14.TIME AND WORK

Work

Work to be done is generally considered as one unit. It may be digging a trench, constructing or Painting a wall, filling up or emptying a tank, reservoir or a cistern.

General rules to be followed in the problems on Time and Work

1. If A can do a piece of work in *n* days, then work done by A in 1 day is *1/n.*
i.e. if a person can do some work in 12 days, he does 1/ 12th of the work in one day.

2. If A's 1 day's work =1/n , then A can finish the whole work in *n* days.
i.e. if a person's one day's work is 1/10, then he can finish the whole work in 10 days.

3. If *A* is thrice as good a workman as *B,* then ratio of work done by *A* and B = 3 : 1. i.e. if a man works three times as fast as a woman does, then when the work is complete, 3 parts of the work has been done by the man and 1 part by the woman.

4. If *A* is thrice as good a workman as *B,* then ratio of time taken by A and *B* = 1 : 3. i.e. if the woman takes 15 days to complete the work, then the man takes 5 days to complete the same work.

5. If two persons *A* and *B* can individually do some work in *a* and *b* days respectively, then *A* and *B* together can complete the same work in *ab/(a + b)* days.

6. The fundamental rules on variation also apply in Time and Work.

(i) Work and men are directly proportional to each other i.e. if the work increases, the no. of men required to do it, also increases, if the work is to be completed in the same number of days.

(ii) Men and days are inversely proportional, i.e. if the number of men increases, the number of days required to complete the same work decreases and vice versa.

(iii) Work and days are directly proportional, i.e. if the work increases, the number of days required also increases, if the work is to be completed with the same number of men and vice versa.

Example 1: Ravi can do a job 10 days. Determine his one day job.

Solution: Ravi's 10 days work = 1

Ravi's 1 day work = $\frac{1}{10}$

Example 2: Tuktuki and Rasmani can do a job alone in 20 days and 30 days respectively. In how many days the job will be finished, if they work together.

Solution: Tuktuki's 1 day work = $\frac{1}{20}$

Rasmani's 1 day work = $\frac{1}{30}$

(Tuktuki + Rasmam)'s 1 day work = $\frac{1}{20}+\frac{1}{30}=\frac{5}{60}=\frac{1}{12}$

⇒ (Tuktuki + Rasmani) will complete the job in 12 days.

Alternate: Required no. of days = $\frac{20\times30}{20+30}$ =12

Example 3: Mary and Maurice can do a piece of work in 10 days and 15 days respectively. They work together for 3 days and then Maurice leaves. Mary finishes the remaining work alone. In how many days is the total work finished?

Solution . (Mary + Maurice)'s 1 day work = $\frac{1}{10}+\frac{1}{15}=\frac{5}{30}=\frac{1}{6}$

(Mary + Maurice)'s 3 day work = $\frac{3}{6}=\frac{1}{2}$

Remaining work = $1-\frac{1}{2}=\frac{1}{2}$

Mary does this piece of work in = 10 days

Mary does the 1/2 piece of work in = $\frac{1}{2}\times 10$ = 5 days

Hence, total number of days to finish the work = 3 + 5 = 8 days

Example 4: Ravi and Rishi can do a piece of work in 24 days, Rishi and Ronit in 30 days, Ronit and Ravi in 40 days. In how many days will they finish it together and separately?

Solution: (Ravi and Rishi)'s 1 day's work =1/24

(Rishi and Ronit)'s 1 day's work = 1/30

(Ronit and Ravi)'s 1 day's work = 1/40
On adding, we get
2 (Ravi + Rishi + Ronit)'s 1 day's work = (1/24 + 1/30 + 1/40) = 1/10
So,(Ravi + Rishi + Ronit)'s 1 day's work = 1/20
So, they all together can finish the work in 20 days.
Ravi's 1 day's work = [(Ravi + Rishi + Ronit)'s 1 day's work] - [(Rishi + Ronit)'s 1 day's work] = (1/20 -1/30) = 1/60
So, Ravi alone can finish the work in 60 days.
Similarly, Rishi's 1 day work = (1/20 - 1/40) = 1/40
So, Rishi alone can finish the work in 40 days
And Ronit's 1 day work = (1/20- 1/24) = 1/120
So, Rishi alone can finish the work in 120 days.

