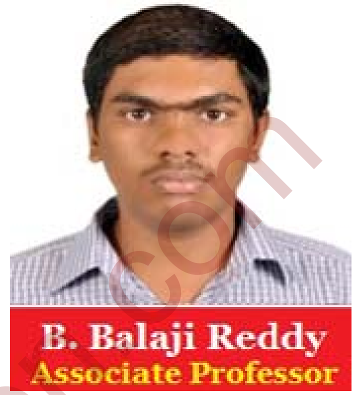
DC Transients

Steady state and transient response:



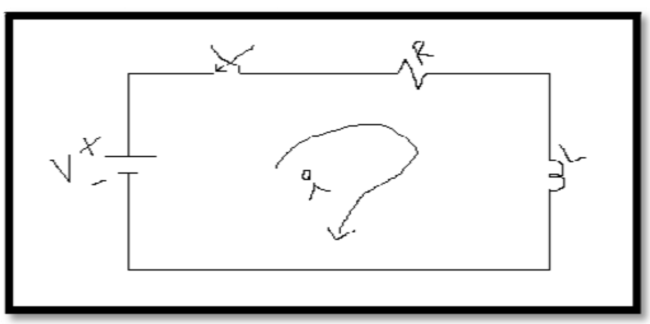
A circuit having constant sources is said to be in steady state of the currents and voltages do not change with time. Thus, circuits with currents and voltages having constant amplitude and constant frequency sinusoidal functions are also considered to be in a steady state. This means that the amplitude or frequency of a sinusoidal never changes in a steady state circuit.

In a network containing energy storage elements, with change in excitation, the currents and voltages change from one state to other state. The behavior of the voltage or current when it is changed from one state to another state is called as transient state. This time is known as transient time.

The application of KVL and KCL to circuits containing energy storage elements results in differential, rather than algebraic equations. When we consider a circuit containing storage elements that are independent of the sources, then the response depends on the nature of the circuit and is called the *natural response*. Storage elements deliver their energy to resistances. Hence the response change with time gets saturated after some time and is referred to as the transient response. When we consider the sources acting on the circuit, the nature depends on the nature of sources and this response is called as forced response. In other words, the complete response of a circuit consists of two parts; they are forced and transient response. When we consider a differential equation, the complete solution consists of two parts, one is complementary solution and another is particular solution. The complementary function dies out after short interval, and is referred to as the transient response or source free response. The particular function is the steady state response or forced response. The first step in finding the complete solution of a circuit is to form a differential equation for the circuit. By obtaining the differential equation, several methods can be used to find out the complete solution.

DC Transients

DC Response of an R-L Circuit:



Consider a circuit consisting of a resistance and inductance as shown in fig. the inductor in the circuit is initially uncharged and is in series with the resistor. When switch S is closed, we can find the complete solution for current. Application of Kirchoff's law to the circuit results in fallowing differential equations.

$$V = iR + L\frac{di}{dt} \text{ ----------- (1)}$$

$$\frac{di}{dt} + \frac{R}{L} I = \frac{V}{L} \text{ -------------- (2)}$$

In the above equation, the current i is the solution to be found and V is the applied constant voltage. The voltage V is applied to the circuit only when the switch S is closed. The above equation is linear differential equation of the first order comparing with the non homogenous differential equation.

$$\frac{dx}{dt} + P X = K \text{ whose solution is } x = e^{-pt}\int Ke^{+pt}\,dt + ce^{-pt} \text{ ----------- (3)}$$

Where c is an arbitrary constant, in similar way we can write the current equation as

$$i = ce^{-(\frac{R}{L})t} + e^{-(\frac{R}{L})t}\int \frac{V}{L} e^{(\frac{R}{L})t}dt \text{ ----------- (4)}$$

$$i = ce^{-(\frac{R}{L})t} + \frac{V}{R} \text{ ------------- (5)}$$

To determine the value of 'c', in equation (5) we use initial conditions. In the circuit shown in fig the switch S is closed at t=0. At $t=0^{-}$, i.e. just before closing the switch S, the current in the inductor is zero. Since the inductor does not allow sudden changes in currents, at $t=0^{+}$ just after the switch is closed, the current remains zero.

