MATHEMATICS PAPER IB

COORDINATE GEOMETRY(2D &3D) AND CALCULUS.

TIME : 3hrs **Max. Marks.75**

Note: This question paper consists of three sections A,B and C.

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS. 10X2 =20

1. Show that the straight lines (a –b)x + (b –c) y = c – a, (b-c) x+ (c-a)y= (a – b) and (c –a)x + (a – b)y b - c are concurrent.

2. Find the angle between the following straight lines y = $-\sqrt{3}x+5, y=\frac{1}{\sqrt{3}}x-\frac{2}{\sqrt{3}}$

3. Find the coordinates of the vertex C of ΔABC if its centroid is the origin and the vertices A, B are (1, 1, 1) and (–2, 4, 1) respectively.

4. Find the equation of the plane passing through the point (1, 1, 1) and parallel to the plane x + 2y + 3z – 7 = 0.

5. Compute $\underset{x\to 0}{Lt}\frac{a^x-1}{b^x-1}$ $a>0, b>0, b\neq 1$.

6. $\underset{x\to 0}{Lt}\left[\frac{3^x-1}{\sqrt{1+x}-1}\right]$

7. $y=\log \ \tan 5x$, find $\frac{dy}{dx}$.

8. If $y=\cot^{-1} \ \cos ec\ 3x$ find $\frac{dy}{dx}$.

9. Find approximate value of $\sqrt[3]{7.8}$

10. Verify Rolle's theorem for the following functions. $x^2 - 1$ on $[-1, 1]$

SECTION B

SHORT ANSWER TYPE QUESTIONS.

ANSWER ANY FIVE OF THE FOLLOWING 5 X 4 = 20

11. Find the equation of locus of a point, the difference of whose distances from (–5, 0) and (5, 0) is 8 units.

12. Show that the axes are to be rotated through an angle of $\frac{1}{2}\mathrm{Tan}^{-1}\left(\frac{2h}{a-b}\right)$ so as to removethexy term from the equation $ax^2+2hxy+by^2=0$, if $a \neq b$ and through the angle $\frac{\pi}{4}$, if a = b

13. Line L has intercepts a and b on the axes of co-ordinates. When the axes are rotated through a given angle, keeping the origin fixed, the same line L has intercepts p and q on the transformed axes. Prove that $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$.

14. Evaluate $\underset{x\to a}{Lt}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a}+x-2\sqrt{x}}\right]$

15. find the derivative of the function $f(x)=\cos^2 x$ from first principle.

16. **Find the value of k so that the length of the sub-normal at any point on the curve $x.y^k = a^{k+1}$ is constant.**

17. The volume of a cube is increasing at the rate of 8 cm^3/sec. How fast is the surface area increasing when the length of an edge is 12 cm?

SECTION C

LONG ANSWER TYPE QUESTIONS.

ANSWER ANY FIVE OF THE FOLLOWING 5 X 7 = 35.

18. Find the equation of the straight lines passing through the point (-3,2) and making an angle of $45°$ with the straight line 3x – y + 4 =0.

19. ***The product of the perpendiculars from*** (α, β) ***to the pair of lines*** $ax^2+2hxy+by^2=0$ ***is*** $\dfrac{\left|a\alpha^2+2h\alpha\beta+b\beta^2\right|}{\sqrt{a-b\ ^2+4h^2}}$

20. Show that the straight lines represented by $3x^2 + 48xy + 23y^2 = 0$ and $3x - 2y + 13 = 0$ form an equilateral triangle of area $\dfrac{13}{\sqrt{3}}$ sq. units.

21. The vertices of a triangle are A(1, 4, 2), B(-2, 1, 2) C(2, 3, -4). Find $\underline{|A}, \underline{|B}, \underline{|C}$

22. If $y=\text{Tan}^{-1}\left(\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$ for $0 < |x| < 1$, find $\dfrac{dy}{dx}$.

23. Find the angle between the curves $\mathbf{x+y+2=0;\ x^2+y^2-10y=0}$

24. Find the point on the graph $y^2 = x$ which is the nearest to the point (4, 0).

SOLUTIONS

SECTION A

1. Show that the straight lines (a –b)x + (b –c) y = c – a, (b-c) x+ (c-a)y= (a – b) and (c –a)x + (a – b)y b - c are concurrent.

Sol. Equations of the given lines are

L_1=(a – b) x + (b – c) y - c + a=0 --- (1)

L_2=(b – c) x + (c – a) y - a + b=0 --- (2)

L_3=(c – a) x + (a – b) y - b + c=0 --- (3)

If three lines L_1, L_2, L_3 are concurrent , then there exists non zero real numbers $\lambda_1, \lambda_2, \lambda_3$, such that $\lambda_1 L_1 + \lambda_2 L_2 + \lambda_3 L_3 = 0$.