Example 5: 4 men and 5 women do a piece of work in 8 days while 5 men and 5 women finish it in 6 days, 3 men and 5 women will finish it in how many days?

Solution: (4 men + 5 women)'s 1 day's work = 1 / 8
(5 men + 5 women)'s 1 day's work 1/6
Subtracting, 1 man's 1 day's work = (1/6 - 1/8) = 1/24
(3 men + 5 women)'s 1 day's works = (1/8 - 1/24) = 1/12
Thus, 3 men and 5 women will finish it in 12 days.

EXERCISE

Directions (1-5): A can do a piece of work in 8 days and B in 12 days. Find how much time they will take to complete the work under the following condition.

1. Working together
(a) 4 days (b) $4\frac{4}{5}$ days
(c) $4\frac{6}{7}$ days (d) $4\frac{3}{7}$ days

2. Working alternately starting with A
(a) 8 days (b)$8\frac{1}{2}$days
(c) 9 days (d) $9\frac{1}{2}$ days

3. Working alternately starting with B
(a) 8 days (b) $8\frac{1}{2}$ days
(c) $9\frac{2}{3}$ days (d) $9\frac{5}{7}$ days

4. If *B* leaves 3 days before the actual completion of the work
(a) 2 days (b) 3 days
(c) 6 days (d) 9 days

5. If *B* leaves 3 days before the scheduled completion of the work
(a) 3 days (b) 6 days
(c) $3\frac{4}{5}$ days (d) $6\frac{4}{5}$ days

6. *A* and *B* together can complete a piece of work in 35 days while *A* alone can complete the same work in 60 days. In how many days, *B* alone will be able to complete the same work?
(a) 80 days (b) 84 days
(c) 88 days (d) 92 days

7. *A* and *B* can do a piece of work in 12 days, *B* and C can do it in 15 days and *C* and *A* can do the same work in 20 days. Find the number of days in which *A* alone can do the same job.
(a) 20 days (b) 30 days
(c) 45 days (d) 60 days

8. A can do a piece of work in 15 days and *B* alone can do it in 10 days. *B* works at it for 5 days and then leaves. In how many days, A alone can finish the remaining work?
(a) 7 days (b) $7\frac{1}{2}$ days
(c) 8 days (d) $8\frac{1}{2}$ days

9. A and *B* can do a piece of work separately in 8 and 12 h. If they work alternate hours, A starting when will the work be finished?
(a) $9\frac{1}{2}$ h (b) 9 h
(c) $8\frac{1}{2}$ h (d) 8 h

10. Two friends A and *B* working together completed a work in 26 days. Their skills of doing the work is in the ratio 8: 5. How many days will *B* take, if engaged alone?

(a) $67\frac{3}{5}$ days (b) 62 days
(c) 67 days (d) $65\frac{3}{5}$ days

11. *A* and *B* together can do a piece of work in 12 days, which *B* and C together can do in 16 days. After *A* has been working at it for 5 days and *B* for 7 days, *C* finishes it in 13 days. In how many days C alone will do the work?
(a) 16 (b) 24
(c) 37 (d) 48

12. *A* and *B* can do a piece of work in 45 days and 40 days respectively. They bE.g.an to do the work together but *A* leaves after some days and then *B* completed the remaining work in 23 days. The number of days after which *A* left the work was:
(a) 11 (b) 7
(c) 9 (d) 12

13. *A* can do a piece of work in 14 days which *B* can do in 21 days. They bE.g.in together but 3days before the completion of the work, A leaves off. The total number of days to complete the work is:
(a) $\mathbf{6}\frac{3}{5}$ (b) $8\frac{1}{2}$
(c) $\mathbf{10}\frac{1}{5}$ (d) $13\frac{1}{2}$

14. *A*, *B* and C can complete a work separately in 24, 36 and 48 days respectively. They started together but C left after 4 days of start and *A* left 3 days before the completion of the work. In how many days will the work be completed?
(a) 15 days (b) 24 days
(c) 25 days (d) 38 days

