13. ALLEGATION OR MIXTURE

Introduction

It is the rule that is used to determine the mean value of the mixture when the prices of the individual items being mixed together and the proportion in which they are being mixed are given. Here, the value of the mixture is always higher than the lowest value and lower than the highest value of the items being mixed.

According to Rule of Allegation

$$\frac{Quantity\ of\ cheaper}{Quantity\ of\ dearer} = \frac{Price\ of\ dearer - Mean\ price}{Mean\ Price - Price\ of\ Cheaper}$$

It can be also expressed as,

Cost price of 1 unit quantity of cheaper (x)

Cost price of 1 unit quantity of dearer (y)

Mean price (m)

(y-m)

(m-x)
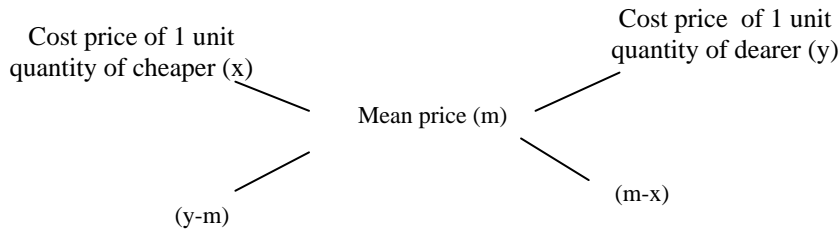
(Cheaper quantity : Dearer quantity) = $(y - m) : (m - x)$

Where, mean price (m) is the cost price of a unit quantity of the mixture.

Also, if a container contains x units of liquid from which y units are taken out and replaced by water.

After n operations, the quantity of pure liquid is $\left[x\left(1 + \frac{y}{x}\right)^n\right]$ unit.

Example 1 : How many kilograms of rice costing Rs. 18 per kg must be mixed with 30 kg of rice costing Rs. 14 per kg, so that the resultant mixture cost Rs. 15 per kg.

Solution: Applying the rule of allegation, we have

(₹ 14)

(₹ 18)

Mean price (₹ 15)

18-15=3

15-14=1
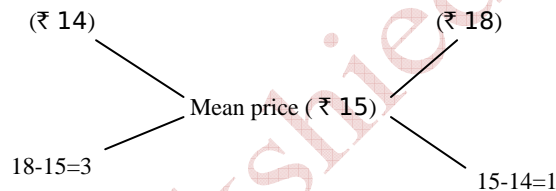
$$\frac{Quantity\ of\ cheaper\ rice}{Quantity\ of\ dearer\ rice} = \frac{3}{1}$$

∴ If cheaper rice is 3 kg, dearer rice is 1 kg.

If cheaper rice is 30 kg, dearer rice = $\left(\frac{30 \times 1}{3}\right)$ kg = 10 kg

Example 2: In what proportion must a person mix rice Rs.12.00 per kg and Rs. 14.40 per kg so as to make a mixture worth Rs. 12.60 per kg?
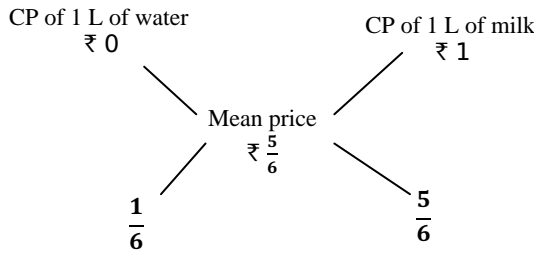
Solution:

1200 paise

CP of 1 kg dearer rice 1440 paise

Mean price 1260 paise

180

60
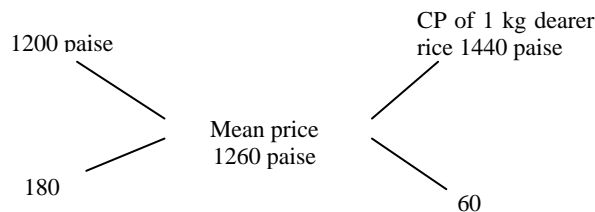
By the allegation rule,

$$\frac{Quantity\ of\ cheaper\ rice}{Quantity\ of\ dearer\ rice} = \frac{180}{60} = \frac{3}{1}$$

∴ He must mix rice in the ratio 3 : 1.