Let $\lambda_1 = 1, \lambda_2 = 1, \lambda_3 = 1$, then $1.L_1 + 1.L_2 + 1.L_3 = 0$

Hence the given lines are concurrent.

2. Find the angle between the following straight lines

$y = -\sqrt{3}x + 5, y = \frac{1}{\sqrt{3}}x - \frac{2}{\sqrt{3}}$

Sol. slope of 1st line is $m_1 = -\sqrt{3}$

Slope of 2nd line is $m_2 = \frac{1}{\sqrt{3}}$. $m_1 m_2 = -\sqrt{3}\ \frac{1}{\sqrt{3}} = -1$.

The lines are perpendicular $\theta = \frac{\pi}{2}$

3. Find the coordinates of the vertex C of ΔABC if its centroid is the origin and the vertices A, B are (1, 1, 1) and (–2, 4, 1) respectively.

Sol. A(1, 1, 1), B(–2, 4, 1) and (x, y, z) are the vertices of ΔABC.

G is the centroid of ΔABC

Coordinates of G are :

$$\left(\frac{1-2+x}{3}, \frac{1+4+y}{3}, \frac{1+1+z}{3}\right) = (0,0,0)$$

$$\frac{x-1}{3} = 0, \frac{y+5}{3} = 0, \frac{z+2}{3} = 0$$

$$x-1=0,\ y+5=0,\ z+2=0$$
$$x=1,\ y=-5,\ z=-2$$

∴ Coordinates of c are (1, –5, –2).

4. Find the equation of the plane passing through the point (1, 1, 1) and parallel to the plane
x + 2y + 3z – 7 = 0.

Sol. Equation of the given plane is

x + 2y + 3z – 7 = 0

Equation of the parallel plane is

x + 2y + 3z = k

This plane passing through the point P(1, 1, 1)

$\Rightarrow 1 + 2 + 3 = k \Rightarrow k = -6$

Equation of the required plane is

$x + 2y + 3z = 6$

5. Compute $\underset{x \to 0}{Lt} \frac{a^x - 1}{b^x - 1}$ $a > 0, b > 0, b \neq 1$.

Sol : For $x \neq 0, \frac{a^x - 1}{b^x - 1} = \frac{\left[\frac{a^x - 1}{x}\right]}{\left[\frac{b^x - 1}{x}\right]}$

$$\underset{x\to 0}{Lt} \frac{a^x - 1}{b^x - 1} = \frac{\underset{x\to 0}{Lt} \frac{a^x - 1}{x}}{\underset{x\to 0}{Lt} \frac{b^x - 1}{x}} = \frac{\log_e^a}{\log_e^b}$$

6. $\underset{x\to 0}{Lt}\left[\frac{3^x - 1}{\sqrt{1+x} - 1}\right]$

$$\underset{x\to 0}{Lt} \frac{3^x - 1}{\sqrt{1+x} - 1}$$

Sol : $= \underset{x\to 0}{Lt} \frac{3^x - 1}{\sqrt{1+x} - 1} \times \frac{\sqrt{1+x} + 1}{\sqrt{1+x} + 1}$

rationalise Dr.

$$= \underset{x\to 0}{Lt} \frac{3^x - 1 \quad \sqrt{1+x} + 1}{1 + x - 1}$$

$$= \underset{x\to 0}{Lt} \frac{3^x - 1}{x} . \underset{x\to 0}{Lt} \quad \sqrt{1+x} + 1$$

$$= \log 3 \quad \sqrt{1+0} + 1 \quad = 2.\log 3$$

7. $y = \log \tan 5x$, find $\frac{dy}{dx}$.

sol :

$$\frac{dy}{dx}=\frac{d}{dx}\left(\log\tan 5x\right)=\frac{1}{\tan 5x}\frac{d}{dx}\left(\tan 5x\right)$$

$$=\frac{5\sec^2 5x}{\tan 5x}=5.\frac{1}{\cos^2 5x.\frac{\sin 5x}{\cos 5x}}$$

$$=\frac{10}{2\sin 5x.\cos 5x}$$

$$=\frac{10}{\sin 10x}=10.\operatorname{cosec}10x$$

8. If $y=\cot^{-1}\left(\operatorname{cosec} 3x\right)$ find $\frac{dy}{dx}$.

sol :

$$\frac{dy}{dx}=\frac{d}{dx}\cot^{-1}\left(\operatorname{cosec} 3x\right)$$

$$=-\frac{1}{1+\operatorname{cosec}^2}\cdot\frac{d}{dx}\left(\operatorname{cosec} 3x\right)$$

$$=-\frac{1}{1+\operatorname{cosec}^2 3x}\left(-\operatorname{cosec} 3x.\cot 3x\right)\frac{d}{dx}\left(3x\right)=\frac{3.\operatorname{cosec} 3x.\cot 3x}{1+\operatorname{cosec}^2 3x}$$