15. *A*, *B* and C together earn Rs. 300 per day, while *A* and *C* together earn Rs.188 and *B* and C together earn Rs.152. The daily earning of C is:
(a) Rs.40 (b) Rs. 70
(c) Rs. 112 (d) Rs. 160

16. *A*, *B* and C are employed to do a piece of work for Rs. 529. *A* and *B* together are supposed to do $\frac{19}{23}$ of the work and *B* and C together $\frac{8}{23}$ of the work. What amount should A be paid?
(a) Rs. 320 (b) Rs. 345
(c) Rs. 355 (d) Rs.380

17. A alone can do a piece of work in 6 days and *B* alone in 8 days. A and *B* undertook to do it for Rs. 3200, with the help of C, they completed the work in 3 days. How much is to be paid to C?
(a) Rs.450 (b) Rs.400
(c) Rs.600 (d) Rs.900

18. A sum of money is sufficient to pay A's wages for 21 days and *B* s wages for 28 days. The same money is sufficient to pay the wages of both for:
(a) 12 days (b) $12\frac{1}{4}$ days
(c) 16 days (d) $24\frac{1}{2}$ days

19. 12 men can complete a piece of work in 4 days, while 15 women can complete the same work in 4 days. 6 men start working on the job and after working for 2 days, all of them stopped working. How many women should be put on the job to complete the remaining work, if it is to be completed in 3 days?
(a) 15 (b) 19
(c) 23 (d) data inadequate

20. Sixteen men can complete a work in twelve days. Twenty-four children can complete the same work in eighteen days. Twelve men and eight children started working and after eight days three more children joined them. How many days will they now take to complete the remaining work?
(a) 5 days (b) 4 days
(c) 10 days (d) 9 days

21. 5 men and 2 boys working together can do four times as much work as a man and a boy. Working capacities of a man and a boy are in the ratio:
(a) 1 :2 (b) 2 : 1
(c) 1: 3 (d) 6 : 7

22. A and *B* can complete a piece of work in 80 and 120 days respectively. They together start the work but A left after 20 days. After another 12 days C joined *B* and now they complete the work in 28 more days. In how many days *C* can complete the work, working alone?
(a) 100 days (b) 112 days
(c) 120 days (d) 126 days

23. 8 men and 10 women are working together on a job and can complete it in

10 days. The work done by each woman is four-fifth of the work by each man. In how many days 10 men alone complete the same work?

(a) 20 days (b) 12 days
(c) 15 days (d) 16 days

24. There are 2 machine tools. The capacity of the 1st machine tool is 20% less than the 2nd one. The 1st machine tool operated for 5 h whereas the 2nd one operated for 4 h and together they machined 3000 work pieces. The number of work pieces machined by the 1st machine is

(a) 950 (b) 1000
(c) 1032 (d) 1050

25. A firm has tractors of four models *A, B, C* and *D*. Four tractors (two of model *B* and one each of models C and *D)* plough a field in 2 days. Two model A tractors and one model C tractor take 3 days to do this job. Three tractors on each of models A, *B* and C take 4 days to do the same task. How long will it take to do the job if a team is made up of four tractors of different models?

(a) $\frac{10}{7}$ days (b) $\frac{14}{7}$ days
(c) $\frac{13}{7}$ days (d) $\frac{12}{7}$ days

26. The efficiency of Ronil is twice that of Sanjay and he can finish a work in 4 h less than the time taken by Sanjay. If both of them work together they can finish the same work in

(a) 8 h (b) 3 h
(c) 8/3 h (d) 4/3 h

Answer Key									
1	b	7	b	13	c	19	a	25	d
2	d	8	b	14	a	20	b	26	c
3	c	9	a	15	a	21	b		
4	c	10	a	16	b	22	b		
5	d	11	b	17	b	23	d		
6	b	12	c	18	a	24	b		

