16. TIME AND DISTANCE

Relation between Time, Speed and Distance

Distance covered, time and speed are related by

$$Time = \frac{distance}{speed} \ldots\ldots\ldots\ldots\ldots(i)$$

$$Speed = \frac{Distance}{Time} \ldots\ldots\ldots\ldots\ldots(ii)$$

$$Distance = Speed \times Time \ldots\ldots\ldots\ldots(iii)$$

• Distance is measured in meters, kilo meters and miles.

• Time in hours, minutes and seconds.

• Speed in km/h, miles/h and m/s.

1. To convert speed of an object from km/h to m/s multiply the speed by $\frac{5}{18}$.

$$\left(1\ kmph = \frac{5}{18} m/s\right)$$

2. To convert speed of an object from m/s to km/h, multiply the speed by $\frac{18}{5}$.

$$\left(1 m/s = \frac{18}{5} kmph\right)$$

Average Speed

It is the ratio of total distance covered to total time of journey.

∴Average Speed = $\frac{Total\ distance\ covered}{\text{Total time of journey}}$

General Rules for Solving Time & Distance Problems

Rule 1

If a certain distance is covered with a speed of $x\ km/h$ and another equal distance with a speed of y km/h, then the average speed for the whole journey is the harmonic mean of the two speeds.

Average speed = $\left(\frac{2}{\frac{1}{x}+\frac{1}{y}}\right)$km/h = $\left(\frac{2xy}{x+y}\right)$km/h

Rule 2

If three equal distances are covered by three different speeds *x, y* and *z* km/h, then average speed for the whole journey is given by

Average speed = $\left(\frac{3}{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}\right)$km/h = $\left(\frac{3xyz}{xy+yz+zx}\right)$km/h

Rule 3

If a certain distance is covered with a speed of $x\ km/h$ and another distance with a speed of y km/h but time interval for both journeys being same, then average speed for the whole journey is given by

Average Speed = $\left(\frac{x+y}{2}\right)$km / h

Rule 4

If a certain distance is covered with a speed of *x, y* and *z* km/h, but time interval for the three journeys being equal, then average speed is given by

Average speed = $\left(\frac{x+y+z}{3}\right)$km/h

Rule 5

If the ratio of speeds *A* and *B* is *x: y,* then the ratio of times taken by them to cover the same distance is $\frac{1}{x}:\frac{1}{y}$

Relative Speed

(i) If two bodies are moving in the same direction at *x* km/h and *y* km/h, where *(x > y),* then their relative speed is given by *(x - y)* km/h.

(ii) If two bodies are moving in the opposite direction at *x* km/h and *y* km/h, then their relative speed is given by (x + *y)* km/h.

General Rules for Solving Circular Tracks

Rule 1

When two people are running around a Circular Track starting at the same point and at the same time, then whenever the two people meet, the person moving with a greater speed covers one round more than the person moving with lesser speed.

Rule 2

When two people with speeds of *x* km/h and *y* km/h start at the same time and from the same point in the same direction around a circular track of circumference 'c' km, then,

The time taken to meet for the first time anywhere on the track = $\frac{c}{x-y}\boldsymbol{h}$

The time taken to meet for the first time at the starting point = LCM of $\left(\frac{c}{x},\frac{c}{y}\right)\boldsymbol{h}$

Rule 3

When two people with speeds of *x* km/h and *y* km/h respectively start at the same time and from the same point but in opposite direction around a circular track of circumference 'c' km, then

The time taken to meet for the first time anywhere on the track = $\frac{c}{x+y}\boldsymbol{h}$

The time taken to meet for the first time at the starting point = LCM of $\left(\frac{c}{x},\frac{c}{y}\right)\boldsymbol{h}$

Example 1: Convert 90 km/h into m/s.

Solution: 90 km/h = $\left(\mathbf{90}\times\frac{5}{18}\right)$m/s= 25 m/s

Example 2: Convert 10 m/s into km/h.

Solution: 10 m/s = $\left(\mathbf{10}\times\frac{18}{5}\right)$m/s = 36km/h

Example 3: A man can cover a certain distance in 1 h 30 min by covering one-third of the distance at 6 km/h and the rest at 15 km/h. Find the total distance.