Substituting above conditions we get,

$$0 = c + \frac{V}{R}$$

Therefore, $c = -\frac{V}{R}$

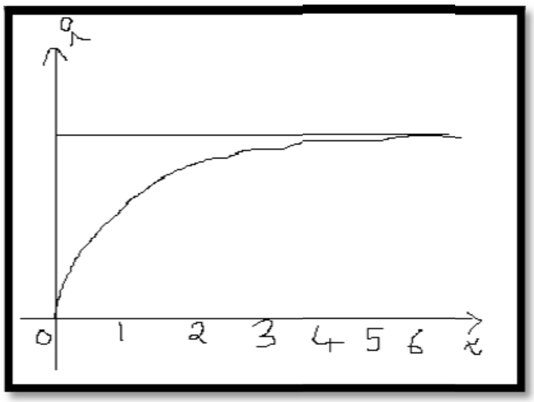
Hence from equation (5),

$$i = -\frac{V}{R} e^{-(\frac{R}{L})t} + \frac{V}{R}$$

$$i = \frac{V}{R}(1 - e^{-(\frac{R}{L})t}) \text{ ------------ (6)}$$

Equation (6) consists of two parts, the steady state part $\frac{V}{R}$ and other is transient part. When switch S is closed the response reaches a steady state value after a time interval as shown in fig.

hence the transient period is defined as the time taken for the current to reach its final or steady state value from its initial value. In the transient part of the solution, the quantity $\frac{L}{R}$ is important in describing the curve since $\frac{L}{R}$ is the time required for the current to reach from its initial value of zero to the final value $\frac{V}{R}$. The time constant of a function $\frac{V}{R}$ $ce^{-(\frac{R}{L})t}$ is the time at which the exponent of e is unity, where e is the base of the natural logarithms. The term $\frac{L}{R}$ is called time constant and is denoted by τ.



$$\tau = \frac{L}{R} \text{ seconds}$$

The transient part of solution is, i(τ) = - $\frac{V}{R}$ $e^{-\frac{t}{\tau}}$

At time constant is one, i(τ) = - $\frac{V}{R}$ $e^{-\frac{t}{\tau}}$ = - $\frac{V}{R}$ e^{-1} = - 0.368$\frac{V}{R}$

The transient response reaches 36.8 percent of its initial value.

Similarly, i(2τ) = - $\frac{V}{R}$ $e^{-\frac{t}{\tau}}$ = - $\frac{V}{R}$ e^{-2} = - 0.135$\frac{V}{R}$

i(3τ) = - $\frac{V}{R}$ $e^{-\frac{t}{\tau}}$ = - $\frac{V}{R}$ e^{-3} = - 0.0498$\frac{V}{R}$

i(5τ) = - $\frac{V}{R}$ $e^{-\frac{t}{\tau}}$ = - $\frac{V}{R}$ e^{-5} = - 0.0067$\frac{V}{R}$

After 5, the transient part reaches more than 99 percent of its final value. In fig we can find out the voltages and powers across each element by using the current.

Voltage across the resistor, V_R = R i = R * $\frac{V}{R}$ (1- $e^{-(\frac{R}{L})t}$)

$$V_R = V\,(1- e^{-(\frac{R}{L})t})$$

Similarly, the voltage across the inductor, V_L = L $\frac{di}{dt}$

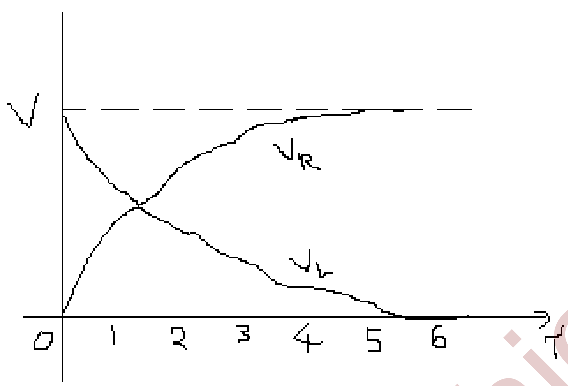
$$V_L = \text{L} * \frac{V}{R}\ e^{-(\frac{R}{L})t}\ \frac{R}{L} = \ \text{V}e^{-(\frac{R}{L})t}$$