Example 3: In what proportion must water be mixed with milk to gain 20% by selling it at cost price?

CP of 1 L of water ₹ 0   CP of 1 L of milk ₹ 1

Mean price ₹ $\frac{5}{6}$

$\frac{1}{6}$ $\frac{5}{6}$

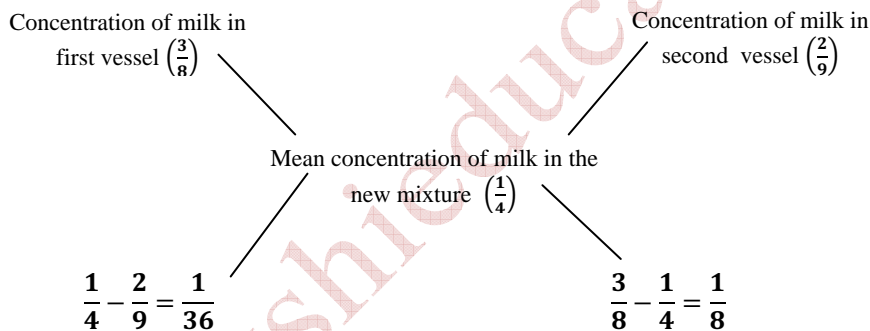**Solution:** Let CP of milk = Rs. 1 per litre

∴ SP of 1 L of mixture = Rs. 1,

Profit = 20%

∴ CP of 1 L mixture = $\frac{1}{120} \times 100$ = Rs.$\frac{5}{6}$

∴ Required ratio = $\frac{1}{6} : \frac{5}{6}$ = 1 : 5

Example 4: Two vessels contain mixture of milk and water in the ratio of 3 : 5 in the first vessel and in the ratio of 2 : 7 in the second. In what ratio should the contents of these two vessels be mixed such that the resultant mixture has milk and water in the ratio 1 : 3?

Solution . Here, we can apply the allegation rule taking the concentration of the mixtures. The concentration of milk in the first vessel is 3/8 and that in the second is $\frac{2}{9}$

Concentration of milk in first vessel $\left(\frac{3}{8}\right)$ Concentration of milk in second vessel $\left(\frac{2}{9}\right)$

Mean concentration of milk in the new mixture $\left(\frac{1}{4}\right)$

$\frac{1}{4} - \frac{2}{9} = \frac{1}{36}$ $\frac{3}{8} - \frac{1}{4} = \frac{1}{8}$

The ratio in which the two mixture should be mixed is — $\frac{1}{36} : \frac{1}{8} = 8 : 36 =$ 2 : 9

EXERCISE

1. In what ratio, tea at Rs. 90 per kg should be mixed with another tea at Rs. 120 per kg to get a tea of Rs.100 per kg
   (a) 2 : 1   (b) *3 :1*
   (c) 3 : 2   (d) 4 :3

2. Two varieties of rice at Rs. 10 per kg and Rs.12 per kg are mixed together in the ratio 1 : 2. Find the average price of the resulting mixture
   (a) Rs.11 per kg   (b) Rs. 11.22 per kg
   (c) Rs. 11.33 per kg   (d) Rs. 11.44 per kg

3. On combining two groups of students having 30 and 40 average marks respectively in an exam, the resultant group has an average score of 34. Find the ratio of the number of students in the first group to the number of students in the second group.
   (a) 2 : 1   (b) 3 : 2
   (c) 3 : 1   (d) 4 :3

