20.COMPOUND INTEREST

In compound interest, the interest is added to the principal at the end of each period and the amount thus obtained becomes the principal for the next period. The process is repeated till the end of the specified time.

If P = Principal

R = Rate per cent p.a

Time = n years

A = Amount; CI = Compound Interest

When the interest is compounded annually

Amount after n years = (A) = $\boldsymbol{P\left(1+\frac{R}{100}\right)^n}$

Compound Interest = $\boldsymbol{P\left(1+\frac{R}{100}\right)^n - P = P\left[\left(1+\frac{R}{100}\right)^n - 1\right]}$

Important Formulae

1. If the rate of interest differs from year to year i.e. $\boldsymbol{R_1}$ in the first year, $\boldsymbol{R_2}$ in the second year,$\boldsymbol{R_3}$ in the third year. Then, $A = \boldsymbol{P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)}$
2. When the principal changes every year, we say that the interest is compounded annually. Then,

$$A = \boldsymbol{P\left(1+\frac{R}{100}\right)^n}$$

3.When the principal changes every six months, we say that the interest is compounded half yearly or semi-annually. Then,

$$A = \boldsymbol{P\left(1+\frac{\frac{R}{2}}{100}\right)^{2n}}$$

4. When the principal changes every three months, we say that the interest is compounded quarterly Then,

$$A = \boldsymbol{P\left(1+\frac{\frac{R}{4}}{100}\right)^{4n}}$$

5. When the principal changes after every month, we say that the interest is compounded monthly Then,

$$A = \boldsymbol{P\left(1+\frac{\frac{R}{12}}{100}\right)^{12n}}$$

6. When the interest is compounded annually but time is in fraction say $\boldsymbol{2\frac{3}{4}}$ year Then,

$$\boldsymbol{A = P\left(1+\frac{R}{100}\right)^2\left(1+\frac{\frac{3R}{4}}{100}\right)}$$

7. The difference between the simple interest and compound interest for 2 years (or terms) is given by the formula

$$\boldsymbol{D = P\left(\frac{R}{100}\right)^2}$$

Where D is the difference, P is the principal and R is the rate of interest.

8. Present worth of x % due n years, hence is given by

Present worth = $\boldsymbol{\frac{X}{\left(1+\frac{R}{100}\right)^n}}$

Example1: Find the compound interest on Rs. 5500 at 9% per annum for 2 years, if the interest is compounded annually?

Solution: P = Rs. 5500, *R* = 10% per annum and *n* = 2 years

$$A = 5500\left(1+\frac{9}{100}\right)^2 = 5500\frac{109}{100}\times\frac{109}{100}$$

= Rs. 6534.56

$$\therefore CI = A - P = 6534.55 - 5500$$

=Rs. 1034.55

Example 2: Find the compound interest on Rs. 12000 for 3 years, if the rate of interest for first year is 5%, second year is 6% and third year is 7%.

Solution: *P*= Rs. 12000, R_1= 5%, R_2 = 6% and R_3= 7%

Amount $= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$

$= \text{Rs. } \left[12000\left(1+\frac{5}{100}\right)\left(1+\frac{6}{100}\right)\left(1+\frac{7}{100}\right)\right]$

$= \text{Rs. } \left[12000\times\frac{105}{100}\times\frac{106}{100}\times\frac{107}{100}\right]$

$= \text{Rs. } 14290.92$

Compound interest = Rs. (14290.92 - 12000) = Rs. 2290.92

Example 3: If simple interest on a sum of money at $7\frac{1}{2}$% per annum for 3 year is Rs. 1800. Find the compound interest on the same sum for same period at the same rate.

