

## 11. RATIO & PROPORTION

### Ratio

The ratio of two quantities  $a$  and  $b$  is the fraction and is expressed as  $a : b$ . Here  $a$  is the first term or antecedent and  $b$  is the second term or consequent. Since the ratio expresses the number of times one quantity contains the other, it is an abstract (without units) quantity.

A ratio remains unaltered if its numerator and denominator are multiplied or divided by the same number, e.g.  $4 : 3$  is the same as  $(4 \times 10) : (3 \times 10)$  i.e.  $40 : 30$ .

$20 : 15$  is the same as  $\left(\frac{20}{5}\right) : \left(\frac{15}{5}\right)$  i.e.  $4 : 3$

### Kinds of Ratios

Duplicate Ratio :  $a^2 : b^2$  is called duplicate ratio of  $a : b$ .

Triplicate Ratio :  $a^3 : b^3$  is called triplicate ratio of  $a : b$ .

Sub -Duplicate Ratio :  $\sqrt{a} : \sqrt{b}$  is called sub-duplicate ratio of  $a : b$ .

Sub -triplicate Ratio:  $\sqrt[3]{a} : \sqrt[3]{b}$  is called sub-triplicate ratio of  $a : b$ .

**Compound Ratio:**  $ab : cd$  is the compound ratio of  $a : c$  and  $b : d$ . It is the ratio of the product of the antecedents to that of the consequents of two or more given ratios.

**Inverse Ratio :**  $\frac{1}{a} : \frac{1}{b}$  is the inverse ratio of  $a : b$ .

**Componendo and Dividendo:** If  $\frac{a}{b} = \frac{c}{d}$ , then  $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

### Proportion:

When two ratios are equal, they make a proportion, i.e. if  $\frac{a}{b} = \frac{c}{d}$ , then  $a, b, c$  and  $d$  are in proportion.

This is represented as  $a : b :: c : d$  and is read as "a is to b as c is to d"

When  $a, b, c$  and  $d$  are in proportion, then  $a$  and  $d$  are called the Extremes and  $b$  and  $c$  are called the Means, also Product of the Means = Product of the Extremes i.e.  $bc = ad$ .

### Continued Proportion

If three quantities  $a, b$  and  $c$  are such that  $a : b :: b : c$ , then  $b^2 = ac$  and  $a, b$  and  $c$  are in continued proportion. Also, the quantity  $c$  is called the third proportion of  $a$  and  $b$ .

### Fourth Proportion

If four quantities  $a, b, c$  and  $x$  are such that  $a : b :: c : x$ , then  $ax = bc$  and  $x$  is called the fourth proportion of  $a, b$ , and  $c$ .

### Mean or Second Proportion

If three quantities  $a, b$  and  $x$  are such that  $a : x :: x : b$ , then  $x^2 = ab$  and  $x$  is called the mean of  $a$  and  $b$ . Also, If  $a : b = c : d$ , then the following properties hold good.

- (i)  $b : a = d : c$  (Invertendo)
- (ii)  $a : c = b : d$  (Alter nendo)
- (iii)  $(a + b) : b = (c + d) : d$  (Componendo)
- (iv)  $(a - b) : b = (c - d) : d$  (Dividendo)
- (v)  $\frac{a+b}{a-b} = \frac{c+d}{c-d}$  (Componendo - Dividendo)

### Variation

If two quantities  $x$  and  $y$  are related in such a way that as the quantity  $x$  changes it also brings a change in the second quantity  $y$ , then the two quantities are in variation.

### Direct Variation

The quantity  $x$  is in direct variation to  $y$  if an increase in  $x$  makes  $y$  to increase proportionally. Also a decrease in  $x$  makes  $y$  to decrease proportionally it can be expressed as  $x = ky$ , where,  $k$  is called the constant of proportionality.

e.g. Cost is directly proportional to the number of articles bought.

### Inverse Variation

The quantity  $x$  is in inverse variation to  $y$  if an increase in  $x$  makes  $y$  to decrease proportionally. Also, a decrease in  $x$  makes  $y$  to increase proportionally. It can be expressed as  $x = \frac{k}{y}$  where,  $k$  is called the constant of proportionality, e.g. The time taken by a vehicle in covering a certain distance is inversely proportional to the speed of the vehicle.

### Joint Variation

If there are more than two quantities  $x$ ,  $y$  and  $z$  and  $x$  varies with both  $y$  and  $z$ , then  $x$  is in joint variation to  $y$  and  $z$ . It can be expressed as  $x = kyz$ , where,  $k$  is the constant of proportionality.

e.g. Men doing a work in some number of days working certain hours a day!