4. On mixing two classes of students having average marks 25 and 40 respectively, the overall average obtained is 30 marks. Find the number of students in first class if the second class has 30 students.
   (a) 45   (b) 60
   (c) 70   (d) 80

5. 4 kg of rice at Rs. 5 per kg is mixed with 8 kg of rice at Rs. 6 per kg. Find the average price of the mixture
   (a) Rs. 5 per kg   (b) Rs. 5.55 per kg
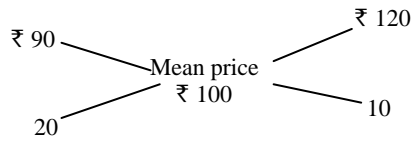   (c) Rs. 5.66 per kg   (d) Rs. 6 per kg

6. 5 kg of rice at 6 per kg is mixed with 4 kg of rice to get a mixture costing Rs. 7 per kg. Find the price of costlier rice.
(a) Rs. 7.00   (b) Rs. 7.50
(c) Rs. 8.00   (d) Rs. 8.25

7. In what proportion must water be mixed with spirit to gain $12\frac{1}{2}$ % by selling it at CP?
(a) 2 : 7   (b) 1 : 8
(c) 1 : 9   (d) 2 : 9

8. In what ratio a grocer mix tea at Rs. 22 per kg and Rs. 32 per kg, so that by selling that mixture at Rs. 28 he may gain 12%?
(a) 3 : 5   (b) 7 : 3
(c) 8 : 3   (d) 3 : 11

9. A mixture of 70 L of milk and water contains 10% water. How many litres of water must be added, so that water may be 30% of the mixture?
(a) 35 L   (b) 33 L
(c) 20 L   (d) 25 L

10. Gold is 20 times heavier than water and copper 9 times. In what ratio should these metals be mixed so that the mixture may be 15 times as heavy as water?
(a) 6 : 5   (b) 3 : 2
(c) 5 : 1   (d) 4 : 3

11. A mixture of 150 L of wine and water contains 20% water. How many litres of water must be added so that water may be 25% of the mixture?
(a) 12 L   (b) 8 L
(c) 10 L   (d) 6 L

12. Pure milk costs Rs. 21.50 per litre. A milk man adds water to 60 L of pure milk and the resultant mixture costs Rs. 20 per litre. How many litres of water does he add?
(a) $3\frac{2}{5}$ L   (b) $2\frac{1}{3}$ L
(C) $4\frac{1}{2}$ L   (d) $1\frac{3}{4}$ L

13. A dealer mixes tea costing Rs. 50 per kg with a high quality tea and sells the mixture at Rs. 54 per kg. If the ratio in which the two quality tea were mixed is 2 : 1, what is the cost of higher quality tea?
(a) Rs. 62 per kg   (b) Rs. 58 per kg
(c) Rs. 66 per kg   (d) Rs. 72 per kg

14. One type of liquid contains 25% of milk, the other contains 30% of milk. A container is filled with 6 parts of the first liquid and 4 parts of the second liquid. The percentage of milk in the mixture is
(a) 27%   (b) 31%
(c) 29 %   (d) 33%

15. A mixture of 45 L of spirit and water contain 20% of water in it. How much water must be added to it make the water 25% in' the new mixture?
(a) 5 L   (b) 3 L
(c) 4 L   (d) 6 L

16. A mixture of 20 kg of spirit and water contains 10% water. How much water (in kg) must be added to this mixture to raise the percentage of water to 25%?
(a) 30   (b) 8
(c) 5   (d) 4

17. A person travels 340 km in 8 h. The first part of the journey, he travels by car at the speed of 50 km/h and in the second part of the journey he travels by train at the speed of 38 km/h. How many km did he travel by train?
(a) 120 km   (b) 190 km
(c) 150 km   (d) 210 km

18. A merchant lent out Rs. 6440 in two parts, one at 8% and the other at 12% interest. If the yearly average interest comes out to be 9%. Find the amount lent at 12% interest?
(a) Rs. 1610   (b) Rs. 4830
(c) Rs.2640   (d) Rs. 3610