SOLUTIONS

1. A's one day work = $\frac{1}{8}$

B's one day work = $\frac{1}{12}$

(A + *B*)'s one day work = $\frac{1}{8}+\frac{1}{12}=\frac{5}{24}$

∴ Required number of days = $\frac{24}{5}$

$= 4\frac{4}{5}$ days

2. Work done in first 2 days

$=\frac{1}{8}+\frac{1}{12}=\frac{5}{24}$

∴ Work done in first 8 days

$=\frac{5}{24}\times 4=\frac{20}{24}$

Remaining work = $1-\frac{20}{24}=\frac{4}{24}=\frac{1}{6}$

Work done by A in 9th day = $\frac{1}{8}$

After 9 days, the remaining work has to be done $=\frac{1}{6}-\frac{1}{8}=\frac{1}{24}$

In the 10th day *B* will come to do the work and he will complete all the remaining work. Time taken by *B* for it

$=\frac{1}{24}\times 12=\frac{1}{2}$ day

So, total required time = $8+1+\frac{1}{2}$

$= 9\frac{1}{2}$ days

3. Here, there will be no difference in work completed by the 8th day.

On the 9th day *B* works alone and does

$=\frac{1}{12}$ of the work

So, after 9 days remaining work to be done $=1-\left(\frac{5}{6}+\frac{1}{12}\right)=1-\frac{11}{12}=\frac{1}{12}$

Time taken by A to do the remaining work $=\frac{1}{12}\times 8=\frac{2}{3}$ days

So, total required time =$8+1+\frac{2}{3}=9\frac{2}{3}$

4. If *B* leaves 3 days before the actual completion of the work, then *A* will work alone for the last 3 days.

So, in last 3 days work done by A = $\frac{3}{8}$

∴ A and B have done $\left(1-\frac{3}{8}\right)=\frac{5}{8}$ of the work

∴ Time taken by A and B together to do 5/8 of the work = $\frac{5}{8}\times\frac{12\times8}{12+8}$ = 3 days

∴ Total required time = 3 + 3 = 6 days

5. Time taken by A and B working together to finish the work = $4\frac{4}{5}$ days

B's work of 3 days = $\frac{3}{12}=\frac{1}{4}$

Since, B leaves 3 days before the scheduled completion of the work. So, this work $\left(\frac{1}{4}\ \boldsymbol{of\ the\ work}\right)$ $\boldsymbol{will}$ be done by A. So, time taken to finish 1/4 of the work by

$A=\frac{1}{4}\times 8$ = 2 days

∴ Total required time

$$=4\frac{4}{5}+2=6\frac{4}{5}\boldsymbol{days}$$

6. B's one day work = $\frac{1}{35}-\frac{1}{60}=\frac{12-7}{60}=\frac{5}{420}=\frac{1}{84}$

∴ Required number of days =84

7. Work done by A and B in one day= $\frac{1}{12}$

Work done by B and C in one day = $\frac{1}{15}$

Work done by C and A in one day = $\frac{1}{20}$

Adding all these, 2 (A + B + C)'s work of one day = $\frac{1}{12}+\frac{1}{15}+\frac{1}{20}=\frac{1}{5}$

(A + B + C)'s one day work = $\frac{1}{10}$

∴ A's one day work = (A + B + C)'s one day work - (B + C)'s one day work

⇒ A's one day work

$$=\frac{1}{10}-\frac{1}{15}=\frac{1}{30}$$

Hence, A can do this work in 30 days.

8. Work done by B in 5 days =5*$\frac{1}{10}=\frac{1}{2}$

Remaining work=1-$\frac{1}{2}=\frac{1}{2}$

Time taken by A to finish the remaining work=$\frac{1}{2}\boldsymbol{815=7\frac{1}{2}\ days}$

9. (A + B)'s 2 h work = $\frac{1}{8}+\frac{1}{12}=\frac{5}{24}$

(A + B)'s 8 h work = $\frac{5\times4}{24}=\frac{5}{6}$

Work done by A in 9th h = $\frac{1}{8}$

Total work done up to 9^{th} h = $\frac{5}{6}+\frac{1}{8}=\frac{23}{24}$

Remaining work = $\frac{1}{24}$

B's 1 h work = $\frac{1}{12}$

B can do $\frac{1}{24}$ of the work in $\frac{1}{24}\times 12=\frac{1}{2}\boldsymbol{h}$