### Distribution of an Amount

If an amount  $A$  is distributed in the ratio  $a : b$ , then

$$\text{First part} = \frac{a}{a+b} \times A$$

$$\text{Second part} = \frac{b}{a+b} \times A$$

**Example 1:** Divide 60 in the ratio of 1 : 3

**Solution .** We have  $1 + 3 = 4$

$$\text{first part} = \frac{1}{4} \times 60 = 15$$

$$\text{second part} = \frac{3}{4} \times 60 = 45$$

Thus, the required parts are 15 and 45.

**Example 2:** The ratio of boys to girls in a science class of 28 is 16 : 12. Express it in simplest ratio.

**Solution:**  $16 : 12 = 4 : 3$  Which is the ratio in its simplest form.

**Example 3:** Compare the ratio of the third proportion of 6 and 5 with the fourth proportion of 4, 3 and 10.

**Solution:** Third proportion of 6 and 5 is  $6 : 5 :: 5 : x$

$$\Rightarrow x = \frac{25}{6}$$

Fourth proportion of 4, 3 and 10 is  $4 : 3 :: 10 : y$

$$\Rightarrow y = \frac{3 \times 10}{4} = \frac{15}{2}$$

$$\therefore \text{Required ratio} = \frac{25}{6} : \frac{15}{2} = 5 : 9$$

**Example 4:** Two numbers are in the ratio 2 : 3. If 10 is added to each, they are in the ratio 4 : 5. Find the two numbers?

**Solution:** Let the two numbers be  $x$  and  $y$ .

$$\frac{x}{y} = \frac{2}{3} \dots\dots (i)$$

$$\frac{x+10}{y+10} = \frac{4}{5} \dots\dots (ii)$$

Solving (i) and (ii) we get  $x = 10$  and  $y = 15$

$\therefore$  The two numbers are 10 and 15.

**Example 5:** The ratio between two numbers is 3 : 7. If their LCM is 210, find the numbers.

**Solution:** Let the numbers be  $3x$  and  $7x$

$$\text{LCM is } 3 \times 7 \times x = 21x$$

$$\therefore 21x = 210$$

$\therefore$  Numbers are 30 and 70.

## EXERCISE

- The mean proportional between 234 and 104 is:  
(a) 16 (b) 40  
(c) 54 (d) None of these
- The fourth proportional to 5, 8, 15 is :  
(a) 22 (b) 24  
(c) 23 (d) 20
- The third proportional to 0.36 and 0.48, is:  
(a) 0.64 (b) 0.1728  
(c) 0.44 (d) 0.82
- In a ratio, which is equal to 3 : 4, if the antecedent is 12, then the consequent is:  
(a) 10 (b) 16  
(c) 20 (d) 22
- Ratio of the earnings of A and B is 4:7. If the earnings of A increase by 50% and those of B decrease by 25%, the new ratio of their earnings become 8: 7. What are A's earnings?  
(a) Rs. 25,000 (b) Rs. 26,000  
(c) Rs. 29,000 (d) Data incorrect
- What least number must be subtracted from each of the numbers 14, 17, 34 and 42 so that the remainders may be proportional?  
(a) 4 (b) 3 (c) 2 (d) 9
- The compounded ratio of (2 : 3), (6 : 11) and (11 : 2) is :  
(a) 7 : 2 (b) 2 : 1  
(c) 11 : 24 (d) 58 : 121
- 7 is what part of 8.  
(a)  $\frac{1}{7}$  (b)  $\frac{1}{8}$   
(c)  $\frac{7}{8}$  (d)  $\frac{8}{7}$
- If  $3x = 8y$  and  $5y = 9z$ , then  $\frac{x}{z}$  is equal to  
(a)  $\frac{72}{15}$  (b)  $\frac{83}{15}$   
(c)  $\frac{9}{8}$  (d)  $\frac{11}{83}$
- If  $a : b = 5 : 9$ ,  $b : c = 6 : 11$ , find  $a : b : c$   
(a) 11 : 18 : 33 (b) 10 : 19 : 34  
(c) 11 : 19 : 34 (d) 10 : 18 : 33
- Duplicate ratio of  $x : 2y$   
(a)  $x^2 : 4y^2$  (b)  $x^2 + 2y^2$   
(c)  $\sqrt{x} : \sqrt{2y}$  (d)  $2y : x$
- If  $A : B = 1 : 3$  and  $B : C = 4 : 5$ , Find A : C.  
(a) 4 : 9 (b) 15 : 4  
(c) 4 : 15 (d) 9 : 15
- A bag contains 25 paise, 10 paise and 5 paise coins in the ratio 1: 2 : 3. If their total value is 60, the number of 5 paise coins is  
(a) 100 (b) 500  
(c) 300 (d) 400
- The monthly salary of A, B and C is in the ratio of 4 : 5 : 7. If C's monthly salary is Rs. 300 more than that of A. Then, B's annual salary is  
(a) Rs. 6000 (b) Rs. 8500  
(c) Rs. 4000 (d) Rs. 6500
- Ratio between two numbers is 5 : 6 and sum of their squares is 549. The numbers are  
(a) 10, 12 (b) 15, 18  
(c) 20, 24 (d) 30, 36
- A club consists of 24 members. The ratio of men to women can be  
(a) 2:3 (b) 3:4  
(c) 1:3 (d) 2:5
- Some money is divided among three workers A, B and C such that 5 times A's share is equal to 12 times B's share which is equal to 6 times C share. The ratio between the shares of A, B, C is  
(a) 5 : 10 : 12 (b) 12 : 5 : 10  
(c) 10 : 12 : 5 (d) 5 : 12 : 10
- A man is 20 yr older to his son. The present age of the son is 30 yr. How many years ago was the ratio of their ages 1 : 2 ?  
(a) 18 yr (b) 20 yr  
(c) 10 yr (d) 15 yr
- The ratio of Anita's age to the age of her mother is 4:9. The difference between their ages is 25 yr. The ratio of their ages after 10 yr will be  
(a) 10:6 (b) 6 : 10  
(c) 6 : 11 (d) 11 : 6
- The electricity bill of a certain establishment is partly fixed and partly varies as the number of units of electricity consumed. When in a certain month 540 units are consumed, the bill is Rs. 1800. In another month 620 units are consumed and the bill is Rs. 2040. In yet another month 500 units are consumed. The bill for that month would be:

