15. PIPES AND CISTERNS

Pipes and Cisterns

A pipe or a tap connected with a tank (or a cistern or a reservoir) is called an inlet, if it fills it.

A pipe or a tap connected with a tank (or a cistern or a reservoir) is called an outlet, if it empties it.

General rules to be followed in the problems on Pipes and Cisterns

1. If a pipe can fill a tank in a hours, then the part filled in 1 h =1/a

2. If a pipe can empty *a* tank in *b* hours, then the part of the full tank emptied in 1 *h* = 1 / *b*

3. If a pipe can fill a tank in a hours and another pipe can empty the full tank in b hours, then the net part filled in 1 h, when both the pipes are opened = (1/a -1/b)

Thus, in Pipes and Cisterns a filling pipe or tap does positive work and an emptying pipe or a leak does negative work.

Example 1: A pipe can fill $\frac{3}{4}$ of cistern in 12 min. In how many minutes it fills $\frac{1}{2}$ of cistern.

Solution: ∵ Time taken to fill $\frac{3}{4}$ of cistern = 12 min

Time taken to fill full cistern = $12 \times \frac{4}{3}$ = 16min

Time taken to fill 1/2 of cistern = $\frac{1}{2} \times 16$ =8 min

Example 2: A tap can fill a cistern in 20 min and another tap can fill in 30 min. If both are opened simultaneously, find the time when the cistern will be full.

Solution . Total part that can be filled in 1 min = $\frac{1}{20} + \frac{1}{30} = \frac{3+2}{60} = \frac{1}{12}$

Hence Tank will be filled in 12 min.

Example 3: Pipe A can fill a tan k in 15 min, pipe B in 18 min and a leak can empty the tank in 90 min. If all of them work together, find the time taken to fill the empty tank?

Solution: Net part filled in 1 min = (1/15 + 1/18 - 1/90) = 1/9

So, the empty tank will be filled in 9 min.

Example 4: Two taps *A* and *B* can fill a tank in 20 and 40 min respectively. Both the taps are opened and after 10 min tap *A* was shut. In how much more time will the tank be full?

Solution: Part filled by *(A+ B)* in 10 min = 10 (1/20 + 1/40) = 10-3/40 = 3/4

Part of the tank to be filled = (1- 3/*4)* = 1/4

Now, l/4th of the tank can be filled by B in (l/4)*40 = 10 min

So, the tank would be full in 10 more min.

EXERCISE

1. Two pipes *A* and *B* can fill a tank in 6 hours and 4 hours respectively. If they are opened on alternate hours and if pipe *A* is opened first, in how many hours, the tank shall be full?

   (a) 8 (b) $4\frac{1}{2}$

   (c) 5 (d) $5\frac{1}{2}$

2. Two pipes A and *B* fill a tank in 15 h and 20 h respectively while a third pipe C can empty the full tank in 25 h. All the three pipes are opened in the beginning. After 10 h, C is closed. In how much time will the tank be full?

   (a) 10 h (b) 11 h

   (c) 11.5 h (d) 12h

3. A pump can fill a tank with water in 2 hours. Because of a leak, it took $2\frac{1}{3}$ hours to fill the tank. The leak can drain all the water of the tank in:

   (a) 43 hrs (b) 9 hrs

   (c) 10 hrs (d) 14 hrs

4. Two taps *A* and *B* can fill a tank in 5 hours and 20 hours respectively. If both the taps are open then due to a leakage, it took 30 minutes more to fill the tank. If the tank is full, how long will it take for the leakage alone to empty the tank?

   (a) 44 hrs (b) 12 hrs

   (c) 18 hrs (d) 36 hrs

5. Two pipes *A* and *B* together can fill a cistern in 4 hours. Had they been opened separately, then *B* would have taken 6 hours more than *A* to fill the cistern. How much time will be taken by *A* to fill the cistern separately?
(a) 10 hr (b) 4 hrs
(c) 6 hrs (d) 8 hrs.