19. A sum of Rs. 7.50 is made up of 21 coins which are either 25 paise or 50 paise coins. How many coins are there of 50 paise?
(a) 9   (b) 12
(c) 7   (d) 10

20. In a zoo, there are some pigeons and some rabbits. If their heads are counted these are 100 and if their lE.g.s are counted these are 320. How many pigeons are there?
(a) 66   (b) 60
(c) 40   (d) 45

Answer Key									
1	a	5	c	9	c	13	a	17	b
2	c	6	d	10	a	14	a	18	a
3	b	7	b	11	c	15	b	19	a
4	b	8	b	12	c	16	d	20	c

SOLUTIONS

1.

₹ 90   Mean price ₹ 100   ₹ 120

20   10

Required ratio = 20 : 10 = 2 : 1

2.

₹ 10   Mean price ₹ x ₹ 12
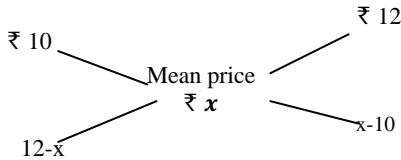
12-x x-10

$$\frac{\textbf{Quantity of cheaper rice}}{\textbf{Quantity of dearer rice}} = \frac{12-x}{x-10}$$

$$\Rightarrow \frac{1}{2} = \frac{12-x}{x-10}$$

$\Rightarrow x =$ Rs. 11.33 per kg

3. Required ratio $= \frac{40-34}{34-30}$

$$= \frac{6}{4} = \frac{3}{2}$$

4. $\frac{\textit{Students in first class}}{\textit{Students in second class}}$

$$= \frac{40-30}{30-25} = \frac{10}{5} = \frac{2}{1}$$

$\Rightarrow$ Students in first class $= \frac{2}{1} \times 30 = 60$

5. Using the cross method if average price is
Rs. x per kg.

Then

$$\frac{4}{8} = \frac{6-x}{x-5}$$
$$\Rightarrow 4x - 20 = 48 - 8x$$
$$\Rightarrow 12x = 68$$

$\Rightarrow$ x= Rs. 5.66 per kg

6.

₹ 6   Mean price ₹ 7   ₹ x

5   4
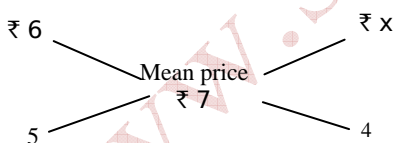
By the cross method,

$$\frac{5}{4} = \frac{x-7}{7-6} \Rightarrow 4x - 28 = 5$$
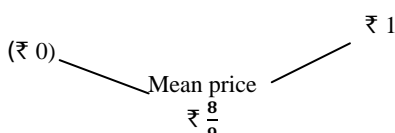
$\Rightarrow 4x = 33 \Rightarrow x =$ Rs. 8.25

7. Let the CP of the spirit be Rs. 1 per litre
Then SP of 1 L of mixture = Rs. 1 per litre

Gain $= 12\frac{1}{2}\% = \frac{25}{2}\%$

CP of 1 L of mixture $= Rs. \left(\frac{100}{100+12\frac{1}{2}}\right) = Rs. \frac{8}{9}$

By the rule of allegation.

(₹ 0)   Mean price ₹ $\frac{8}{9}$ ₹ 1

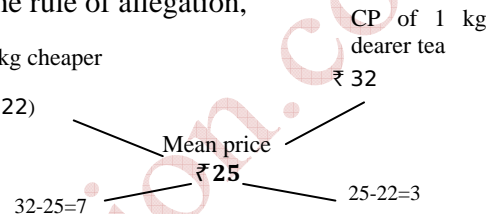$1 - \frac{8}{9} = \frac{1}{9}$ $\frac{8}{9} - 0 = \frac{8}{9}$

