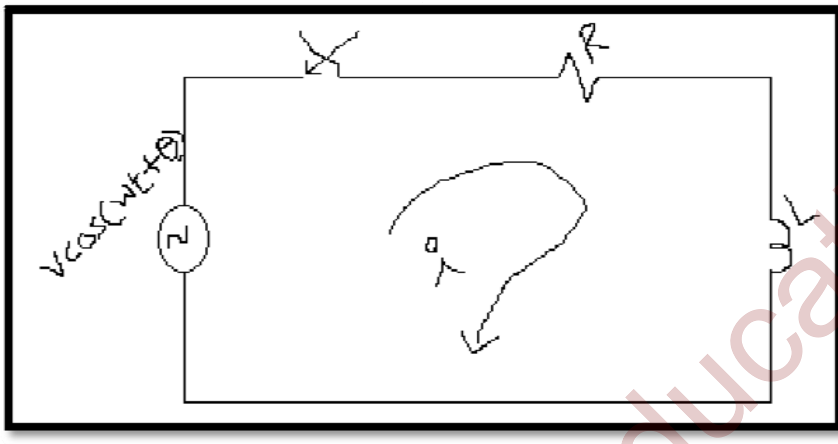
AC Transients

Sinusoidal Response of R-L Circuit:

Consider a circuit consisting of resistance and inductance are connected as shown in fig. the switch S is closed at t=0. At t=0, sinusoidal voltage V$\cos(wt+\theta)$ applied to RL circuit, where V is the amplitude of the wave and θ is the phase angle. Application of Kirchoff's laws we can determine the differential equations.



$$V\cos(wt+\theta) = iR + L\frac{di}{dt} \text{ ----------- (1)}$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L}\cos(wt+\theta) \text{ -------------- (2)}$$

The corresponding characteristic equation is

$$(D + \frac{R}{L})\, i = \frac{V}{L}\cos(wt+\theta) \text{ -------------- (3)}$$

For the above equation, the solution consists of two parts. One is complementary function and other is particular integral.

The complementary function of the solution is

$$i_c = c\, e^{-t(\frac{R}{L})} \text{ ----------- (4)}$$

The particular solution can be obtained by using undetermined co-efficient.

By assuming,

$$i_p = A\cos(wt+\theta) + B\sin(wt+\theta) \text{ ---------- (5)}$$

$$i_p{}^1 = -Aw\sin(wt+\theta) + Bw\cos(wt+\theta) \text{ -------- (6)}$$

Substituting equations 5 & 6 in 3 we get,

$$\{-Aw\sin(wt+\theta) + Bw\cos(wt+\theta) + \frac{R}{L}[A\cos(wt+\theta) + B\sin(wt+\theta)]\} = \frac{V}{L}\cos(wt+\theta)$$

$$(-Aw + \frac{BR}{L})\sin(wt+\theta) + (Bw + \frac{AR}{L})\cos(wt+\theta) = \frac{V}{L}\cos(wt+\theta)$$

Comparing cosine terms and sine terms, we get

$$-Aw + \frac{BR}{L} = 0$$

$$Bw + \frac{AR}{L} = \frac{V}{L}$$

From the above equations we have

$$A = V\frac{R}{R^2+(wL)^2}$$

$$B = V\frac{wL}{R^2+(wL)^2}$$

Substituting the A and B values in equation (5) we get

$$i_p = V\frac{R}{R^2+(wL)^2}\cos(wt+\theta) + V\frac{wL}{R^2+(wL)^2}\sin(wt+\theta) \text{ -------- } (7)$$

Putting $M\cos\emptyset = V\frac{R}{R^2+(wL)^2}$

$$M\sin\emptyset = V\frac{wL}{R^2+(wL)^2}$$

To find M and $\emptyset$, we divide one equation by the other

$$\tan\emptyset = \frac{\sin\emptyset}{\cos\emptyset} = \frac{wL}{R}$$

Squaring both equations and adding, we get

$$M^2\,cos^2\emptyset + M^2\,sin^2\emptyset = V\frac{V}{R^2+(wL)^2}$$

$$M = \frac{V}{\sqrt{R^2+(wL)^2}}$$

The particular current becomes

$$i_p = \frac{V}{\sqrt{R^2+(wL)^2}}\cos(wt+\theta-\tan^{-1}\frac{wL}{R}) \text{ ----------- } (8)$$