6. One pipe can fill a tank three times as fast as another pipe. If together the two pipes can fill the tank in 36 minutes, then the slower pipe alone will be able to fill the tank in:
(a) 92 min (b) 112 min
(c) 144 min (d) 192 min

7. A tank is filled in 5 hours by three pipes A, *B* and C. The pipe C is twice as fast as *B* and *B* is twice as fast as *A*. How much time will pipe *A* alone take to fill the tank?
(a) 22 hrs (b) 27 hrs
(c) 35 hrs (d) cannot be determined

8. A tank is filled by three pipes with uniform flow. The first two pipes operating simultaneously fill the tank in the same time during which the tank is filled by the third pipe alone. The second pipe fills the tank 5 hours faster than the first pipe and 4 hours slower than the third pipe. The time required by the first pipe is:
(a) 8 hrs (b) 13 hrs
(c) 15 hrs (d) 32 hrs

9. 12 buckets of water fill a tank when the capacity of each bucket is 13.5 litres. How many buckets will be needed to fill the same tank, if the capacity of each bucket is 9 litres?
(a) 9 (b) 112 (c) 24 (d) 18

10. A booster pump can be used for filling as well as for emptying a tank. The capacity of the tank is 2400 m^3. The emptying capacity of the tank is 10 m3 per minute higher than its filling capacity and the pump needs 8 minutes lesser to empty the tank than it needs to fill it. What is the filling capacity of the pump?
(a) 50 m^3/min (b) 80 m^3/min
(c) 36m^3/min (d) 48m^3/min

11. A leak in the bottom of a tank can empty the full tank in 6 h. An inlet pipe fills water at the rate of 4 L a minute. When the tank is full, the inlet is opened and due to the leak the tank is empty in 8 h. Find capacity of the tank (in litres).
(a) 5000 L (b) 5670 L
(c) 5700 L (d) 5760 L

12. A tap can fill a cistern in 12 h. After half the tank is filled, 2 more similar taps are opened. What is the total time taken to fill the tank complete by?
(a) 9 h (b) 6 h
(c) 8 h (d) 4 h

13. A tank 9 ft by 5 ft by 2 ft has a supply pipe pouring in 576 of water in a minute and an exhaust pipe emptying it in 3 h. If the tank is full and both pipes are open, how many hour will it take to empty it?
(a) 9h (b) $9\frac{1}{5}$h
(c) $9\frac{1}{7}$ h (d) $9\frac{3}{8}$ h

14. A cistern has three pipes *A*, *B* and C. The pipes *A* and *B* can fill it in 5 h and 6 h respectively and C can empty it in 3 h. If the pipes are opened in order at 3 pm, 4 pm an d 5 Pm respectively at what time will the tank be filled?
(a) 6 am (b) 5 am
(c) 4 am (d) 7 am

Answer Key					
1	c	6	c	11	d
2	d	7	c	12	c
3	d	8	c	13	a
4	d	9	d	14	a
5	c	10	a		

SOLUTIONS

1. A's work in 1 hour $=\frac{1}{6}$,

B's work in 1 hour $=\frac{1}{4}$

(A + B)'s 2 hour's work when opened alternately $=\left(\frac{1}{6}+\frac{1}{4}\right)=\frac{5}{12}$

(A + B)'s 4 hour's work when opened alternately $=\frac{10}{12}=\frac{5}{6}$

Remaining part $=\left(1-\frac{5}{6}\right)=\frac{1}{6}$

Now, it is *A's* turn and $\frac{1}{6}$ part is filled by *A* in 1 hour.

Total time taken to fill the tank

= (4 + 1) hrs. = 5 hrs.

2. Work done $=\frac{x}{15}+\frac{x}{20}-\frac{10}{25}=1$

Or $x = 12$ h

3. Work done by the leak in 1 hour $=\left(\frac{1}{2}-\frac{3}{7}\right)=\frac{1}{14}$

Leak will empty the tank in 14 hrs.

4. Part filled by *(A + B)* in 1 hour

$=\left(\frac{1}{5}+\frac{1}{20}\right)=\frac{1}{4}$

So, A and B together can fill the tank in 4 hours.