Ratio of water and spirit $= \frac{1}{9} : \frac{8}{9} = 1 : 8$

8. SP of 1 kg mixture = Rs. 28 Gain = 12%

CP of 1 kg mixture $= Rs. \left(\frac{100}{112} \times 28\right)$
$= Rs.$ 25

By the rule of allegation,

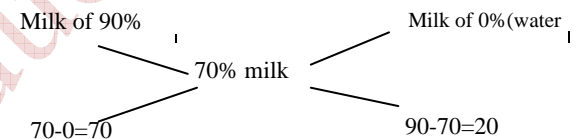CP of 1 kg cheaper (₹ 22)   Mean price ₹ 25   CP of 1 kg dearer tea ₹ 32

32-25=7   25-22=3

Ratio of cheaper tea and dearer tea =7 : 3

9. The mixture contains 10% water and 90% milk

By the rule of allegation

Milk of 90%   70% milk   Milk of 0%(water)

70-0=70   90-70=20
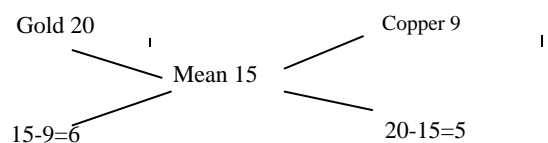
Ratio of milk and water = 70 : 20 = 7 : 2

For 7 L milk, 2 L water is to be added.

∴ For 70 L milk, $\left(\frac{70 \times 2}{7}\right)L = 20$ L water is to added.

10. By the rule of allegation,
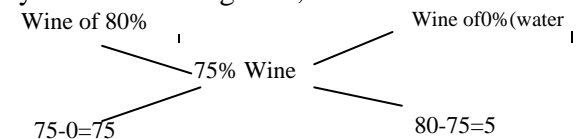
Gold 20   Mean 15   Copper 9

15-9=6   20-15=5

Ratio of gold and copper = 6 : 5

11. The mixture contains 20% water and 80% of wine.

By the rule of allegation,
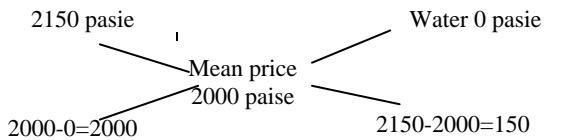
Wine of 80%   75% Wine   Wine of0%(water)

75-0=75   80-75=5

Ratio of wine : water = 75 : 5 = 15 : 1

For 15 L wine, 1 L water is to be added.

∴ For 150 L wine, $\left(\frac{150 \times 1}{15}\right)L=10$ L to be added.

12. By the rule of allegation

Cost per litre of pure milk

2150 pasie   Water 0 pasie

Mean price
2000 paise

2000-0=2000   2150-2000=150

$$\frac{Quantity\ of\ pure\ milk}{Quantity\ of\ water} = \frac{2000}{150} = \frac{40}{3}$$

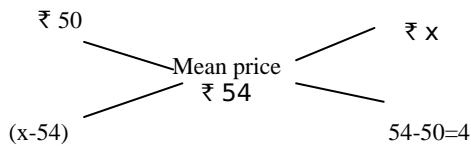For every 40 L of pure milk he adds 3 L of water.

For 60 L of milk he adds $= \frac{60\times3}{40} = \frac{9}{2}$

$= 4\frac{1}{2}$ L of water

13. Let the rate of second quality be Rs. x per kg.

CP of 1 kg tea of 1ˢᵗ Kind   **CP of 1 kg tea of 2ⁿᵈ Kind**

₹ 50   ₹ x

Mean price
₹ 54

(x-54)   54-50=4

Now $\frac{x-54}{4} = \frac{2}{1}$

$$\Rightarrow x - 54 = 4 \times 2 = 8$$
$$\Rightarrow x = 54 + 8 = 62$$

∴ CP of second quality tea is Rs. 62 per kg

14. Milk in mixture $= 6 \times \frac{25}{100} + 4 \times \frac{30}{100} = \frac{270}{100}$=27%

15. 45 L of mixture has 20% water so 45 L of mixture has 9 L of water. suppose x litres must be added.
Then
$$\frac{x+9}{x+45} \times 100 = 25$$
$$4x + 36 = x + 45 \Rightarrow x = 3L$$