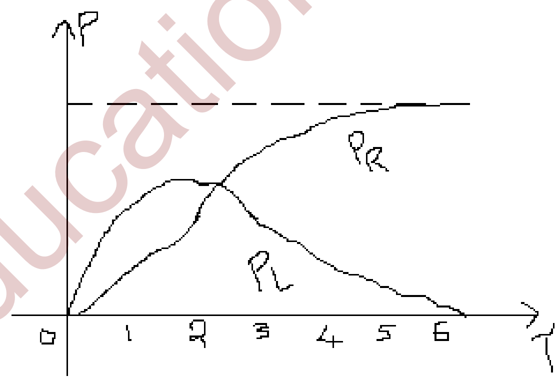
Power in the resistor, $P_R = V_R\ i$ = V (1- $e^{-(\frac{R}{L})t}$) * $\frac{V}{R}$ (1- $e^{-(\frac{R}{L})t}$)

$$= \frac{V^2}{R}\,(1 - 2\,e^{-(\frac{R}{L})t}) + \ e^{-(\frac{2R}{L})t}$$

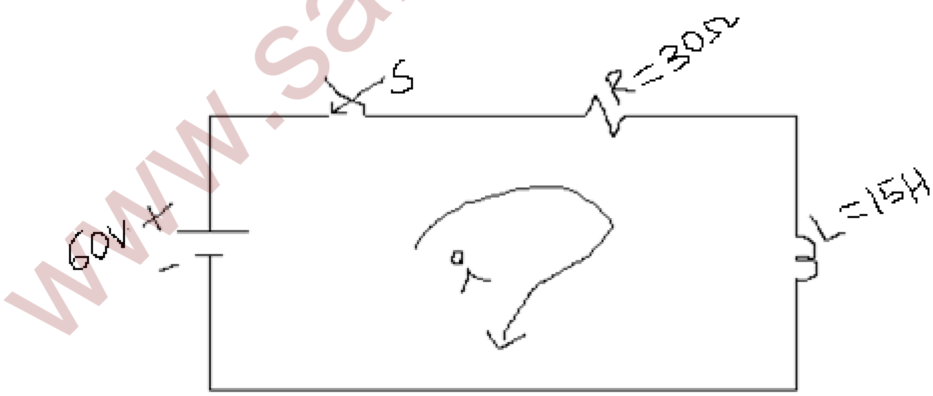
Power in the resistor, $P_R = V_L\ i$ = V$e^{-(\frac{R}{L})t}$ * $\frac{V}{R}$ (1- $e^{-(\frac{R}{L})t}$)

$$= \frac{V^2}{R}\,(\ e^{-(\frac{R}{L})t}\ -\ e^{-(\frac{2R}{L})t})$$





Example: A series RL circuit with R=30Ω and L=15H has a constant voltage V=60v applied at t=0 as shown in fig. determine the current i, the voltage across the inductor.



By applying KVL we get,

$$60 = 30\text{i} + 15\,\frac{di}{dt}$$

$\frac{di}{dt}$ + 2i = 4

The general solution for a linear differential equation is,

$$i = e^{-pt} \int K e^{+pt}\, dt + c e^{-pt}$$

$$i = e^{-2t} \int 4 e^{2t}\, dt + c e^{-2t}$$

At t=0, the switch S is closed. Since the inductor never allows sudden changes in currents. At $t=0^+$ the current in the circuit is zero.