The complete solution for the current, i = $i_c + i_p$

$$i = c\, e^{-t(\frac{R}{L})} + \frac{V}{\sqrt{R^2+(wL)^2}}\cos(wt+\theta-\tan^{-1}\frac{wL}{R})$$

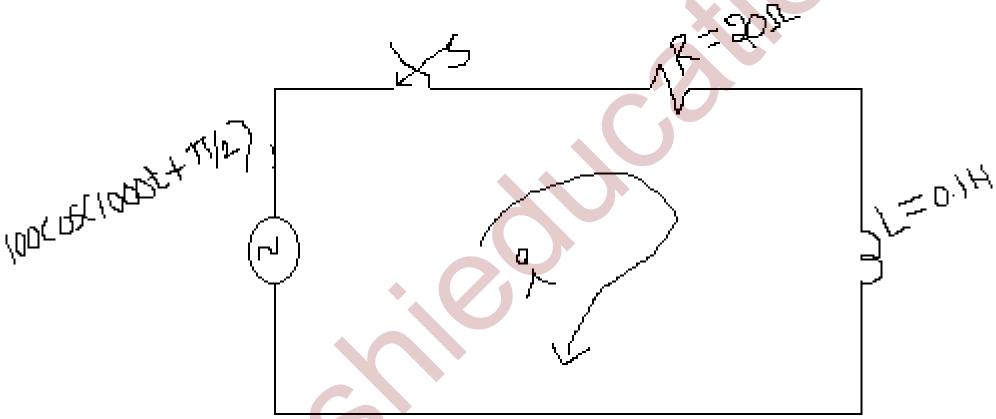
Since the inductor does not allow sudden changes in currents, at t=0, i=0

$$c = \frac{-V}{\sqrt{R^2+(wL)^2}}\cos(\theta-\tan^{-1}\frac{wL}{R})$$

The complete solution for the current is,

$$i = e^{-t(\frac{R}{L})}[\frac{-V}{\sqrt{R^2+(wL)^2}}\cos(\theta-\tan^{-1}\frac{wL}{R})] + \frac{V}{\sqrt{R^2+(wL)^2}}\cos(wt+\theta-\tan^{-1}\frac{wL}{R})$$

Example: In the circuit shown in fig. determine the complete solution for the current, when the switch S is closed at t=0. Applied voltage is V (t) = 1000cos$(1000t+\frac{\pi}{2})$, Resistance=20Ω, and inductance= 0.1H.



$$100\cos(1000t+\frac{\pi}{2}) = 20i + 0.1\frac{di}{dt}$$

$$\frac{di}{dt} + 200\, i = 1000\cos(1000t+\frac{\pi}{2})$$

The corresponding characteristic equation is

$$(D + 200)\, i = 1000\cos(1000t+\frac{\pi}{2})$$

The complementary function of the solution is

$$i_c = c\, e^{-t(\frac{R}{L})} = c\, e^{-200t}$$

The particular current becomes

$$i_p = \frac{100}{\sqrt{20^2+(1000*0.1)^2}}\cos(1000t+\frac{\pi}{2}-\tan^{-1}\frac{100}{20})$$

i_p =0.98 $\cos(1000t + \frac{\pi}{2} - 78.6^{\circ})$

The complete solution for the current, i = $i_c + i_p$

i = c e^{-200t} + 0.98 $\cos(1000t + \frac{\pi}{2} - 78.6^{\circ})$

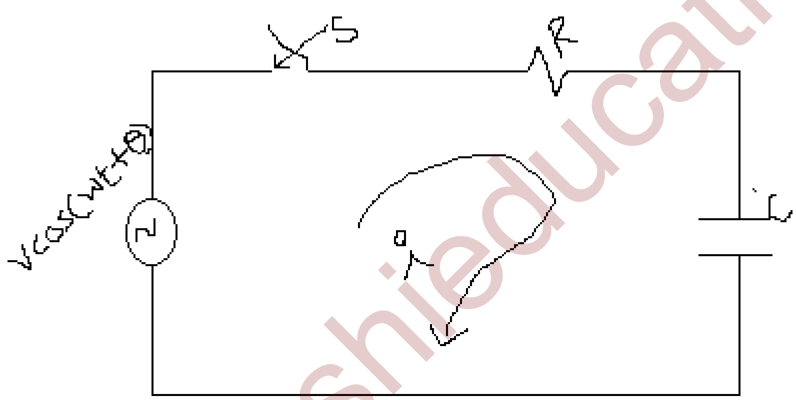
At t=0, the current flowing through the circuit is zero, i = 0