16. Water in mixture = 2 kg
Let x kg of water should be mixed.
$$\therefore \frac{2+x}{20+x} \times 100 = 25 \Rightarrow \frac{2+x}{20+x} = \frac{1}{4}$$
$$\Rightarrow 8 + 4x = 20 + x$$
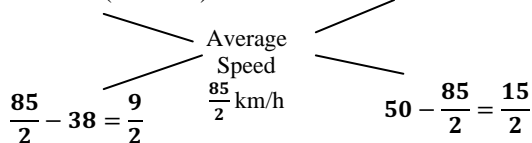$$\Rightarrow 3x = 12 \Rightarrow x = 4$$

17. Average speed of the journey
$$= \frac{340}{8} = \frac{85}{2}\ km/h$$

By the rule of allegation
Speed of car (50 km/h)   Speed of train (38 km/h)

Average Speed
$\frac{85}{2}$ km/h

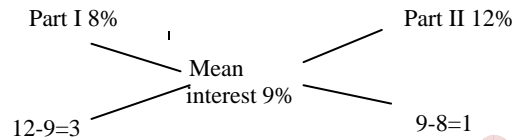$\frac{85}{2} - 38 = \frac{9}{2}$ $50 - \frac{85}{2} = \frac{15}{2}$

Time spent in car : Time spent in train

$$= \frac{9}{2} : \frac{15}{2} = 3:5$$

Time spent in train $= \frac{5}{8} \times 8 = 5$ h

Distance travelled by train =38×5= 190 km

18. By the rule of allegation

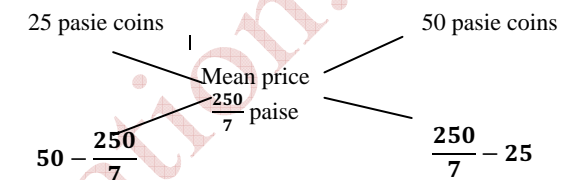Part I 8%   Part II 12%

Mean interest 9%

12-9=3   9-8=1

Ratio of Part I : Part II = 3 : 1
Amount lent on 12% interest
$= \frac{1}{4} \times 6440 =$ Rs. 1610

19. Average price $= \frac{750}{21}$ paise $= \frac{250}{7}$ paise
By the rule of allegation

25 pasie coins   50 pasie coins

Mean price
$\frac{250}{7}$ paise

$50 - \frac{250}{7}$ $\frac{250}{7} - 25$

Ratio of 25 paise coins to 50 paise coins
$= \frac{100}{7} : \frac{75}{7} = 4 : 3$

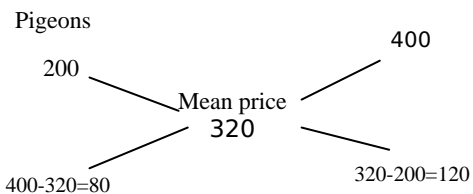∴ no. of 50 paise coins $= \frac{3}{7} \times 21 = 9$

20. Since there are 100 heads, it means that the number of pigeons and rabbits are 100. A pigeon has 2 legs and a rabbit has 4 legs. If all are pigeons, number of legs $= 2 \times 100 = 200$
If all are rabbits, number of legs $= 4 \times 100 = 400$
Actual number of legs = 320
By the rule of allegation, Rabbits

Pigeons
200   400

Mean price
320

400-320=80   320-200=120

Ratio of pigeons and rabbit
= 80 : 120 = 2 : 3
Number of pigeons $= \frac{2}{5} \times 100 = 40$