Work done by the leak in 1 hour $=\frac{1}{4}-\frac{2}{9}=\frac{1}{36}$

Leak will empty the tank in 36 hrs,

5. Let the cistern be filled by pipe *A* alone in *x* hours.

Then, pipe *B* will fill it in (x + 6) hours.

$$\frac{1}{x}+\frac{1}{x+6}=\frac{1}{4}$$

$$\Rightarrow x^2-2x-24=0$$

$$(x-6)(x+4)=0$$

$$x=6$$

[neglecting the -ve value of x]

6. Let the slower pipe alone fill the tank in *x* minutes.

Then, faster pipe will fill it in $\frac{x}{3}$ minutes.

$$\frac{1}{x}+\frac{3}{x}=\frac{1}{36}\Leftrightarrow\frac{4}{x}=\frac{1}{36}$$

$$\Leftrightarrow x=144\,min.$$

7. Suppose pipe A alone takes x hours to fill the tank.

Then, pipes *B* and *C* will take $\frac{x}{2}$ and $\frac{x}{4}$ hours respectively to fill the tank.

$$\frac{1}{x}+\frac{2}{x}+\frac{4}{x}=\frac{1}{5}\Leftrightarrow\frac{7}{x}=\frac{1}{5}$$

$$\Leftrightarrow x=35hrs$$

8. Suppose, first pipe alone takes x hours to fill the tank.

Then, second and third pipes will take $(x-5)$ and $(x-9)$ hours respectively to fill the tank.

$\frac{1}{x}+\frac{1}{x-5}=\frac{1}{x-9}$

$\Rightarrow (2x-5)(x-9)=x(x-5)$

$\Rightarrow x^2-18x+45=0$

$x=15$ *or* 3. If $x=3$, $(x-9)$ becomes negative. Hence $x=15$

9. Capacity of the tank= (12 × 13.5) litres = 162 litres.

Capacity of each bucket = 9 litres.

Number of buckets needed =

$=\left(\frac{162}{9}\right)=18$.

10. Let the filling capacity of the pump be x m^3/min.

Then, emptying capacity of the pump

$=(x+10)\ m^3/min$.

So, $\frac{2400}{x}-\frac{2400}{(x+10)}=8$

$\Leftrightarrow x^2+10x-3000=0$

$\Leftrightarrow (x-50)(x+60)=0$

$\Leftrightarrow x=50$

[neglecting the -ve value of x]

11. Efficiency of leak = $\frac{1}{6}$

Combined efficiency of leak and inlet pipe = $\frac{1}{8}$

So, efficiency of inlet pipe $=\frac{1}{6}-\frac{1}{8}=\frac{4-3}{24}=\frac{1}{24}$

Inlet pipe can fill the tank in 24 h,

So, capacity = 24 × 60 × 4 = 5760 L

12. $\frac{1}{2}$ of the tank can be filled by the tap in 6 h.

Part filled by one tap in 1 h = $\frac{1}{12}$

Part filled by three taps in 1 h = $\frac{1}{4}$

$\therefore \frac{1}{2}$ tank can be filled by the three pipes

In $\frac{1}{4}\times 2$ = 2 h

Total time taken to fill the tan k completely = 6 + 2 = 8 h

13. Volume of the tank = 9 × 5 × 2 × 12 × 12 × 12 m^3

Volume of water exhausted in 1 min

$=\frac{90\times12\times12\times12}{3\times60}$ = 864 m^3

Hence, combined effect of the two pipes in 1 min is (864 - 576), ie 288 m^3 of water is removed in 1 min.

$\therefore$ Time required to empty the tank

$=\frac{90\times12\times12\times12}{288\times60}$ = 9 h

14. By 5 pm, the water filled by two pipes = 2 h work done by pipe A + 1 h work done by pipe *B*

$=2\times\frac{1}{5}+1\times\frac{1}{6}=\frac{2}{5}+\frac{1}{6}=\frac{17}{30}$

After 5 pm all the three pipes are working and the work done by all the three pipes in 1h $=\frac{1}{5}+\frac{1}{6}-\frac{1}{3}=\frac{1}{30}$

After 5 pm only $\frac{13}{30}$ of the tank is to be filled. time taken in filling $\frac{13}{30}$ of the tank

$=30\times\frac{13}{30}$ = 13 h

$\therefore$ 13 h after 5 pm= 6 am