∴ Both can finish the job in $\boldsymbol{9\frac{1}{2}h}$

10. Skill ratio of A and B = *8 :* 5

∴ Work done by B is $\frac{5}{13}$ of the whole which he does in 26 days

∴ Number of days taken by B to do the whole work $=\frac{26\times13}{5}=67\frac{3}{5}$ days

11. A's 5 days' work + B's 7 days' work + C's 13 days work = 1

⇒ (A + B)'s 5 days' work + (B + C)'s 2 days' work + C's 11 days' work = 1

⇒ $\frac{5}{12}+\frac{2}{16}$ + C's 11 days' work = 1

⇒ C's 11 day's work

$$=1-\left(\frac{5}{12}+\frac{2}{16}\right)=\frac{11}{24}$$

⇒ C's 1 day's work = $\left(\frac{11}{24}\times\frac{1}{11}\right)=\frac{1}{24}$

C alone can finish the work in 24 days.

12. (A + B)'s 1 day's work

$=\left(\frac{1}{45}+\frac{1}{40}\right)=\frac{17}{360}$

Work done by B in 23 days

$=\left(\frac{1}{40}\times 23\right)=\frac{23}{40}$

Remaining work = $\left(1-\frac{23}{40}\right)=\frac{17}{40}$

Time taken by A and B to finish the remaining work=$\left(\frac{360}{17}\times\frac{17}{40}\right)=\boldsymbol{9days}$

∴ A left after 9 days.

13. B's 3 day's work = $\left(\frac{1}{21}\times 3\right)=\frac{1}{7}$

Remaining work = $\left(1-\frac{1}{7}\right)=\frac{6}{7}$

(A + B)'s 1 day's work

$$=\left(\frac{1}{14}+\frac{1}{21}\right)=\frac{5}{42}$$

∴ $\frac{6}{7}$ work is done by A and B in

$$=\left(\frac{42}{5}\times\frac{6}{7}\right)=\frac{36}{5}\boldsymbol{days}$$

Hence, total time taken = $\left(3+\frac{36}{5}\right)$ days

$=10\frac{1}{5}$ days.

14. (A+ B + C)' s 1 day's work

$$=\left(\frac{1}{24}+\frac{1}{36}+\frac{1}{48}\right)=\frac{13}{144}$$

Work done by (A + B + C) in 4 days

$=\left(\frac{13}{144}\times 4\right)=\frac{13}{36}$

Work done by B in 3 days

$=\left(\frac{1}{36}\times 3\right)=\frac{1}{12}$

Remaining work = $\left[1-\left(\frac{13}{36}+\frac{1}{12}\right)\right]=\frac{5}{9}$

(A + B)'s 1 day's work

$$=\left(\frac{1}{24}+\frac{1}{36}\right)=\frac{5}{72}$$

Time taken by A and B to finish 5/9th of the work=$\frac{5}{9} * \frac{72}{5}$=8 days

Hence, total time taken = (4+ 3+8) days = 15days.

15. B's daily earning = Rs. (300 - 188)
= Rs.112.
= Rs. [152 - 11)]= Rs.40.

16. Work done by A = $\left(1-\frac{8}{23}\right)=\frac{15}{23}$

A : (B +C) = $\frac{15}{23}:\frac{8}{23}$ = 15 : 8

So, A's share =Rs. $\left(\frac{15}{23}\times 529\right)$ = Rs. 345.

17. C's l day's work = $\frac{1}{3}-\left(\frac{1}{6}+\frac{1}{8}\right)$

$$=\frac{1}{3}-\frac{7}{24}=\frac{1}{24}$$

A's wages : B's wages : C's wages

$$=\frac{1}{6}:\frac{1}{8}:\frac{1}{24}=4:3:1$$

∴ C's share = Rs. $\left(\frac{1}{8}\times 3200\right)$ = Rs. 400.

18. Let total money be Rs. x.

A's 1 day's wages = Rs. $\frac{x}{21}$

B s 1 day's wages = Rs. $\frac{x}{28}$

∴ (A + *B*)'s 1 day's wages

= Rs. $\left(\frac{x}{21}+\frac{x}{28}\right)$ = Rs. $\frac{x}{12}$

∴Money is sufficient to pay the wages of both for 12 days.