Solution: Let the total distance be x km. Then, $\frac{\frac{x}{3}}{6}+\frac{\frac{2x}{3}}{15}=\frac{3}{2}\Rightarrow\frac{x}{18}+\frac{2x}{45}=\frac{3}{2}\Rightarrow\frac{9x}{90}=\frac{3}{2}\Rightarrow\frac{x}{10}=\frac{3}{2}$

$$\Rightarrow x=\frac{(3\times 10)}{2}=15\ km$$

Example 4: An aero plane started one hour later than the scheduled departure from a place 1200 km away from its destination. To reach the destinations on time, the pilot had to increase its speed by 200 km/h. What was the normal speed of the aero plane?

Solution: Let the time taken by the aero plane in second case be x hour. Then,

$$\frac{1200}{x}=\frac{1200}{x+1}+200\Rightarrow\frac{6}{x}=\frac{6}{x+1}+1$$

$$\Rightarrow 6x+6=6x+x^2+x\Rightarrow x^2+x-6=0$$

$$\Rightarrow(x+3)(x-2)=x\Rightarrow x=2h \qquad (\because -3\ is\ not\ posible)$$

∴ Time taken in second case = 2 h

So, Speed = $\frac{1200}{2}$ =600 km/h

Hence, normal speed = 600 - 200 = 400 km/h

Example 5: Speed of three cars are in the ratio 2: 3: 4. What is the ratio of time taken by them in covering the same distance

Solution: Let the speeds of three cars be 2x, 3x and 4x km/h, covered distance be *d*, then ratio of time taken by them

$$=\frac{d}{2x}:\frac{d}{3x}:\frac{d}{4x}=\frac{1}{2}:\frac{1}{3}:\frac{1}{4}=6:4:3$$

Example 6: Two men *A* and *B* start together from the same point to walk around a circular path 8 km long. *A* walks 2 km and *B* walks 4 km an hour. When will they next meet at the starting point, if they walk in the same direction?

Solution: Time to complete one revolution by *A* and *B* is $\left(\frac{8}{2}\right)\boldsymbol{h}$ and $\left(\frac{8}{4}\right)\boldsymbol{h}$ or 4h and 2h

∴ The required time is the LCM of 4 and 2 which is 4 h.
Thus, they will meet next time at the starting point after 4 h.

EXERCISE

1. Which of the following speed is the fastest?
(a) 40 m/s (b) 144 km/h
(c) 2400 m/min (d) All are equal

2. Mac travels from *A* to *B* a distance of 250 miles in $\mathbf{5\frac{1}{2}}$ hours. He returns to A in 4 hours 30 minutes. His average speed is:
(a) 42 mph (b) 49 mph
(c) 48 mph (d) 50 mph

3. A, *B* and C are on a trip by a car. A drives during the first hour at an average speed of 50 km/hr. *B* drives during the next 2 hours at an average speed of 48 km/hr. C drives for the next 3 hours at an average speed of 52 km/hr. They reached their destination after exactly 6 hours. Their mean speed was:
(a) 50 km/hr (b) $50\frac{1}{3}$ km/hr
(c) 51 km/hr (d) 52 km/hr

4. A car travels the first one-third of a certain distance with a speed of 10 km/hr, the next one-third distance with a speed of 20 km/hr, and the last one-third distance with a speed, of 60 km/hr. The average speed of the car for the whole journey is:
(a) 18 km/hr (b) 34 km/hr
(c) 35 km/hr (d) 39 km/hr

5. Mary jogs 9 km at a speed of 6 km per hour. At what speed would she need to jog during the next 1.5 hours to have an average of 9 km per hour for the entire jogging session?
(a) 9 kmph (b) 13 kmph
(c) 12 kmph (d) 15 kmph

6. A car travelling with 5/7 of its actual speed covers 42 km in 1 hr 40 min 48 sec. find the actual speed of the car.
(a) 17 km/hr (b) 32 km/hr
(c) 31 km/hr (d) 35 km/hr

7. Starting from his house one day, a student walks at a speed of $2\frac{1}{2}$ kmph and reaches his school 6 minutes late. Next day he increases his speed by 1 kmph and reaches the school 6 minutes early. How far is the school from his house?
(a) 1 km (b) 15 km
(c) 14 km (d) $1\frac{3}{4}$ km