Solution: Here, Rate = $\frac{15}{2}$ % per annum, Time = 3 years, SI = Rs. 1800

Principal = $\text{Rs. } \left(\frac{1800\times100}{3\times\frac{15}{2}}\right)$ = Rs. 8000

Amount = $P\left(1+\frac{R}{100}\right)^n = \text{Rs. } \left[8000\left(1+\frac{\frac{15}{2}}{100}\right)^3\right] = \text{Rs. } \left[8000\left(\frac{215}{200}\right)^3\right]$

$= \text{Rs. } \left[8000\times\frac{43}{40}\times\frac{43}{40}\times\frac{43}{40}\right] = \text{Rs. } 1938.38$

CI = Rs. [9938.38 - 8000] = Rs. 1938.38

Example 4: The difference between the compound interest and simple interest on a certain sum at 10% per annum for 2 year is Rs. 150. Find the sum.

Solution: Here, *D* = *Rs.* 150, *T* = 2 year, *R* = 10%

$$\therefore D = \frac{PR^2}{100^2} \Rightarrow 150 = \frac{P\times100}{100\times100} \Rightarrow P = 15000$$

Example 5: In what time will Rs. 64000 invested at 5% per annum fetch an interest of Rs. 4921, the interest being compounded half yearly.

Solution : A= P + CI = $P\left(1+\frac{R}{100}\right)^n$

$$\Rightarrow 64000 + 4921 = 64000\left[1+\frac{5}{100\times2}\right]^{2t}$$

$$\Rightarrow \frac{68921}{64000} = \left(\frac{41}{40}\right)^{2t}$$

$$\Rightarrow \left(\frac{41}{40}\right)^3 = \left(\frac{41}{40}\right)^{2t}$$

$$\Rightarrow 2t = 3 \Rightarrow t = \frac{3}{2}\ year$$

Example 6: The value of a TV that was purchased in January 1999, depreciates at 12% per annum. If its value in January 2001 is *Rs.* 4840, then what was the purchase price of TV?

Solution: Let P be the price of TV in January 1999, then the value of TV in January 2001, i.e. after two years.

$$= P\left(1 - \frac{R}{100}\right)^2$$

$$\Rightarrow 4840 = P\left(1 - \frac{12}{100}\right)^2$$

$$\Rightarrow 4840 = P\left(\frac{22}{25}\right)^2$$

$$P = \frac{4840 \times 25 \times 25}{22 \times 22}$$

$$\Rightarrow P = \text{Rs. } 6250$$

EXERCISE

1. Find the compound interest on Rs. 9375 at 8% per annum for 2 yr.
   (a) Rs.1560 (b) *Rs.*1512
   (c) Rs.1590 (d) Rs.1548
2. The compound interest on Rs.10240 at $6\frac{1}{4}$% per annum for 2 yr 73 days is
   (a) Rs. 1464.50 (b) Rs. 1664.50
   (c) Rs. 1480 (d) Rs. 1580.50
3. The difference between compound Interest and simple interest on an amount of Rs. 15,000 for 2 years is Rs. 96. What is the rate of interest per annum?
   (a) 8 (b) 11
   (c) 12 (d) None of these
4. The difference between the simple interest the compound interest on Rs. 8000 at 10% per annum 3 yr is
   (a) Rs. 260 (b) Rs. 352
   (c) Rs.248 (d) Rs. 310
5. The present worth of Rs. 2809 due 2 yr hence at 6% per annum, is
   (a) Rs. 2100 (b) Rs. 2600
   (c) Rs. 2400 (d) Rs. 2500

9. The effective annual rate of interest corresponding to a nominal rate of 6% per annum payable half-yearly is:
   (a) 6.10% (b) 6.11 %
   (c) 6.08% (d) 6.09%

10. A sum of money placed at compound interest doubles itself in 5 years. It will amount to eight times itself at the same rate of interest in:
   (a) 7 years (b) 12 years
   (c) 15 years (d) 30 years

8. A sum of Rs. 12,000 deposited at compound interest becomes double after 5 years. After 20 years, it will become:
   (a) Rs. 1,10,000 (b) Rs. 1,30,000
   (c) Rs. 1,24,000 (d) Rs. 1,92,000