19. 1 man 1 day's work = $\frac{1}{48}$;

1 woman 1 day's work= $\frac{1}{60}$

6 men's 2 day's work = $\left(\frac{6}{48}\times 2\right)=\frac{1}{4}$

Remaining work = $\left(1-\frac{1}{4}\right)=\frac{3}{4}$

Now, $\frac{1}{60}$ work is done in 1 day by 1 woman.

So, $\frac{3}{4}$ work will be done in 3 days by $\left(60\times\frac{3}{4}\times\frac{1}{3}\right)$ = 15 women.

20. 1 man's 1 day's work = $\frac{1}{192}$;

1 child's 1 day's Work= $\frac{1}{432}$

Work done in 8 days = $8\left(\frac{12}{192}+\frac{8}{432}\right)$

$$=8\left(\frac{1}{16}+\frac{1}{54}\right)=\frac{35}{54}$$

Remaining work = $\left(1-\frac{35}{54}\right)=\frac{19}{54}$

(12 men + 11 children)'s 1 day's work

$$=\left(\frac{12}{192}+\frac{11}{432}\right)=\frac{19}{216}$$

Now, $\frac{19}{216}$ work is done by them in 1 day.

∴ $\frac{19}{54}$ work will be done by them in $\left(\frac{216}{19}\times\frac{19}{54}\right)$ = 4 days.

21. Let 1 man's 1 day's work = *x* and 1 boy's 1 day's work = *y*.

Then, $5x + 2y = 4(x + y)$

∴ $x = 2y$

$$\Rightarrow \frac{x}{y}=\frac{2}{1}$$

22. A did the work for 20 days, *B* did the work for (20 + 12 + 38) = 60 days, C did the work for 28 days.

Let C alone can complete the work in *x* days.

Fraction of work did by A + Fraction of work did by *B* + Fraction of work did by C = 1

Or $\frac{20}{80}+\frac{60}{120}+\frac{28}{x}=1$

Or $x = 28 \times 4$

= 112 days

C alone can complete the work in 112 days

23. 1 Woman =$\frac{4}{5}$ men

8 Men + 10 Women

= 8 Men + $\frac{4}{5}$ ×10 Men = 16 Men

16 Men can complete the work in 10 days.

∴10 Men can complete the work in

$\frac{16\times 10}{10}$ = 16 days

24. Let the capacity of the second machine be x work pieces per hour.

Capacity of the 1st machine tool

= 0.8 x.

$$\Rightarrow 5(0.8x) + 4x = 2000$$
$$\Rightarrow 8x = 2000$$
$$\Rightarrow x = 250$$

Number of work machined by the first Tool = 5 × 0.8 x = 1000

25. $\frac{2}{B}+\frac{1}{C}+\frac{1}{D}=\frac{1}{2}$ ……(i)

$\frac{2}{A}+\frac{1}{C}=\frac{1}{3}$……(ii)

and $\frac{1}{A}+\frac{1}{B}+\frac{1}{C}=\frac{1}{4}$ ……...(iii)

Subtracting Eq. (i) from Eq. (ii), we get$\frac{2}{A}-\frac{2}{B}-\frac{1}{D}=\frac{1}{6}$...(iv)

Subtracting Eq. (iii) from Eq. (ii), we get

$$\frac{1}{A}-\frac{1}{B}=\frac{1}{12}$$

$\Rightarrow \frac{2}{A}-\frac{2}{B}=\frac{1}{6}$……..(v)

Subtracting Eq. (v) from Eq. (iv), we get

$\frac{1}{D}=\frac{1}{3}$ ……..(vi)

Adding Eqs. (iii) and (iv), we get

$$\frac{1}{A}+\frac{1}{B}+\frac{1}{C}+\frac{1}{D}=\frac{1}{4}+\frac{1}{3}=\frac{7}{12}$$

Hence, A, B, C and D will take $\frac{12}{7}$ days to do the required job.

26. As the efficiency of Ronil is twice that of Sanjay, Ronil will take hours for completion of the work, when Sanjay is taking 2*x* h.

Given, $2x - x = 4 \Rightarrow x = 4$

Hence, they take $\frac{1}{\frac{1}{4}+\frac{1}{8}}=\frac{8}{3}$h