8. If a train runs at 40 kmph, it reaches its destination late by 11 minutes but if it runs at 50 kmph, it is late by 5 minutes only. The correct time for the train to complete its journey is:
(a) 13 min. (b) 17 min.
(c) 19 min. (d) 22 min

9. Walking 5/7 of his usual rate, a boy reaches his school 6 min late. Find his usual time to reach school.
(a) 10 min (b) 12 min
(c) 15 min (d) 18 min

10. If I walk at 4 km/h, I miss the bus by 10 min. If I walk at 5 km/h, I reach 5 min before the arrival of the bus. How far I walk to reach the bus stand?
(a) 5 km (b) 5.5 km
(c) 6 km (d) 7.5 km

11. A man travels on a scooter from *A* to *B* at a speed of 30 km/h and returns back from *B* to *A* at 20 km/h. The total journey was performed by him in 10 h. Find the distance from *A* to *B.*
(a) 100 km (b) 110 km
(c) 120 km (d) 125 km

12. A man walks 7.5 km at a speed of 3 km/h. At what speed would the man need to walk during the next 2 h to have an average of 4 km/h for the entire session.
(a) 3.65 km/h (b) 4.75 km/h
(c) 5.25 km/h (d) 6.50 km/h

13. An express train travelled at an average speed of 75 km/h stopping for 5 min every 125 km. How long did it take to reach its destination 375 km from the starting point?
(a) 6 h 30 min (b) 4 h 45 min

(c) 3 h 15 min (d) 5 h 10 min

14. A man performs 2/25 of his total journey by bus, 21/50 by car and the remaining 2 km on foot. Find the total journey.
(a) 4 km (b) 2.7 km
(c) 3.4 km (d) 3.8 km

15. A long distance runner runs 9 laps of a 400 m track every day. His timings (in min) for four consecutive days are 88, 96, 89 and 87 respectively. On an average, how many m/min does the runner cover?
(a) 39 m/min (b) 40 m/min
(c) 41 m/min (d) 43 m/min

16. I started on my bicycle at 7 a.m. to reach a certain place. After going a certain distance, my bicycle went out of order. Consequently, I rested for 35 minutes and came back to my house walking all the way. I reached my house at 1 p.m. If my cycling speed is 10 kmph and my walking speed is 1 kmph, then on my bicycle I covered a distance of:
(a) $4\frac{61}{66}$ km (b) $13\frac{4}{3}$ km
(c) $5\frac{3}{6}$ km (d) $15\frac{7}{8}$ km

17. The ratio between the speed of Meena and Teena is 2 : 3. Meena takes 20 min more than Teena to walk from A to B. If Meena had walked at double the speed, find the time she would take to walk from A to B.
(a) 30 min (b) 60 min
(c) 45 min (d) 110 min

18. Two men starting from the same place walk at the rate of 4 km/h and 4.6 km/h respectively. What time will they take to be 3 km apart, if they walk in the same direction?
(a) 8 h (b) 4 h
(c) 5 h (d) 6 h

19. A walks at 2 km/h and 5 h after his start, *B* cycles after him at 4 km/h. How far from the start does *B* catch up with *A*?
(a) 20 km (b) 18 km
(c) 16 km (d) 14 km

20. Three persons A, B and C run around a circular track of length 1 km, with respective speeds of 10, 20 and 25 km/h. If they start at the same point and at the same time in the same direction, when will they will meet again at the starting point.
(a) after 12 min (b) after 14 min
(c) after 16 min (d) after 18 min

21. A can give *B* a 40 m start and C 70 m start in a km race. How many metres start can *B* give C in a km race?
(a) 31 m start (b) $31\frac{1}{2}$ m start
(c) $31\frac{1}{4}$ m start (d) $31\frac{1}{7}$ m start

22. Two trains 121 m long and 99 m long are running in opposite directions, first at 40 km/h and the second at 32 km/h. In what time will they completely clear of each other from the moment they meet?
(a) 10 s (b) 11 s
(c) 13 s (d) 14 s

23. A train running at 8/11 of its own speed reached a place in $5\frac{1}{2}$ h. How much time could be saved, if the train would have run at its own speed.
(a) $2\frac{1}{2}$ h (b) 2 h
(c) $1\frac{1}{2}$ h (d) 1 h

Answer Key							
1	d	7	d	13	d	19	a
2	d	8	c	14	a	20	a
3	b	9	c	15	b	21	c
4	a	10	a	16	a	22	b
5	c	11	c	17	a	23	c
6	d	12	c	18	c		

SOLUTIONS

1. 40 m/s = $\left(40 \times \frac{18}{5}\right)$km /h =144 km/h

2400 m/min = $\frac{2400\times60}{1000}$ = 144 km/h

So, all speeds are equals.