- (a) Rs. 1605 (b) Rs. 1680  
 (c) Rs. 1840 (d) Rs. 2050
21. An amount of Rs. 2430 is divided among A, B and C such that if their shares be reduced by 5, 10 and 15 respectively, the remainders shall be in the ratio of 3 : 4 : 5. Then, B's share was  
 (a) Rs. 609 (b) Rs. 798  
 (c) Rs. 845 (d) Rs. 810

22. An amount of Rs. 735 was divided between A, B and C. If each of them had received Rs. 25 less, their shares would have been in the ratio of 1:3:2. The money received by C was:  
 (a) Rs. 198 (b) Rs. 228  
 (c) Rs. 225 (d) Rs. 245

ANSWER KEY

1	d	6	c	11	a	16	c	21	d
2	b	7	b	12	c	17	b	22	d
3	a	8	c	13	c	18	c		
4	b	9	a	14	a	19	c		
5	d	10	d	15	b	20	b		

SOLUTIONS

1. Required mean proportional  
 $= \sqrt{234 \times 104}$   
 $= \sqrt{13 \times 9 \times 2 \times 13 \times 8}$   
 $= (13 \times 3 \times 4) = 156.$
2. Let the fourth proportional to 5, 8, 15 be x, Then,  $5 : 8 :: 15 : x$   
 $\Leftrightarrow 5x = (8 \times 15) \Leftrightarrow x = \frac{(8 \times 15)}{5} = 24.$
3. Let the third proportional to 0.36 and 0.48 be x.  
 Then,  $0.36 : 0.48 :: 0.48 : x$   
 $\Leftrightarrow x = \left( \frac{0.48 \times 0.48}{0.36} \right) = 0.64$
4. (b) let  $3:4 = 12: x$   
 $\Rightarrow 12 \times 4 = x \times 3$   
 $\Rightarrow x = \frac{12 \times 4}{3} = 16.$
5. Let the original earnings of A and B be Rs.  $4x$  and Rs.  $7x$ .  
 New earnings of A = 150% of Rs.  $4x$   
 $= \text{Rs. } 6x$   
 New earnings of B = 75% of Rs.  $7x$   
 $= \text{Rs. } \left( \frac{75}{100} \times 7x \right) = \frac{525x}{100} = \text{Rs. } \frac{21x}{4}$   
 This does not give x. So, the given data is inadequate.
6. Let the required number be x.  
 Then,  
 $(14 - x) : (17 - x) :: (34 - x) : (42 - x)$   
 $\therefore \frac{14 - x}{17 - x} = \frac{34 - x}{42 - x} \Leftrightarrow (14 - x)(42 - x)$   
 $= (17 - x)(34 - x)$