Therefore, $t=0^+$, i=0

0 = c + 2

c=-2

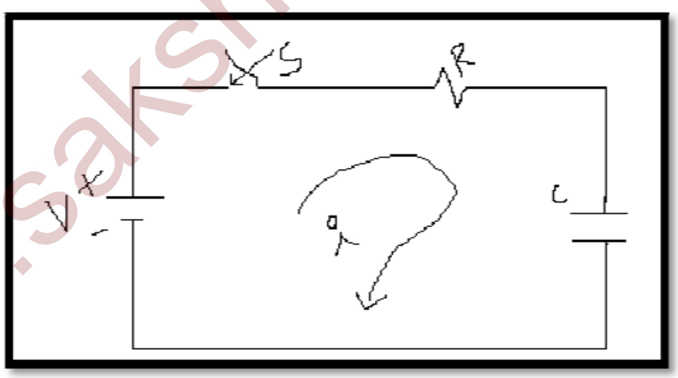
Substituting the value of c in the current equation, we have

$$i = 2(1-e^{-2t})\text{A}$$

Voltage across resistor, V_R =iR= $2(1-e^{-2t})*30=60(1-e^{-2t})$V

Voltage across the inductor, $V_L = L\frac{di}{dt} = 15 * \frac{d}{dt}(2(1-e^{-2t}) = 60e^{-2t}$V

DC Response of an R-C Circuit:



Consider a circuit consisting of resistance and capacitance as shown in fig. the capacitor in the circuit is initially uncharged, and is in series with resistor. When the switch S is closed at t=0, we can determine the complete solution for current. Application of Kirchoff's laws we can determine the differential equations.

$$V = R\,i + \frac{1}{C}\int i\, dt \text{ ----- (1)}$$

By differentiating the above equation we get,

$$0 = R\frac{di}{dt} + \frac{i}{C} \text{ ------------- } (2)$$

$$\frac{di}{dt} + \frac{i}{RC} = 0 \text{ --------- } (3)$$

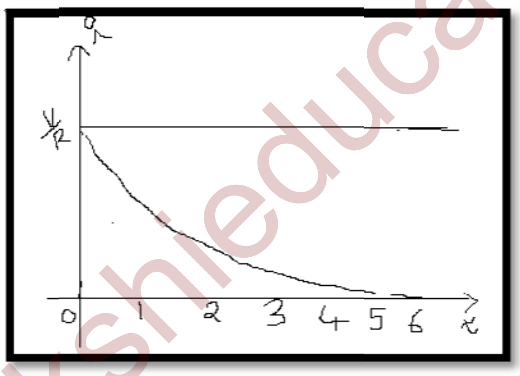
Equation (3) is linear differential equation with only the complementary function. The particular solution for the above equation is zero. The solution for this type of differential equation is

$$i = c\, e^{-\frac{t}{RC}} \text{ --------- } (4)$$

Here, to find the value of c, we use the initial conditions. In the circuit shown in fig switch S is closed at t=0. Since the capacitor never allows sudden changes in voltage, it will act as short at $t=0^+$. So, the current in the circuit at $t=0^+$ is V/R.

Substituting the i value in equation (4) we get,

$$i = \frac{V}{R} e^{-\frac{t}{RC}} \text{ ----------- } (5)$$



When switch S is closed, the response decays with time as shown in fig. In the solution, the quantity RC is the time constant, and is denoted byτ, where τ= RC seconds.

After 5, the transient part reaches more than 99 percent of its final value. In fig we can find out the voltage across each element by using the current equation.

Voltage across the resistor V_R = R i = R $* \frac{V}{R} e^{-\frac{t}{RC}}$ = V $e^{-\frac{t}{RC}}$

Similarly, voltage across the capacitor $V_C = \frac{1}{C}\int i\, dt$

$$= \frac{1}{C}\int \frac{V}{R}\, e^{-\frac{t}{RC}}\, dt$$

$$= -\left(\frac{V}{RC} * RC\, e^{-\frac{t}{RC}}\right) + c$$

$$=-V\, e^{-\frac{t}{RC}} + c$$

At t=0, voltage across the capacitor is zero.