c = - 0.98 $\cos(\frac{\pi}{2} - 78.6^{\circ})$

The complete solution for the current, i = $i_c + i_p$

i = [- 0.98$\cos(\frac{\pi}{2} - 78.6^{\circ})$] e^{-200t} + 0.98 $\cos(1000t + \frac{\pi}{2} - 78.6^{\circ})$

Sinusoidal Response of R-C Circuit:



Consider a circuit consisting of resistance and capacitance in series as shown in fig. the switch S is closed at t=0. At t=0, sinusoidal voltage V$\cos(wt + \theta)$ applied to RL circuit, where V is the amplitude of the wave and θ is the phase angle. Application of Kirchoff's laws we can determine the differential equations.

V$\cos(wt + \theta)$ = Ri + $\frac{1}{C}\int i\, dt$ ----------- (1)

- Vw$\sin(wt + \theta)$ = R$\frac{di}{dt}$ + $\frac{i}{C}$ ------------- (2)

(D +$\frac{1}{RC}$) i = $\frac{-\text{Vw}}{R}\sin(wt + \theta)$ --------- (3)

The complementary function, i_C = c $e^{\frac{-t}{RC}}$

The particular solution can be obtained by using undetermined coefficients.

i_p = A $\cos(wt+\theta)$ + B $\sin(wt+\theta)$ ---------- (4)

$i_p{}^1$ = -Aw $\sin(wt+\theta)$ + Bw $\cos(wt+\theta)$ -------- (5)

Substituting equation 4 & 5 in 3 we get

$$\{-Aw \sin(wt+\theta) + Bw \cos(wt+\theta)\} +$$

$$\frac{1}{RC}\{ A\cos(wt+\theta) + B\sin(wt+\theta)\} = \frac{-Vw}{R}\sin(wt+\theta)$$

Comparing both sides,

$$-Aw + \frac{B}{RC} = \frac{-Vw}{R}$$

$$Bw + \frac{A}{RC} = 0$$

So we get,

$$A = \frac{VR}{R^2+(\frac{1}{wc})^2}$$

$$B = \frac{-V}{wC[R^2+(\frac{1}{wc})^2]}$$

Substituting A and B values in 4 we get

$$i_p = \frac{VR}{R^2+(\frac{1}{wc})^2}\cos(wt+\theta) - \frac{V}{wC[R^2+(\frac{1}{wc})^2]}\sin(wt+\theta)$$

Putting $M\cos\emptyset = \frac{VR}{R^2+(\frac{1}{wc})^2}$

$$M\sin\emptyset = \frac{V}{wC[R^2+(\frac{1}{wc})^2]}$$

To find M and$\emptyset$, we divide one equation by the other

$$\tan\emptyset = \frac{\sin\emptyset}{\cos\emptyset} = \frac{1}{wCR}$$

Squaring both equations and adding, we get

$$M^2\cos^2\emptyset + M^2\sin^2\emptyset = V\frac{V}{R^2+(\frac{1}{wc})^2}$$

$$M = \frac{V}{\sqrt{R^2+(\frac{1}{wc})^2}}$$

The particular current becomes

$$i_p = \frac{V}{\sqrt{R^2+(\frac{1}{wc})^2}} \cos(wt + \theta + \tan^{-1}\frac{1}{wCR}) \text{ ----------- } (8)$$

The complete solution for the current, i = $i_c + i_p$

$$i = c\, e^{-t(\frac{1}{RC})} + \frac{V}{\sqrt{R^2+(\frac{1}{wc})^2}} \cos(wt + \theta + \tan^{-1}\frac{1}{wCR})$$

Since the capacitor does not allow sudden changes in voltages, at t=0, i= $\frac{V}{R}\cos\theta$