- $\Leftrightarrow x^2 - 56x + 588 = x^2 - 51x + 578$   
 $\Leftrightarrow 5x = 10 \Leftrightarrow x = 2$   
 $\therefore$  Required number = 2.
7. Required ratio =  $\left( \frac{2}{3} \times \frac{6}{11} \times \frac{11}{2} \right)$   
 $= \frac{2}{1} = 2:1$
8. Required part =  $\frac{7}{8}$
9.  $3x = 8y \Rightarrow x = \frac{8y}{3}$   
 $9z = 5y \Rightarrow z = \frac{5y}{9}$   
 $\therefore \frac{x}{z} = \frac{8y}{3} \times \frac{9}{5y} = \frac{72}{15}$
10.  $a : b = 5 : 9$   
 $b : c = 6 : 11 = 6 \times \frac{9}{6} : 11 \times \frac{9}{6} = 9 : \frac{33}{2}$   
 $\therefore a : b : c = 5 : 9 : \frac{33}{2} = 10 : 18 : 33$
11. Duplicate ratio of.  
 $x : 2y = x^2 : (2y)^2 = x^2 : 4y^2$
12.  $A : B = 1 : 3 = (1 \times 4) : (3 \times 4) = 4 : 12$   
 $B : C = 4 : 5 = (4 \times 3) : (5 \times 3)$   
 $= 12 : 15$   
 $\therefore A : B : C = 4 : 12 : 15 \Rightarrow A : C = 4 : 15$
13. Let the number of 25-p, 10-p and 5-p coins be x, 2x and 3x respectively. Then,  $0.25x + 0.2x + 0.05x = 60$   
 $0.6x = 60 \Rightarrow x = 100$   
 $\therefore$  the number of 5-p coins =  $3 \times 100 = 300.$
14. Let the monthly salaries of A, B and C be  $4x$ ,  $5x$  and  $7x$  respectively.  
 $4x + 300 = 7x \Rightarrow x = 100$

The monthly salary of  $B = \text{Rs. } 500$  and his annual salary  
 $= 12 \times 500 = \text{Rs. } 6000$ .

15. Let the two numbers be  $5x$  and  $6x$  respectively.

$$\begin{aligned}(5x)^2 + (6x)^2 &= 549 \\ \Rightarrow 25x^2 + 36x^2 &= 549 \\ \Rightarrow 61x^2 &= 549 \\ \Rightarrow x^2 &= 9 \\ \Rightarrow x &= 3\end{aligned}$$

So, the two numbers are 15 and 18 respectively.

16. On dividing 24 into two whole numbers, the sum of the terms of the ratio must be a factor of 24. So, 1 : 3 is the required ratio.

17. Let  $5A = 12B = 6C = k$

$$\text{Then, } A = \frac{k}{5}, B = \frac{k}{12}, C = \frac{k}{6}$$

$$\therefore A : B : C = \frac{k}{5} : \frac{k}{12} : \frac{k}{6} = 12 : 5 : 10$$

18. Present age of son = 30 years  
 Present age of man =  $(30 + 20)$  years  
 $= 50$  years

Then,

$$\frac{30 - x}{50 - x} = \frac{1}{2} \Rightarrow 2(30 - x) = 50 - x \\ \Rightarrow x = 10$$

So, 10 years ago the ratio of their ages was 1 : 2

19. Let Anita's present age be  $4x$  and her mother's present age be  $9x$  years.

$$\text{Then, } (9x - 4x) = 25 \\ \Rightarrow 5x = 25 \Rightarrow x = 5$$

Ratio of their ages after 10 years

$$\begin{aligned}\frac{4x + 10}{9x + 10} &= \frac{4 \times 5 + 10}{9 \times 5 + 10} \\ &= \frac{20 + 10}{45 + 10} = \frac{30}{55} = \frac{6}{11} = 6 : 11\end{aligned}$$

20. Let the fixed amount be  $\text{Rs. } x$  and the cost of each unit be  $\text{Rs. } y$ . Then,

$$540y + x = 1800 \dots \dots (i)$$

$$\text{And } 620y + x = 2040 \dots \dots (ii)$$

On subtracting (i) from (ii), we get

$$80y = 240$$

$$\Leftrightarrow y = 3.$$

Putting  $y = 3$  in (i), we get:

$$540 \times 3 + x = 1800$$

$$\Leftrightarrow x = (1800 - 1620) = 180.$$

$\therefore$  Fixed charges =  $\text{Rs. } 180$ , Charge per unit  
 $= \text{Rs. } 3$ .

Total charges for consuming 500 units  
 $= \text{Rs. } (180 + 500 \times 3) = \text{Rs. } 1680$ .

$$\begin{aligned}21. \text{ Remainder} &= \text{Rs. } [2430 - (5 + 10 + 15)] \\ &= \text{Rs. } 2400.\end{aligned}$$

$$\therefore B \text{ 's share} = \text{Rs. } \left[ \left( 2400 \times \frac{4}{12} \right) + 10 \right] \\ = \text{Rs. } 810.$$

$$\begin{aligned}22. \text{ Remainder} &= \text{Rs. } [735 - (25 \times 3)] \\ &= \text{Rs. } 660.\end{aligned}$$

$$\therefore \text{ Money received by } C \\ = \text{Rs. } \left[ \left( 660 \times \frac{2}{6} \right) + 25 \right] = \text{Rs. } 245.$$