9. Find approximate value of $\sqrt[3]{\mathbf{7.8}}$

Sol: Let x = 8, $\Delta x=-0.2, f(x)=\sqrt[3]{x}$

$$f(x+\delta x)=f(x)+f^1(x)\delta x$$

$$=\sqrt[3]{x}+\frac{1}{3}x^{-\frac{2}{3}}.\Delta x=\sqrt[3]{8}+\frac{1}{3.8^{\frac{2}{3}}}\left(-0.2\right)$$

$$=2-0.0166=1.9834$$

10. Verify Rolle’s theorem for the following functions. $x^2 - 1$ on [–1, 1]

Sol. Let $f(x) = x^2 - 1$

f is continuous on [–1, 1]

since f(–1) = f(1) = 0 and

f is differentiable on [–1, 1]

$\therefore$ By Rolle's theorem $\exists c \in (-1,1)$

Such that $f'(c) = 0$

$f'(x) = 2x = 0$

$\therefore = f'(c) = 0$

$2c = 0 \Rightarrow c = 0$

The point $c = 0 \in (-1, 1)$

Then Rolle's theorem is verified.

SECTION B

11. Find the equation of locus of a point, the difference of whose distances from (–5, 0) and (5, 0) is 8 units.

Sol. Given points are A(5, 0), B(–5, 0)

Let P(x, y) be any point in the locus

Given $|PA - PB| = 8$

$\Rightarrow PA - PB = \pm 8$

$\Rightarrow PA = \pm 8 + PB$

Squaring on both sides

$PA^2 = 64 + PB^2 \pm 16PB$

$\Rightarrow (x-5)^2 + y^2 - (x+5)^2 - y^2 - 64 = \pm 16PB$

$-4 \cdot 5 \cdot x - 64 = \pm 16PB$

$-5x - 16 = \pm 4PB$

Squaring on both sides

$25x^2 + 256 + 160x = 16(PB)^2$

$= 16[(x+5)^2 + y^2]$

$= 16x^2 + 400 + 160x + 16y^2$

$9x^2 - 16y^2 = 144$

Dividing with 144, locus of P is

$$\frac{9x^2}{144} - \frac{16y^2}{144} = 1 \Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$$

12. Show that the axes are to be rotated through an angle of $\frac{1}{2}\mathrm{Tan}^{-1}\left(\frac{2h}{a-b}\right)$ so as to removethexy term from the equation $ax^2+2hxy+by^2=0$, if $a \neq b$ and through the angle $\frac{\pi}{4}$, if a = b

Sol: Given equation is $ax^2+2hxy+by^2=0$

Since the axes are rotated through an angle θ, then $x = X\cos\theta - Y\sin\theta$, $y = X\sin\theta + Y\cos\theta$

Now the transformed equation is

$$a(X\cos\theta - Y\sin\theta)^2 + 2h(X\cos\theta - Y\sin\theta)(X\sin\theta + Y\cos\theta) + b(X\sin\theta + Y\cos\theta)^2 = 0$$

$$\Rightarrow a(X^2\cos^2\theta + Y^2\sin^2\theta - 2XY\cos\theta\sin\theta) + 2h\left[X^2\cos\theta\sin\theta + XY(\cos^2\theta - \sin^2\theta) - Y^2\sin\theta\cos\theta\right]$$

$$+ b(X^2\sin^2\theta + Y^2\cos^2\theta + 2XY\cos\theta\sin\theta) = 0$$

$\Rightarrow$ It is in the form $AX^2 + 2XY\left[-a\cos\theta\sin\theta + h(\cos^2\theta - \sin^2\theta) + b\cos\theta\sin\theta\right] + BY^2 = 0$

Since XY term is to be eliminated, $(b-a)\cos\theta\sin\theta + h(\cos^2\theta - \sin^2\theta) = 0$

$\Rightarrow 2h\cos 2\theta = 2(a-b)\sin\theta\cos\theta = (a-b)\sin 2\theta$ -------------(1)

$$\Rightarrow \tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{2h}{a-b}$$

$\Rightarrow$ Angle of rotation $\theta = \frac{1}{2}Tan^{-1}\left(\frac{2h}{a-b}\right)$

If a=b, then from (1) , $\Rightarrow 2h\cos 2\theta = 0 \Rightarrow \cos 2\theta = 0 \Rightarrow 2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$

13. Line L has intercepts a and b on the axes of co-ordinates. When the axes are rotated through a given angle, keeping the origin fixed, the same line L has intercepts p and q on the transformed axes. Prove that $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$.

