (b) $\frac{3}{15}$ (c) $\frac{6}{17}$
(d) None of these

21. A family has 2 children. What is the probability that both the children are girls given that at least one of them is a girl?
(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$
(d) $\frac{1}{8}$ (e) $\frac{1}{52}$

Answer Key					
1	a	8	a	15	a
2	b	9	a	16	c
3	a	10	d	17	a
4	c	11	a	18	b
5	c	12	a	19	c
6	d	13	e	20	c
7	a	14	d	21	c

SOLUTIONS

1. Sample space, S = {HH, HT, TH, TT}

n(S) = 4

There is only one chance to get two heads. That is {HH}

So, n(E) = 1

Probability = $\frac{n(E)}{n(S)} = \frac{1}{4}$

2. Ans: (b) $\frac{1}{2}$

Sample spaces = {HH, HT, TH, TT}

Number of elements in the sample space n(S) = 4

Probability of exactly 1 tail occurs in 2 ways. That is {HT, TH}

So, n(E) = 2

Probability = $\frac{n(E)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

3. Ans: (a) $\frac{1}{8}$

Sample spaces = {HHH, HHT, HTH, THH, TTH, HTT, TTT, THT}

Number of sample spaces = n(S) = 8

Probability of getting all heads occurs in {HHH} 1 way

$n(E_1) = 1$

Probability = $\frac{n(E1)}{n(S)} = \frac{1}{8}$

4. Ans: (c) $\frac{15}{16}$

Sample spaces = {HHHH, HHHT, HHTH, HTHH, THHH, TTHH, THHT, HHTT, TTTH, TTHT, THTT, HTTT, TTTT, HTHT, HTTH, THTH}

n(S) = 16 (2^n possibilities, here n = 4)

Favorable cases to get at least 1 head = {HHHH, HHHT, HHTH, HTHH, THHH, TTHH, THHT, HHTT, TTTH, TTHT, THTT, HTTT, HTHT, HTTH, THTH}

n(E) = 15

Probability = $\frac{n(E)}{n(S)} = \frac{15}{16}$

5. Ans: (c) $\frac{1}{2}$

Sample spaces = {1, 2, 3, 4, 5, 6}

n(S) = 6

Numbers divisible by 2 are 2, 4 and 6

n(E) =3

P(E) = $\frac{3}{6} = \frac{1}{2}$

6. Ans: d) $\frac{1}{6}$

Sample spaces = {(1, 1), (1, 2) … (1, 6)

(2, 1), (2, 2) ... (2, 6)

(3, 1), (3, 2) ... (3, 6)

(4, 1), (4, 2) ... (4, 6)

(5, 1), (5, 2) ... (5, 6)

(6, 1), (6, 2) ... (6, 6)}

n(S) = 36

Favorable cases to get doublets are = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

n(E) = 6

Probability = $\frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

7. Ans: (a)

$\mathbf{n(s) = \{(1,1),(1,2),\ldots\ldots(6,6)\}}$

$\mathbf{n(s) = 36}$

Favorable cases to get prime numbers as sum= {(1,1) (1,2) (1,4) (1,6) (2,1), (2,3) (2,5) (3,2), (3,4) (4,1) (4,3) (5,2) (5,6) (6,1), (6,5)}

n(A)= 15

$\mathbf{P(A) = \frac{n(A)}{n(s)} = \frac{15}{36} = \frac{5}{12}}$

8. Ans: $\frac{1}{3}$

$\mathbf{n(S) = 6 \times 6 = 36}$

Event of getting a sum of numbers shown on the two faces divisible by 5 or 6 = [(1,4), (1,5), (2,3), (2,4), (3,2), (3,3), (4,1), (4,2),(4,6),(5,1), (5,5),(6,4), (6,6)]

n(E) =13

P(E) = $\frac{n(E)}{n(S)} = \frac{13}{36}$

9. Ans: (a) $\frac{2}{13}$

There are 52 playing cards in a pack of cards. So, n(S) = 52

(a) Getting a queen:

There are 4 queen cards, one from each verity of symbols.

So, n (A) = 4

b) Getting an ace:

There are 4 aces in a pack. So, n (B) = 4

P (A) = $\frac{n(A)}{n(S)} = \frac{4}{52}$ and P(B) = $\frac{n(B)}{n(S)} = \frac{4}{52}$

P (A∩B) = $\frac{0}{52}$ (There is no cards common between them)

So, P (A∪B) = P (A) + P (B) – P (A∩B)

= $\frac{4}{52} + \frac{4}{52} - \frac{0}{52}$

= $\frac{8}{52} = \frac{2}{13}$

10. Ans: (d) $\frac{25}{52}$

Let A=letter card and B=Heart

Required probability = (A or B)

P(A∪B) = P(A)+P(B)-P(A∩B)

= $\frac{16}{52} + \frac{13}{52} - \frac{4}{52} = \frac{25}{52}$

11. Ans: (a) $\frac{1}{4}$

Total number of ways drawing one card from 52 cards=$^{52}C_1$

One diamond card can be chosen in $^{13}C_1$ ways.