9. The least number of complete years in which a sum of money put out at 20% compound interest will be more than doubled is:
   (a) 7 (b) 4
   (c) 5 (d) 8

10. Sum of money becomes Rs.13,380 after 3 years and Rs. 20,070 after 6 years on compound interest. The sum is:
   (a) Rs. 9200 (b) Rs. 9000
   (c) Rs. 8920 (d) Rs. 9040

11. A sum of money invested at compound interest amounts to Rs. 800 in 3 years and to Rs. 840 in 4 years. The rate of interest per annum is:
   (a) 4% (b) 8%

(c) 5% (d) 6%

14. In a factory, the production of cement rises to 2420 tons from 2000 tons in two years. Find the rate of growth per annum.
(a) 8% (b) 9%
(c) 10% (d) 11%

13. What is the difference between SI and CI for 2 yr on Rs.10000, when the rate of interest is 11% for the first year and 12% for second year?
(a) Rs. 120 (b) Rs. 132
(c) Rs. 144 (d) Rs. 128

14. A man borrows Rs. 2550 to be paid back with compound interest at the rate of 4% per annum by the end of 2 years in two equal yearly installments. How much will each installment be?
(a) Rs. 1275 (b) Rs.1383
(c) Rs.1352 (d) Rs. 1287

15. Mr. Dua invested money in two schemes A and B offering compound interest @ 8 p.c.p.a. and 9 p.c.p.a. respectively. If the total amount of interest accrued through two schemes together in two years was Rs. 4818.30 and the total amount invested was Rs. 27,000, what was the amount invested in Scheme A?
(a) Rs. 12,000 (b) Rs. 14,500
(c) Rs. 16,000 (d) cannot be determined

16. The compound interest on a sum of money for 2 years is Rs. 832 and the simple interest on the same sum for the same period is Rs. 800. The difference between the compound interest and the simple interest for 3 years will be:
(a) Rs. 50 (b) Rs. 6 7
(c) Rs. 98.56 (d) Rs. 75.45

17. The difference between the simple Interest on a certain sum at the rate of 10% per annum for 2 years and compound interest which is compounded every 6 months is Rs. 124.05. What is the principal sum?
(a) Rs. 9000 (b) Rs.8000
(c) Rs.10,000 (d) Rs. 13,000

18. On a sum of money, the simple interest for 2 years is Rs. 660, while the compound interest is Rs. 696.30, the rate of interest being the same in both the cases. The rate of interest is:
(a) 13% (b) 14%
(c) 12% (d) 11%

19. A person lent out a certain sum on simple interest and the same sum on compound interest at a certain rate of interest per annum. He noticed that the ratio between the difference of compound interest and simple interest of 3 years and that of 2 years is 25: 8. The rate of interest per annum, is
(a) 13% (b) 14%
(c) 12% (d) $\mathbf{12\frac{1}{2}}$%

20. A man borrows Rs. 12,500 at 20% compound interest. At the end of every year he pays Rs. 2000 as part repayment. How much does he still owe after three such installments?
(a) Rs. 14,000 (b) Rs. 13,684
(c) Rs. 15,600 (d) None of these

21. A man deposited a total sum of Rs. 88400 in the name of his two sons aged 19 and 17 yr, so that at the age of 21 , both will get equal amounts. If the money is invested at the rate of 10% compound interest per annum. What are the shares of his two sons?
(a) Rs. 48800, Rs. 40000
(b) Rs. 48400, Rs. 48000
(c) Rs. 48400, Rs.40000
(d) Rs. 48000, Rs. 40000

22. If the compound interest, on a certain sum for 2 yr at 3 percent be Rs. 101.50, what would be the simple interest?
(a) Rs. 100 (b) Rs. 98
(c) Rs. 96.50 (d) Rs. 94