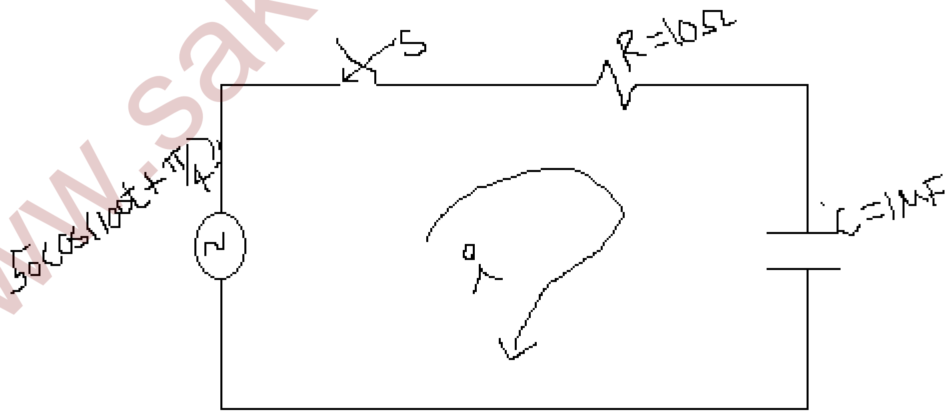
$$\frac{V}{R}\cos\theta = c + \frac{V}{\sqrt{R^2+(\frac{1}{wc})^2}} \cos(\theta + \tan^{-1}\frac{1}{wCR})$$

Therefore, $c = \frac{V}{R}\cos\theta - \frac{V}{\sqrt{R^2+(\frac{1}{wc})^2}} \cos(\theta + \tan^{-1}\frac{1}{wCR})$

The complete solution for the current is,

$$i = e^{-t(\frac{1}{RC})} [\frac{V}{R}\cos\theta - \frac{V}{\sqrt{R^2+(\frac{1}{wc})^2}} \cos(\theta + \tan^{-1}\frac{1}{wCR})] + \frac{V}{\sqrt{R^2+(\frac{1}{wc})^2}} \cos(wt + \theta + \tan^{-1}\frac{1}{wCR})$$

Example: In the circuit shown in fig. determine the complete solution for the current, when the switch S is closed at t=0. Applied voltage is V (t) = $50\cos(100t + \frac{\pi}{4})$, Resistance=10Ω, and capacitance= 1μF.



$$50\cos(100t + \frac{\pi}{4}) = 10i + \frac{1}{1*10^{-6}}\int i\, dt$$

$$-500\sin(100t + \frac{\pi}{4}) = 10\frac{di}{dt} + \frac{i}{1*10^{-6}}$$

$$(D + \frac{1}{1*10^{-5}})\ i = -\ 500\sin(100t + \frac{\pi}{4})$$

The complementary function, i_C = c $e^{\frac{-t}{RC}}$ = c $e^{\frac{-t}{10^{-5}}}$

The particular current becomes

$$i_p = \frac{V}{\sqrt{R^2+(\frac{1}{wc})^2}}\cos(wt + \theta + \tan^{-1}\frac{1}{wCR})$$

$$i_p = \frac{50}{\sqrt{10^2+(\frac{1}{100*10^{-6}})^2}}\cos(wt + \frac{\pi}{4} + \tan^{-1}\frac{1}{100*10^{-6}*10})$$

$$= 4.99*10^{-3}\cos(100t + \frac{\pi}{4} + 89.94^{\circ})$$

At t=0, the current flowing through the circuit is,

$$\frac{V}{R}\cos\theta = \frac{50}{10}\cos\frac{\pi}{4} = 3.53\text{A}$$

The complete solution for the current, i = i_c + i_p

$$i = c\ e^{\frac{-t}{10^{-5}}} + 4.99*10^{-3}\cos(100t + \frac{\pi}{4} + 89.94^{\circ})$$

