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III

Total No. of Questions – 24

Total No. of Printed Pages - 3

Regd. No.

Part – III
MATHEMATICS, Paper – I (A)
(English Version)

Time : 3 Hours] [Max. Marks : 75

Note : This question paper consists of **three** Sections – **A**, **B** and **C**.

SECTION – A

$10 \times 2 = 20$

I. Very Short Answer Type questions.

(i) Attempt **all** questions.

(ii) Each question carries **two** marks.

1. If $f : R \to (0, \infty)$ defined by $f(x) = 5^x$, then find $f^{-1}(x)$.

2. Find the domain of the real valued function $f(x) = \dfrac{1}{\sqrt{x^2 - a^2}}$ $(a > 0)$.

3. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$ and $2X + A = B$, then find X.

4. For any square matrix A, show that AA' is symmetric.

5. If $\bar{a} = 2i + 4j - 5k$, $\bar{b} = i + j + k$ and $\bar{c} = j + 2k$. Find the unit vector in the opposite direction of $\bar{a} + \bar{b} + \bar{c}$.

6. Find the vector equation of the line passing through the point $2i + 3j + k$ and parallel to the vector $4i - 2j + 3k$.

7. If $\bar{a} = i + 2j - 3k$ and $\bar{b} = 3i - j + 2k$, then show that $\bar{a} + \bar{b}$ and $\bar{a} - \bar{b}$ are perpendicular to each other.

8. Find the period of the function $\tan(x + 4x + 9x + \dots + n^2x)$ (n any positive integer).

9. Prove that $\dfrac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \cot 36^\circ$

10. Show that $\tan h^{-1}\left(\dfrac{1}{2}\right) = \dfrac{1}{2}\log_e 3$

SECTION – B $\quad 5 \times 4 = 20$

II. Short Answer Type questions.

(i) Attempt any **five** questions.

(ii) Each question carries **four** marks.

11. If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then show that $(aI + bE)^3 = a^3I + 3a^2bE$, where I is unit matrix of order 2.

12. ABCDEF is a regular hexagon with Centre 'O' Show that

$\overline{AB} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{AF} = 3\overline{AD} = 6\overline{AO}$

13. If $\bar{a} = 2i + j - k$, $\bar{b} = -i + 2j - 4k$ and $\bar{c} = i + j - k$, then find $(\bar{a} \times \bar{b}) \cdot (\bar{b} \times \bar{c})$.

14. Prove that

$$\left(1 + \cos\frac{\pi}{10}\right)\left(1 + \cos\frac{3\pi}{10}\right)\left(1 + \cos\frac{7\pi}{10}\right)\left(1 + \cos\frac{9\pi}{10}\right) = \frac{1}{16}$$

15. Solve $\sqrt{2}\,(\sin x + \cos x) = \sqrt{3}$

16. Prove that $\tan^{-1}\left(\dfrac{1}{2}\right) + \tan^{-1}\left(\dfrac{1}{5}\right) + \tan^{-1}\left(\dfrac{1}{8}\right) = \dfrac{\pi}{4}$

17. In a ΔABC show that

$$\frac{b^2 - c^2}{a^2} = \frac{\sin(B - C)}{\sin(B + C)}$$

SECTION – C

$5 \times 7 = 35$

III. Long Answer Type questions :

(i) Attempt any **five** questions.

(ii) Each question carries **seven** marks.

18. If $f : A \to B$ is a bijection, then prove that $fof^{-1} = I_B$ and $f^{-1}of = I_A$

19. Using Mathematical Induction, prove that statement for all $n \in N$.
$$\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\cdots\cdots\cdots\left(1+\frac{2n+1}{n^2}\right) = (n+1)^2$$

20. Without expanding the determinant show that
$$\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

21. Solve $3x + 4y + 5z = 18$, $2x - y + 8z = 13$ and $5x - 2y + 7z = 20$ by using "matrix inversion method".

22. If $\bar{a}, \bar{b}, \bar{c}$ are three vectors, then prove that

(i) $\left(\bar{a} \times \bar{b}\right) \times \bar{c} = \left(\bar{a} \cdot \bar{c}\right)\bar{b} - \left(\bar{b} \cdot \bar{c}\right)\bar{a}$

(ii) $\bar{a} \times \left(\bar{b} \times \bar{c}\right) = \left(\bar{a} \cdot \bar{c}\right)\bar{b} - \left(\bar{a} \cdot \bar{b}\right)\bar{c}$

23. If $A + B + C = \pi$, then prove that
$$\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2} = 2\left(1 + \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)$$

24. If $a = 13$, $b = 14$, $c = 15$, show that $R = \frac{65}{8}$, $r = 4$, $r_1 = \frac{21}{2}$, $r_2 = 12$ and $r_3 = 14$.