2. Speed from *A* to *B* = $\left(250 \times \frac{2}{11}\right)$ mph

= $\left(\frac{500}{11}\right)$ mph.

Speed from *B* to A =$\left(250 \times \frac{2}{9}\right)$ mph

= $\left(\frac{500}{9}\right)$mph

Average speed = $\left(\frac{2\times\frac{500}{11}\times\frac{500}{9}}{\frac{500}{11}+\frac{500}{9}}\right)$mph

= $\left(\frac{500000}{4500+5500}\right)$mph =50mph

3. Total distance traveled

= (50 × 1 + 48 × 2 + 52 × 3) km

= (50 + 96 + 156) km = 302 km.

Total time taken = 6 hrs.

∴ Mean speed = $\left(\frac{302}{6}\right)$ km/hr

= 50 $\frac{1}{3}$ km/hr.

4. Let the whole distance traveled be *x* km and the average speed of the car for the whole journey be *y* km/hr.

Then $\frac{(x/3)}{10}+\frac{(x/3)}{20}+\frac{(x/3)}{60}=\frac{x}{y}$

$$\Leftrightarrow \frac{x}{30}+\frac{x}{60}+\frac{x}{180}=\frac{x}{y}$$

$$\Leftrightarrow \frac{1}{18}y=1 \Leftrightarrow y=18\frac{km}{hr}$$

5. Let speed of jogging be *x* km/hr.

Total time taken = $\left(\frac{9}{6}\right.$hrs+ 1.5hrs)

= 3 hrs.

Total distance covered = (9 + 1.5x) km.

$$\therefore \frac{9+1.5x}{3}=9 \Leftrightarrow 9+1.5x=27$$

$$\Leftrightarrow \frac{3}{2}x=18 \Leftrightarrow x=\left(18\times\frac{2}{3}\right)=12kmph$$

6. Time taken = 1hr 40 min 48 sec = 1 hr 40 $\frac{4}{5}$ min = $1\frac{51}{75}$ hrs = $\frac{126}{75}$ hrs

Let the actual speed be *x* km/hr,

Then, $\frac{5}{7}x\times\frac{126}{75}$ = 4 2

Or $x=\left(\frac{42\times7\times75}{5\times126}\right)$ = 35 km/hr.

7. Let the distance be *x* km.

Difference in timings = 12 min

= $\frac{12}{60}$ hr = $\frac{1}{5}$ hr.

$$\therefore \frac{2x}{5}-\frac{2x}{7}=\frac{1}{5} \Leftrightarrow 14x-10x=7$$

$$\Leftrightarrow 4x=7 \Leftrightarrow x=\frac{7}{4}$$

$$\Leftrightarrow x=1\frac{3}{4}\text{ km.}$$

8. Let the correct time to complete the journey be *x* min.

Distance covered in (x + 11) min. at 40 kmph = Distance covered in (*x* + 5) min. at 50 kmph

$$\therefore \frac{(x+11)}{60}\times40=\frac{(x+5)}{60}\times50$$

$$\frac{40x+440}{60}=\frac{50x+250}{60}$$

Or, $50x-40x=440-250$

Or, 10x=190

Or, $x=\frac{190}{10}=19min$

9. Since, the boy now walks at $\frac{5}{7}$ of usual speed, he will take $\frac{7}{5}$ of his usual time

⇒ Extra time = $\left(\frac{7}{5}-1\right)$ of usual time

= 6 min (known)

⇒ $\frac{2}{5}$ × usual time = 6

⇒ Usual time = 15 min

10. Suppose the required distance be d km/h

Then, $\frac{d}{4}-\frac{d}{5}=15min=\frac{1}{4}h$

$$\Rightarrow d\frac{1}{20}=\frac{1}{4} \Rightarrow d=5km$$

11. Let the distance be *d* km

$\therefore \frac{d}{30}+\frac{d}{20}=t_1+t_2=10$ (given)