At t=0, i=3.53A then we get c value

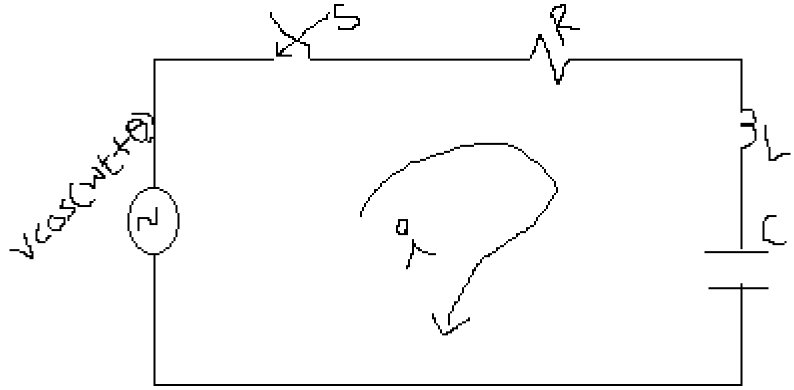
$$3.53 = c\ e^{\frac{-t}{10^{-5}}} + 4.99*10^{-3}\cos(100t + \frac{\pi}{4} + 89.94^{\circ})$$

$$c = [3.53 - 4.99*10^{-3}\cos(100t + \frac{\pi}{4} + 89.94^{\circ})]$$

The complete solution for the current,

$$i = [3.53 - 4.99*10^{-3}\cos(100t + \frac{\pi}{4} + 89.94^{\circ})]\ e^{\frac{-t}{10^{-5}}} + 4.99*10^{-3}\cos(100t + \frac{\pi}{4} + 89.94^{\circ})$$

Sinusoidal Response of R-L-C Circuit:



Consider a circuit consisting of resistance, inductance and capacitance in series as shown in fig. Switch s is closed at t=0. At t=0, a sinusoidal voltage V$\cos(wt+\theta)$ applied to RLC circuit, where V is the amplitude of the wave and θ is the phase angle. Application of Kirchoff's laws we can determine the differential equations.

$$V\cos(wt+\theta) = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \text{ ---------- (1)}$$

By differentiating above equation we get,

$$-Vw\sin(wt+\theta) = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i \text{ ------------ (2)}$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = -Vw\sin(wt+\theta) \text{ ----------- (3)}$$

$$(D^2 + \frac{R}{L}D + \frac{1}{LC})i = -\frac{Vw}{L}\sin(wt+\theta) \text{ ---------- (4)}$$

The particular solution can be obtained by using undetermined coefficients.

$$i_p = A\cos(wt+\theta) + B\sin(wt+\theta) \text{ ---------- (5)}$$

$$i_p{}^1 = -Aw\sin(wt+\theta) + Bw\cos(wt+\theta) \text{ -------- (6)}$$

$$i_p{}^{11} = -Aw^2\cos(wt+\theta) - Bw^2\sin(wt+\theta) \text{ ---------- (7)}$$

Substituting 5,6 & 7 in equation 4 we get,

$$\{-Aw^2\cos(wt+\theta) - Bw^2\sin(wt+\theta)\} + \frac{R}{L}\{-Aw\sin(wt+\theta) + Bw\cos(wt+\theta)\}$$

$$+\frac{1}{LC}\{A\cos(wt+\theta) + B\sin(wt+\theta)\} = -\frac{Vw}{L}\sin(wt+\theta)$$

Comparing both the sides, sine and cosine coefficients we get,

$$- Bw^2 - A\frac{wR}{L} + \frac{B}{LC} = -\frac{Vw}{L}$$

$$A\left(\frac{wR}{L}\right) + B\left(w^2 - \frac{1}{LC}\right) = \frac{Vw}{L} \text{ --------- (8)}$$

$$- Aw^2 + B\frac{wR}{L} + \frac{A}{LC} = 0$$

$$A\left(w^2 - \frac{1}{LC}\right) - B\left(\frac{wR}{L}\right) + = 0 \text{ --------- (9)}$$

Solving equations 8 & 9 we get

$$A = \frac{V * \frac{w^2 R}{L^2}}{[\left(\frac{wR}{L}\right)^2 - (w^2 - \frac{1}{LC})^2]}$$

$$B = \frac{(w^2 - \frac{1}{LC}) * Vw}{L[\left(\frac{wR}{L}\right)^2 - (w^2 - \frac{1}{LC})^2]}$$

Substituting A and B values in 5 we get

$$i_p = \frac{V * \frac{w^2 R}{L^2}}{[\left(\frac{wR}{L}\right)^2 - (w^2 - \frac{1}{LC})^2]} \cos(wt + \theta) + \frac{(w^2 - \frac{1}{LC}) * Vw}{L[\left(\frac{wR}{L}\right)^2 - (w^2 - \frac{1}{LC})^2]} \sin(wt + \theta) \text{ ---- (10)}$$