23. The compound interest on a certain sum for 2 yr is Rs. 40.80 and the simple interest is Rs. 40. Find the sum.
(a) Rs.400 (b) Rs. 450
(c) Rs.500 (d) Rs. 550

ANSWER KEY									
1	a	6	d	11	c	16	c	21	c
2	a	7	c	12	c	17	b	22	a
3	a	8	d	13	b	18	d	23	c
4	c	9	b	14	c	19	d		
5	d	10	c	15	a	20	d		

SOLUTIONS

1. A = $P\left(1+\frac{R}{100}\right)^t$

= Rs. $\left[9375\times\left(1+\frac{8}{100}\right)^2\right]$

= Rs. $\left[9375\times\frac{27}{25}\times\frac{27}{25}\right]$ = Rs. 10935

CI = 10935-9375 = Rs. 1560

2. P = Rs. 10240, R = $\frac{25}{4}$%, T = 2 years 73 days

A = $[10240\times\left(1+\frac{25}{4\times100}\right)^2\times(1+\frac{73}{365}\times\frac{25}{4\times100})]$

= Rs. $\left[10240\times\frac{17}{16}\times\frac{17}{16}\times\left(1+\frac{1}{80}\right)\right]$

= Rs. $\left(10240\times\frac{17}{16}\times\frac{17}{16}\times\frac{81}{80}\right)$

= Rs. 11704.50

CI = Rs. (11704.50-10240)

= Rs. 1464.50

3. $\left[15000\times\left(1+\frac{R}{100}\right)^2-15000\right]-\left(\frac{15000\times R\times 2}{100}\right)=96$

$\Leftrightarrow 15000\left[\left(1+\frac{R}{100}\right)^2-1-\frac{2R}{100}\right]=96$

$\Leftrightarrow 15000\left[\frac{(100+R)^2-10000-200R}{10000}\right]=96$

$\Leftrightarrow R^2=\frac{96\times2}{3}=64$

$\Leftrightarrow R=8.$

∴ Rate = 8%

Method-II: $P\left(\frac{r}{100}\right)^2=96$

$15000\times\left(\frac{r}{100}\right)^2=96$

$\Rightarrow r^2=64$

$\Rightarrow r=8$

4. SI = Rs. $\left[\frac{8000\times3\times10}{100}\right]$ = Rs. 2400

CI = Rs. $\left[8000\times\left(1+\frac{10}{100}\right)^3-8000\right]$

= Rs. [10648 - 8000] = Rs. 2648

∴ Difference = Rs. [2648 - 2400] = Rs. 248

5. PW = Rs. $\left\{\frac{2809}{\left(1+\frac{6}{100}\right)^2}\right\}$

= Rs. $\left(2809\times\frac{50}{53}\times\frac{50}{53}\right)$

= Rs. 2500

6. Amount of Rs.100 for 1 year when compounded half-yearly

= Rs. $\left[100\times\left(1+\frac{3}{100}\right)^2\right]$ =Rs. 106.09

Effective rate = (106.09-100) %

= 106.09.

7. $P\left(1+\frac{R}{100}\right)^5=2P\Rightarrow\left(1+\frac{R}{100}\right)^5=2$

Let $\left(1+\frac{R}{100}\right)^n=8P$

$\Rightarrow\left(1+\frac{R}{100}\right)^n=8=2^3$

$=\left\{\left(1+\frac{R}{100}\right)^5\right\}^3=\left(1+\frac{R}{100}\right)^n$

$=\left(1+\frac{R}{100}\right)^{15}\Rightarrow n=15$

∴ Required time = 15 years.