$\Rightarrow d=\frac{10\times30\times20}{(30+20)}$ = 120 km

12. Let speed of walking be *x* km/h.

Total time taken = $\left(\frac{7.5}{3}+2\right)$ h = 4.5 h

Total distance covered = (7.5 + 2*x*) km

$$\therefore \frac{7.5+2x}{4.5}=4 \Rightarrow 7.5+2x=18$$

∴ Speed of walking = 5.25 km/h

13. Time taken to cover 375 km = $\left(\frac{375}{75}\right)$h

= 5h

Number of stoppages =$\frac{375}{125}-1$ = 2

Total time to stoppages = (5 × 2) min

= 10 min

Hence, total time taken = 5 h 10 min

14. Let the total journey be *x* km

$$\therefore \frac{2}{25}x+\frac{21}{50}x+2=x$$

$$\Rightarrow x=4\ km$$

15. Average speed (m/min) = $\frac{\textit{Total distance}}{\textit{Total time}}$ = $\frac{9\times400\times4}{(88+96+89+87)}$ =40 m/min

16. Time taken = 5 hrs 25 min = $\frac{65}{12}$ hrs.

Let the required distance be x km.

Then $\frac{x}{10}+\frac{x}{1}=\frac{65}{12}$

$$\Leftrightarrow 11x=\frac{650}{12}\Leftrightarrow x=\frac{325}{66}=4\frac{61}{66}km$$

17. Ratio of speed of Meena and Teena is 2 : 3.

Ratio of time taken = 3 : 2

If Teena takes x minute to walk from A to B,

then Meena takes x + 20 min

$$\frac{x+20}{x}=\frac{3}{2}$$

$$\Rightarrow 2x+40=3x$$

$$\Rightarrow x=40min$$

Hence, Meena takes 60 min walking at her usual speed.

Hence, at double the speed, she would take 30 min.

18. Let the required time = x hour

Relative speed= (4.6 - 4) km/h

= 0.6 km/h

$$0.6=\frac{3}{x}\Rightarrow x=\frac{3}{0.6}=5h$$

19. Distance covered by A when B starts

= (2 × 5) km = 10 km

Relative speed = (4 - 2) km/h = 2 km/h

Then, the time of travel for B = $\frac{10}{2}$ = 5h

Distance travelled by B = (5× 4) km

=20 km

20. Length of the track = 1000 m

Speed of A = $10\times\frac{5}{18}=\frac{50}{18}$ m/s

Speed of B = $20\times\frac{5}{18}=\frac{100}{18}$ m /s

Speed of C =$25\times\frac{5}{18}=\frac{125}{18}$ m/s

They will meet for the first time at the starting point at a time which is the LCM of $\{\frac{L}{x},\frac{L}{y},\frac{L}{z}\}$ where x, y, z are the speeds of the three persons.

ie, the LCM of $\left\{\frac{1000}{\frac{50}{18}},\frac{1000}{\frac{100}{18}},\frac{1000}{\frac{125}{18}}\right\}$

i.e. the LCM of $\left\{\frac{1000\times18}{50},\frac{1000\times18}{100},\frac{1000\times18}{125}\right\}$

ie, 720 s (because the LCM of fractions = LCM of numerators/HFC of denominators)

Hence, they will meet for the first time at the starting point 12 min from the time they start.

21. A runs 1000 m while B runs (1000 -40) or 960 m

⇒runs 1000 m while C runs (1000 - 70) or 930 m

∴ B runs 960 m while C runs 930 m

∴B can give $C\left(1000-968\frac{3}{4}\right)$

or $31\frac{1}{4}$m start

22. Total distance to be travelled

= 121 + 99 = 220 m

Relative speed = Sum of speeds

= 72 km/h = $72\times\frac{5}{18}$= 20 m/s

Time required = $\frac{220}{20}$ = 11 s

23. New speed = $\frac{8}{11}$ of usual speed

⇒ New time = $\frac{11}{8}$ of usual time

So, $\frac{11}{8}$ of usual time = $\frac{11}{2}$ h

⇒ Usual time = $\left(\frac{11\times8}{2\times11}\right)=4$h

Hence, time saved = $5\frac{1}{2}-4=1\frac{1}{2}$ h