Putting $M \cos \emptyset = \frac{V * \frac{w^2 R}{L^2}}{[\left(\frac{wR}{L}\right)^2 - (w^2 - \frac{1}{LC})^2]}$

$$M \sin \emptyset = \frac{(w^2 - \frac{1}{LC}) * Vw}{L[\left(\frac{wR}{L}\right)^2 - (w^2 - \frac{1}{LC})^2]}$$

To find M and$\emptyset$, we divide one equation by the other

$$\tan \emptyset = \frac{\sin \emptyset}{\cos \emptyset} = \frac{(wL - \frac{1}{wC})}{R}$$

Squaring both equations and adding, we get

$$M^2 \cos^2\emptyset + M^2 \sin^2\emptyset = V \frac{V}{R^2 + (\frac{1}{wc} - wL)^2}$$

$$M = \frac{V}{\sqrt{R^2 + (\frac{1}{wc} - wL)^2}}$$

The particular current becomes

$$i_p = \frac{V}{\sqrt{R^2+(\frac{1}{wc}-wL)^2}}\cos(wt + \theta + \tan^{-1}\frac{\frac{1}{wc}-wL}{R})$$

The complementary function is similar to that of DC series RLC circuit.

$$D^2 + \frac{R}{L}\text{D} + \frac{1}{LC} = 0$$

The roots above equation are,

$$D_1, D_2 = -\frac{R}{2L} \pm \sqrt{(\frac{R}{2L})^2 - \frac{1}{LC}}$$

By assuming, $K_1 = -\frac{R}{2L}$ and $K_2 = \sqrt{(\frac{R}{2L})^2 - \frac{1}{LC}}$

$$D_1 = K_1 + K_2$$

$$D_2 = K_1 - K_2$$

Here K_2 may be positive or negative or zero.

K_2 Is positive, when $(\frac{R}{2L})^2 > \frac{1}{LC}$

The roots are real and unequal, and give the over damped response as shown in fig. then equation (4) becomes

$$[\text{D} - (K_1+K_2)][[\text{D} - (K_1-K_2)]]\text{i}=0$$

The solution for the above equation is,

$$i_c = c_1 e^{(K_1+K_2)\text{t}} + c_2 e^{(K_1-K_2)\text{t}}$$

Therefore the complete solution is, i = $i_c + i_p$

$$\text{i} = c_1 e^{(K_1+K_2)\text{t}} + c_2 e^{(K_1-K_2)\text{t}} + \frac{V}{\sqrt{R^2+(\frac{1}{wc}-wL)^2}}\cos(wt + \theta + \tan^{-1}\frac{\frac{1}{wc}-wL}{R})$$

K_2 Is negative, when $(\frac{R}{2L})^2 < \frac{1}{LC}$. The roots are complex conjugate, and give the under damped the equation as shown in becomes

$$[\text{D} - (K_1+jK_2)][[\text{D} - (K_1-jK_2)]]\text{i}=0$$

The solution for above equation is,

$$i_c = e^{K_1\text{t}}\,[c_1 \cos K_2 t + c_2 \sin K_2 t\,]$$

Therefore the complete solution is, i = $i_c + i_p$

$$i = e^{K_1 t}\,[c_1 \cos K_2 t + c_2 \sin K_2 t\,] + \frac{V}{\sqrt{R^2+(\frac{1}{wc}-wL)^2}}\cos(wt+\theta+\tan^{-1}\frac{\frac{1}{wc}-wL}{R})$$

K_2 Is zero, when $(\frac{R}{2L})^2 = \frac{1}{LC}$. The roots are equal, and give the critically damped response as shown in fig. the equation becomes