$$c = V$$

$$V_C = V\,(1- e^{-\frac{t}{RC}})$$

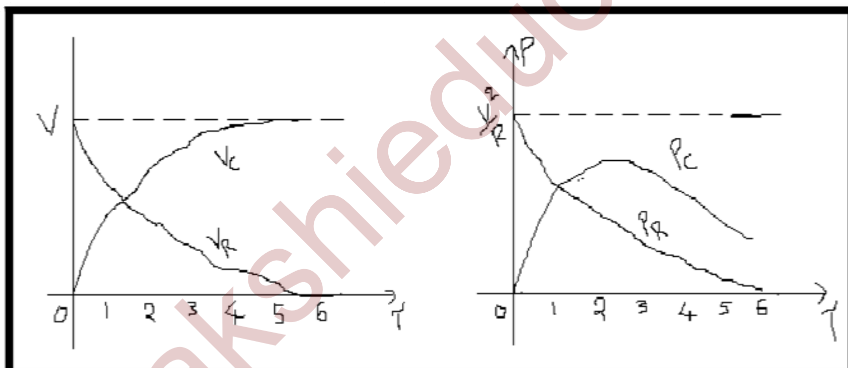
Power in the resistor, $P_R = V_R\, i = V\, e^{-\frac{t}{RC}} * \frac{V}{R}\, e^{-\frac{t}{RC}}$

$$= \frac{V^2}{R}\, e^{-\frac{2t}{RC}}$$

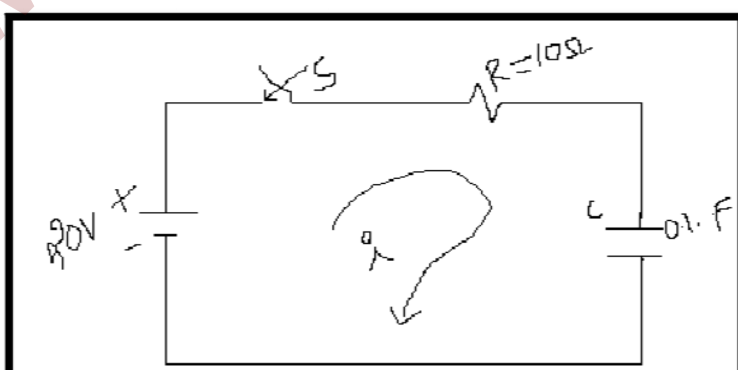
Power in the capacitor, $P_C = V_C\, i = V\,(1- e^{-\frac{t}{RC}}) * \frac{V}{R}\, e^{-\frac{t}{RC}}$

$$= \frac{V^2}{R}\,(e^{-\frac{t}{RC}} - e^{-\frac{2t}{RC}})$$

The responses are shown in fig.



Example: A series RC circuit with R=10Ω and C=0.1F has a constant voltage V=20v applied at t=0 as shown in fig. determine the current i, the voltage across the capacitor.



By applying KVL to the circuit,

$$20 = 10i + \frac{1}{0.1}\int i\,dt$$

By differentiating the above equation we get,

$$0 = 10\,\frac{di}{dt} + \frac{i}{0.1}$$

$$\frac{di}{dt} + \frac{i}{1} = 0$$

The solution for this type of differential equation is

$$i = c\,e^{-\frac{t}{RC}} = c\,e^{-t}$$

At t=0, Switch S is closed. Since the capacitor does not allow sudden changes in voltage, the current in the circuit is, i=V/R=20/10=2A

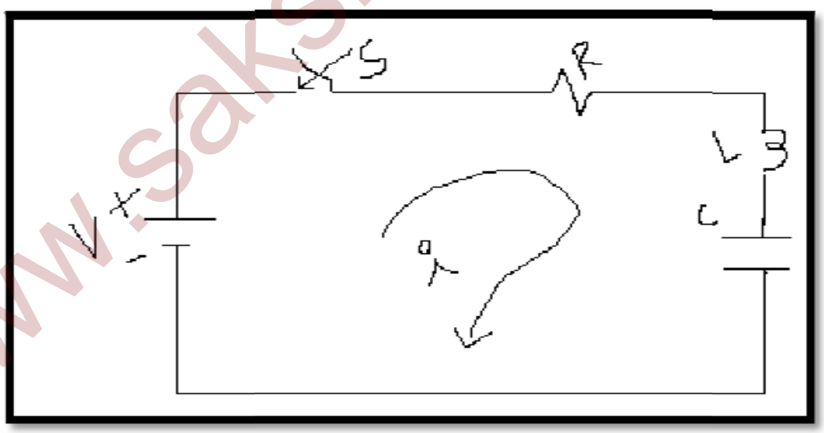
Therefore the current equation is, i= 2 e^{-t}

Voltage across the resistor V_R = R i = = V e^{-t} = 10 * 2 e^{-t} =20 e^{-t} V

Voltage across the capacitor V_C = V (1-2 $e^{-\frac{t}{RC}}$)

$$V_C = 20\,(1- e^{-t})\ \text{V}$$

DC Response of an R-L-C Circuit:



Consider a circuit consisting of resistance, inductance and capacitance as shown in fig. the capacitor and inductor are initially uncharged, and are in series with a resistor. When the switch S is closed at t=0, we can determine the complete solution for current. Application of Kirchoff's laws we can determine the differential equations.