8. $12000\times\left(1\times\frac{R}{100}\right)^5=24000$

$\Rightarrow\left(1+\frac{R}{100}\right)^5=2$

$\therefore\left[\left(1+\frac{R}{100}\right)^5\right]^4=2^4=16$

$\Rightarrow\left(1+\frac{R}{100}\right)^{20}=16$

$$\Rightarrow P\left(1+\frac{R}{100}\right)^{20}=16P$$

$$\Rightarrow 12000\left(1+\frac{R}{100}\right)^{20}=16\times 12000$$

$$=192000$$

9. $P\left(1+\frac{20}{100}\right)^{n}>2P$ *or* $\left(\frac{6}{5}\right)^{n}>2$

Now, $\left(\frac{6}{5}\times\frac{6}{5}\times\frac{6}{5}\times\frac{6}{5}\right)>2$.

So, n = 4 years.

10. Rate of increase from 13380 to 20070=50%

Hence sum must be 8920

11. S.I. on Rs. 800 for 1 year = Rs. (840 - 800)

= Rs. 40.

$$\therefore \ Rate=\left(\frac{100\times 40}{800\times 1}\right)\%=5\%$$

12. Production after two years = 2420

$$\Rightarrow 2000[1+\tfrac{R}{100}]^2=2420$$

$$\Rightarrow \left(1+\frac{R}{100}\right)^2=\frac{2420}{2000}=\left(\frac{11}{10}\right)^2$$

$$\Rightarrow 1+\frac{R}{100}=\frac{11}{10}\Rightarrow R=10\%$$

13. For first year SI = CI

Therefore, the required difference

= SI at 12% for one year and SI at 11% for one year on Rs. 10000

$$=\left(\frac{10000\times 11}{100}\right)\times\frac{12}{100}=\text{Rs. } 132$$

14. Let the value of each installment be Rs. x. Then, (P.W. of Rs. x due 1 year hence) + (P.W. of Rs. x due 2 years hence) = Rs. 2550

$$\Leftrightarrow \frac{x}{(1+\frac{4}{100})}+\frac{x}{(1+\frac{4}{100})^2}=2550$$

$$\Leftrightarrow \frac{25x}{26}+\frac{625x}{676}=2550$$

$$\Leftrightarrow 1275x=2550\times 676$$

$$\Leftrightarrow x=\left(\frac{2550\times 676}{1275}\right)=1352$$

Value of each installment = Rs. 1352.

15. Let the investment in scheme A be Rs. x. Then, investment in scheme B

= Rs. $(27000-x)$.

$$\therefore \begin{bmatrix} x\times\left\{\left(1+\frac{8}{100}\right)^2-1\right\} \\ +(27000-x)\left\{\left(1+\frac{9}{100}\right)^2-1\right\}\end{bmatrix}$$

= 4818.30.

$$\Leftrightarrow \left(x\times\frac{104}{625}\right)+\frac{1881(27000-x)}{10000}=\frac{481830}{100}$$

$$\Leftrightarrow 1664x+1881(27000-x)$$

= 48183000 (188l x – 1664 x)

= (50787000 - 48183000)

217 x = 2604000

$$\Leftrightarrow x=\frac{2604000}{217}=12000$$

16. SI of. the first year = Rs. $\frac{800}{2}$ = Rs. 400

So, we can say that Rs. 32 is the simple interest on Rs. 400 for one year. Therefore,

R = $\frac{SI\times 100}{P\times T}=\frac{0.80\times 100}{20\times 1}$ = 4%

$$=\text{S.I. on } Rs.832=Rs.\left(\frac{832\times 8\times 1}{100}\right)$$

$$=66.56$$

Total Difference = (32 + 66.56)

$$=98.56$$

17. Let the sum be Rs. P. Then

$$P\left[1+\left(\frac{5}{100}\right)^4-1\right]-\frac{P\times 10\times 2}{100}$$

$$=124.05$$

$$\Rightarrow P\left[\left(\frac{21}{20}\right)^4-1-\frac{1}{5}\right]=124.05$$

$$\Rightarrow P\left[\frac{194481}{160000}-\frac{6}{5}\right]=\frac{12405}{100}$$

$$\Rightarrow P=\left(\frac{12405}{100}\times\frac{160000}{2481}\right)=8000$$

18. Difference in C.I. and S.I. for 2 years

= Rs. (696.30 - 660) =Rs. 36.30.