$$[D - K_1][[D - K_2]]i=0$$

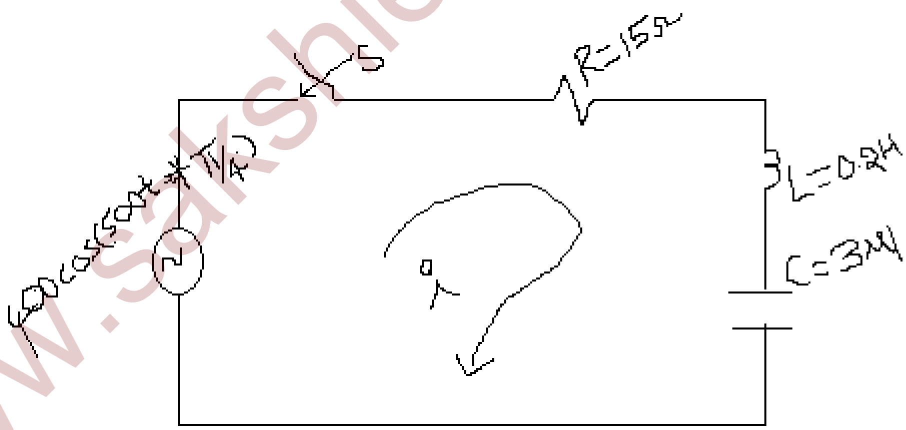
The solution for above equation is

$$i_c = e^{K_1 t}\,[c_1 + c_2 t\,]$$

Therefore the complete solution is, i = $i_c + i_p$

$$i = e^{K_1 t}\,[c_1 + c_2 t\,] + \frac{V}{\sqrt{R^2+(\frac{1}{wc}-wL)^2}}\cos(wt+\theta+\tan^{-1}\frac{\frac{1}{wc}-wL}{R})$$

Example: In the circuit shown in fig. determine the complete solution for the current, when the switch S is closed at t=0. Applied voltage is V (t) = $400\cos(500t+\frac{\pi}{4})$, Resistance=15Ω, inductance= 0.2H and capacitance= 3μF.



By applying Kirchoff's voltage law to the circuit,

$$400\cos(500t+\frac{\pi}{4}) = 15\,i(t) + 0.2\,\frac{di}{dt} + \frac{1}{3*10^{-6}}\int i(t)\,dt$$

By differentiating above equation we get,

$$-2*10^5\sin(500t+\frac{\pi}{4}) = 15\,\frac{di}{dt} + 0.2\,\frac{d^2 i}{dt^2} + \frac{1}{3*10^{-6}}\,i$$

$$(D^2 + 75\,D + 16.7 * 10^5)i = -10 * 10^5 \sin(500t + \frac{\pi}{4})$$

The roots of the characteristic equation are,

D_1 =-37.5+j1290 and D_2= -37.5-j1290

$$[D - (K_1 + jK_2)][[D - (K_1 - jK_2)]]i = 0$$

The solution for above equation is,

$$i_c = e^{-37.5t}\,[c_1 \cos 1290t + c_2 \sin 1290t\,]$$

The particular solution is

$$i_p = \frac{V}{\sqrt{R^2 + (\frac{1}{wc} - wL)^2}} \cos(wt + \theta + \tan^{-1}\frac{\frac{1}{wc} - wL}{R})$$

$$i_p = 0.71 \cos(500t + \frac{\pi}{4} + 88.5^{\circ})$$

Therefore the complete solution is, $i = i_c + i_p$

$$i = e^{-37.5t}\,[c_1 \cos 1290t \ + c_2 \sin 1290t\,] + 0.71 \cos(500t + \frac{\pi}{4} + 88.5^{\circ})$$

At t=0, $i_0 = 0$

$$c_1 = 0.71 \cos(\frac{\pi}{4} + 88.5^{\circ}) = 0.49$$

Differentiating the current equation, we get

$$\frac{di}{dt} = e^{-37.5t}\,[-1290c_1 \sin 1290t \ + 1290c_2 \cos 1290t\,] - 37.5e^{-37.5t}\,[c_1 \cos 1290t \ + c_2 \sin 1290t\,] - 500 * 0.71 \sin(500t + \frac{\pi}{4} + 88.5^{\circ})$$

At t=0, $\frac{di}{dt}$ =1414

$$1414 = e^{-37.5t}\,[-1290c_1 \sin 1290t \ + 1290c_2 \cos 1290t\,] - 37.5e^{-37.5t}\,[c_1 \cos 1290t \ + c_2 \sin 1290t\,] - 500 * 0.71 \sin(500t + \frac{\pi}{4} + 88.5^{\circ})$$

Solving this we get, c_2 = 1.31

Therefore the complete solution is,

$$i = e^{-37.5t}\,[0.49 \cos 1290t \ + 1.31 \sin 1290t\,] + 0.71 \cos(500t + 133.5^{\circ})$$