$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \text{ --------- (1)}$$

By differentiating above equation we get,

$$0 = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i \text{ ------------ (2)}$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0 \text{ ----------- (3)}$$

The above equation is a second order linear differential equation, with only complementary function. The particular solution for the above equation is zero. Characteristic equation for the above differential equation is

$$D^2 + \frac{R}{L}D + \frac{1}{LC} = 0 \text{ ---------- (4)}$$

The roots above equation are,

$$D_1, D_2 = -\frac{R}{2L} \pm \sqrt{(\frac{R}{2L})^2 - \frac{1}{LC}}$$

By assuming, $K_1 = -\frac{R}{2L}$ and $K_2 = \sqrt{(\frac{R}{2L})^2 - \frac{1}{LC}}$

$$D_1 = K_1 + K_2$$

$$D_2 = K_1 - K_2$$

Here K_2 may be positive or negative or zero.

K_2 Is positive, when $(\frac{R}{2L})^2 > \frac{1}{LC}$

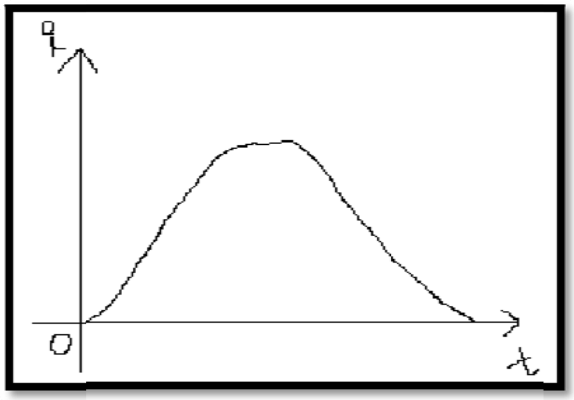
The roots are real and unequal, and give the over damped response as shown in fig. then equation (4) becomes

$$[D - (K_1+K_2)][[D - (K_1-K_2)]]i=0$$

The solution for the above equation is,

$$i = c_1 e^{(K_1+K_2)t} + c_2 e^{(K_1-K_2)t}$$

The current curve for the over damped case is shown in fig.



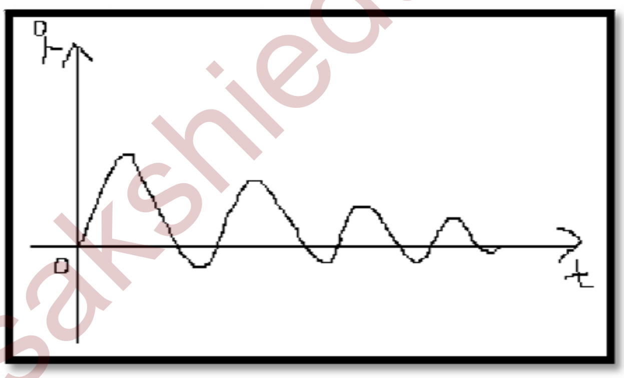
K_2 Is negative, when $(\frac{R}{2L})^2 < \frac{1}{LC}$. The roots are complex conjugate, and give the under damped response as shown in fig. the equation as shown in becomes

$$[D - (K_1 + jK_2)][[D - (K_1 - jK_2)]]i = 0$$

The solution for above equation is,

$$i = e^{K_1 t}\,[c_1 \cos K_2 t + c_2 \sin K_2 t\,]$$

The current curve for the under damped case is shown in fig.