S.I. for one year = Rs. 330.

∴ S.I. on Rs. 330 for 1 year = Rs. 36.30

$$\therefore Rate=\left(\frac{100\times 36.30}{330\times 1}\right)\%=11\%$$

19. Let the principal be Rs. P and rate of interest be R% per annum.

Difference of C.I. and S.I. for 2 years

$= \left[P \times \left(1 + \frac{R}{100}\right)^2 - P\right] - \left(\frac{P \times R \times 2}{100}\right) = \frac{PR^2}{10^4}$

Difference of C.I. and S.I. for 3 years

$= \left[P \times \left(1 + \frac{R}{100}\right)^3 - P\right] - \left(\frac{P \times R \times 3}{100}\right)$

$= \frac{PR^2}{10^4}\left(\frac{300 + R}{100}\right)$

$\therefore \frac{\frac{PR^2}{10^4}\left(\frac{300+R}{100}\right)}{\frac{PR^2}{10^4}} = \frac{25}{8} \Rightarrow \left(\frac{300 + R}{100}\right)$

$= \frac{25}{8} \Rightarrow R = \frac{100}{8} = 12\frac{1}{2}\%$

20. Balance $= \left[\left\{12500 \times \left(1 + \frac{20}{100}\right)^3\right\} - \left\{2000 \times \left(1 \times \frac{20}{100}\right)^2 + 2000 \times \left(1 \times \frac{20}{100}\right) + 2000\right\}\right]$

=Rs. $\left[\left(12500 \times \frac{6}{5} \times \frac{6}{5} \times \frac{6}{5}\right) - \left(2000 \times \frac{6}{5} \times \frac{6}{5} + 2000 \times \frac{6}{5} + 2000\right)\right]$

= Rs. [21600-(2880+2400+2000)]

= Rs. 14320.

21. Let *x* and y be .the shares of elder and younger sons respectively. The amounts invested in their names fetch interest for 21 - 19 = 2 year and 21 - 17 = 4 year respectively. Since, the two sons are to receive equal amounts when they attain 21 year. We have,

$x\left(1 + \frac{10}{100}\right)^2 = y\left(1 + \frac{10}{100}\right)^4$

$\Rightarrow \frac{x}{y} = \left(1 + \frac{10}{100}\right)^2 = \frac{121}{100}$

$\Rightarrow x : y = 121 : 100$

$\therefore x = \frac{121}{221} \times 88400 = 48400$

and y = $\frac{100}{221} \times 88400 = 40000$

Hence, required shared of sons are Rs. 48400 and Rs. 40000 respectively.

22. CI = $P\left[\left(1 + \frac{R}{100}\right)^n - 1\right]$

$\Rightarrow 101.50 = P\left[\left(1 + \frac{3}{100}\right)^2 - 1\right]$

$\Rightarrow 101.50 = P\left[\left(\frac{103}{100}\right)^2 - 1\right]$

$\Rightarrow 101.50 = P \times \frac{609}{10000}$

$\Rightarrow P = \frac{101.50 \times 10000}{609} = \frac{10000}{6}$

Now, SI = $\frac{PRT}{100} = \frac{10000 \times 3 \times 2}{6 \times 100} = 100$

23. Difference between CI and SI

D = Rs. 40.80 - 40.00 = Rs. 0.80

SI of. the first year = Rs. $\frac{40}{2}$ = Rs. 20

So, we can say that Rs. 0.80 is the interest of Rs. 20 for one year. Therefore,

R = $\frac{SI \times 100}{P \times T} = \frac{0.80 \times 100}{20 \times 1}$ = 4%

Now, principal = $\frac{SI \times 100}{RT} = \frac{40 \times 100}{4 \times 2}$ = Rs. 500