P (getting diamond) =$\frac{13C1}{52C1} = \frac{13}{52} = \frac{1}{4}$

12. Ans: (a) $\frac{8}{663}$

Probability (a king and a Queen) =$\frac{4C1 \times 4C1}{52C2} = \frac{4 \times 4}{\frac{52 \times 51}{2}} = \frac{8}{663}$

13. Ans: (e) $\frac{1}{5525}$

P (getting queen) =$\frac{4C3}{52C3} = \frac{4}{22100} = \frac{1}{5525}$

14. Ans: (d) $\frac{1}{10}$

Here n(S) = 100 (As mentioned in the problem)

Favorable cases to get the square cards = {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}

n(E) = 10

Probability = $\frac{n(E)}{n(S)} = \frac{10}{100} = \frac{1}{10}$

15. Ans: (a) $\frac{21}{49}$

Sample space = n(S) = 49

n(A) = favorable cases divisible by 3

= {3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, and 48}

n(A) = 16

P(A) = $\frac{n(A)}{n(S)} = \frac{16}{49}$

n(B) = favorable cases, perfect square = {1, 4, 9, 16, 25, 36, 49}

n(B) = 7

P(B) = $\frac{n(B)}{n(S)} = \frac{7}{49}$

n(A∩B) = {9, 36} = 2

P(A∩B) = $\frac{n(A \cap B)}{n(S)} = \frac{2}{49}$

P(A∪B) = P (A)+P (B)- P (A∩B)

= $\frac{16}{49} + \frac{7}{49} - \frac{2}{49} = \frac{21}{49}$

P (A∪B) = $\frac{21}{49}$

16. Ans: (c)

Probability of drawing white ball from first bag =$\frac{\text{No of white ball}}{\text{total ball}} = \frac{4}{6} = \frac{2}{3}$

Probability of drawing white ball from second bag=$\frac{\text{No of white ball}}{\text{total ball}} = \frac{3}{8}$

Since, these are independent

The probability of both the balls is white = $\frac{2}{3} \times \frac{3}{8} = \frac{2}{8} = \frac{1}{4}$

17. Ans: (a)

A can solve 80% of problems

n(A) = 80% =$\frac{80}{100}$

P( ) = 1-(P(A)) = $1 - \frac{80}{100} = \frac{20}{100}$

B can solve 70% of problems

n (B) = 70% = $\frac{70}{100}$

$P(B) = 1 - \frac{70}{100} = \frac{-30}{100}$

$\textbf{required probability} = P(A)P(\overline{B}) + P(B)P(\ )$

$= \frac{80}{100} \times \frac{30}{100} + \frac{70}{100} \times \frac{20}{100}$

$= \frac{4}{5} \times \frac{3}{10} + \frac{7}{50}$

$= \frac{6}{25} + \frac{7}{50} = \frac{19}{50}$

18. Ans: (b) $\frac{1}{4}$

's' occupies the first place

$$\textbf{required probability} = P(A) = \frac{n(A)}{n(s)}$$

n(s) = no of samples = 4

n(A) = no of letters excluding 's'

= 4-1=3 letters

n(A) = 3!

P(A) = $\frac{3!}{4!} = \frac{1}{4}$

19. Ans: (c)

$$n(S) = 5 + 8 + 7 + 4 = 24$$

$$n(A) = \text{green balls} = 4$$

$$P(A) = \frac{n(A)}{n(s)} = \frac{4}{24} = \frac{1}{6}$$

Required ratio P( ) = 1- P(A)= $1 - \frac{1}{6} = \frac{5}{6}$

20. Ans: (c)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.45 = 0.15 + P(A) - P(A \cap B)$$

$$0.45 = 0.15 + P(B) - (0.15 \times P(B))$$

$$0.45 = 0.15 + 0.85P(B)$$

$$0.85P(B) = 0.30$$

$$P(B) = \frac{0.30}{0.85} = \frac{30}{85} = \frac{6}{17}$$

21. Ans: (c) $\frac{1}{3}$

Let b stand for boy and g for girl. The sample space of the experiment is S= {(b, b), (g, b), (b, g), (g, g)}

There are two events

A---> both the children are girls

B----> at least one of the child is a girl

{(g, g)} and {(g, b),(b, g),(g, g)}

A∩B = {g, g}

P(B)=$\frac{3}{4}$ and P(A∩B)=$\frac{1}{4}$

P(A/B)=$\frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$ ∵ conditional probability