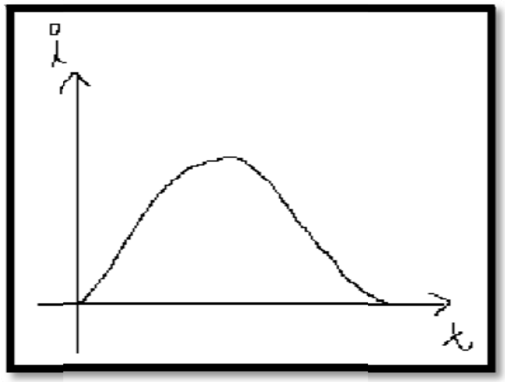
K_2 Is zero, when $(\frac{R}{2L})^2 = \frac{1}{LC}$. The roots are equal, and give the critically damped response as shown in fig. the equation becomes

$$[D - K_1][[D - K_2]]i = 0$$

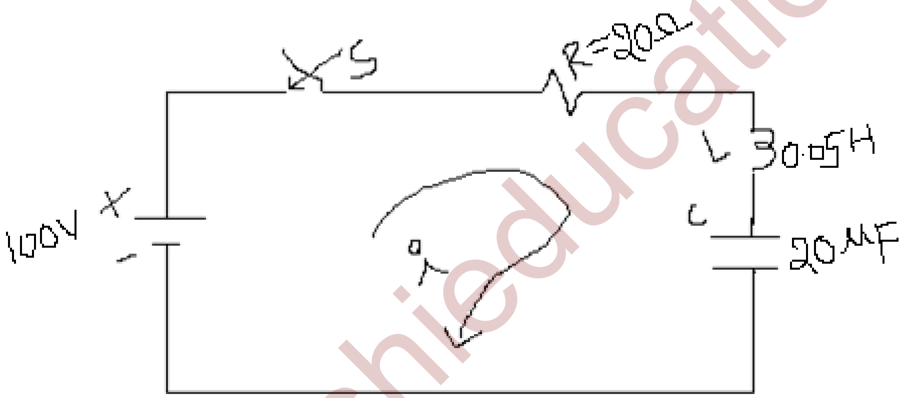
The solution for above equation is

$$i = e^{K_1 t}\,[c_1 + c_2 t\,]$$

The current curve for critically damped case is as shown in fig.



Example: A series RLC circuit with R=10Ω, L=0.05H and C=20μF has a constant voltage V=100v applied at t=0 as shown in fig. determine the current i.



$$100 = 20i + 0.05\frac{di}{dt} + \frac{1}{20*10^{-6}}\int i\,dt$$

By differentiating above equation we get,

$$0 = 20\frac{di}{dt} + 0.05\frac{d^2i}{dt^2} + \frac{1}{20*10^{-6}}\,i$$

$$\frac{d^2i}{dt^2} + 400\frac{di}{dt} + 10^6\,i = 0$$

$$D^2 + 400\,D + 10^6 = 0$$

The roots above equation are,

$$D_1, D_2 = -\frac{400}{2} \pm \sqrt{(\frac{400}{2})^2 - 10^6}$$

$$= -200 \pm \sqrt{(200)^2 - 10^6}$$

D_1 = -200+ j979.8

D_2 = -200- j979.8

Therefore the current, i = $e^{K_1 t}$ [$c_1 \cos K_2 t$ +$c_2 \sin K_2 t$]

i = e^{-200t} [$c_1 \cos 979.8t$ +$c_2 \sin 979.8t$] A

At t=0, the current through the circuit is zero

i=0= 1[$c_1 \cos 0$ +$c_2 \sin 0$]

c_1 =0

i = e^{-200t} [$c_2 \sin 979.8t$] A

Differentiating we have,

$\frac{di}{dt} = c_2[e^{-200t} 979.8 \cos 979.8t$] + e^{-200t}(-200) $\sin 979.8t$

At t=0, the voltage across the inductor is 100V

L $\frac{di}{dt}$ =100 or $\frac{di}{dt}$ = 2000

At t=0, $\frac{di}{dt}$ =2000 = c_2 979.8cos 0

c_2 = 2.04

Therefore the current equation is, i= e^{-200t} [2.04 $\sin 979.8t$] A