Sol. Equation of the line in the old system in intercept form is

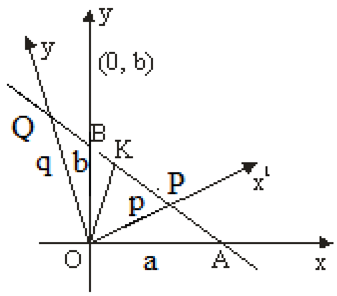
$$\frac{x}{a}+\frac{y}{b}=1 \Rightarrow \frac{x}{a}+\frac{y}{b}-1=0$$

Length of the perpendicular form origin $=\frac{|0+0-1|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}$ ---(1)

Equation of the line in the new system in intercept form is

$\frac{x}{p}+\frac{y}{q}=1 \Rightarrow \frac{x}{p}+\frac{y}{q}-1=0$ Length of the perpendicular $=\frac{|0+0-1|}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}}$ form origin

---(2)



Since the position of origin and the given line remain unchanged ,perpendicular distances in both the systems are same.

$$\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}=\frac{1}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}} \Rightarrow \frac{1}{\left(\frac{1}{a^2}+\frac{1}{b^2}\right)}=\frac{1}{\left(\frac{1}{p^2}+\frac{1}{q^2}\right)}$$

$$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$$

14. $\underset{x\to a}{Lt}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$

Sol. Rationalize both nr.and dr.

$$\underset{x\to a}{Lt}\frac{\sqrt{a+2x}-\sqrt{3x}\quad \sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}} \times \frac{\sqrt{3a+x}+\sqrt{4x}}{\sqrt{3a+x}-\sqrt{4x}\quad \sqrt{3a+x}+\sqrt{4x}}$$

$$=\underset{x\to a}{Lt}\frac{a+2x-3x}{\sqrt{a+2x}+\sqrt{3x}} \times \frac{\sqrt{3a+x}+\sqrt{4x}}{3a+x-4x}=\underset{x\to a}{Lt}\frac{a-x\quad \sqrt{3a+x}+\sqrt{4x}}{\sqrt{a+2x}+\sqrt{3x}\ \ 3\ \ a-x}$$

$$=\frac{2\ \ 2a}{2\ \ \sqrt{3a}\ \ 3}=\frac{2}{3\sqrt{3}}$$

15. $f(x)=\cos^2 x$

sol : $f^1(x) = \underset{h\to 0}{Lt}\dfrac{f(x+h)-f(x)}{h}$

$$f^1(x) = \underset{h\to 0}{Lt}\frac{\cos^2(x+h)-\cos^2 x}{h}$$

$$= \underset{h\to 0}{Lt}\frac{-\left(\cos^2 x-\cos^2(x+h)\right)}{h}$$

$$= \underset{h\to 0}{Lt}\frac{-\sin(x+h+x)\sin(x+h-x)}{h}$$

$$f'(x) = \underset{h\to 0}{Lt}-\sin(2x+h)\ \underset{h\to 0}{Lt}.\frac{\sin h}{h}$$

$$= -\sin 2x.1 = -\sin 2x$$

16. **Find the value of k so that the length of the sub-normal at any point on the curve $x.y^k = a^{k+1}$ is constant.**

Sol: Let P(x_1,y_1) be a point on the curve.

Equation of the curve is $x.y^k = a^{k+1}$.

Differentiating w.r.to x

$$x.k.y^{k-1}\frac{dy}{dx}+y^k.1=0 \Rightarrow k.xy^{k-1}.\frac{dy}{dx}=-y^k$$

$$\Rightarrow \frac{dy}{dx}=\frac{-y^k}{k.x.y^{k-1}}=-\frac{y}{kx}$$

Length of the sub-normal = $|y_1 m|$ **=** $\left|y_1\frac{-y_1}{kx_1}\right| = \dfrac{y_1^2}{kx_1}$

$$=\frac{y_1^2}{k}.\frac{y_1^k}{a^{k+1}}=\frac{y_1^{k+2}}{k.a^{k+1}}$$

Length of the sub-normal is constant at any point on the curve is independent of x_1 **and**

$\dfrac{y_1^{k+2}}{k.a^{k+1}}$ **is independent of y_1** $\Rightarrow k+2=0 \Rightarrow k=-2$

17. The volume of a cube is increasing at the rate of 8 cm^3/sec. How fast is the surface area increasing when the length of an edge is 12 cm?

Sol. Suppose ‘a’ is the edge of the cube and v be the volume of the cube.

$v = a^3$ …(1)

given $\frac{dv}{dt} = 8cm^3/sec$

a = 12 cm

Surface area of cube S = $6a^2$

$\frac{ds}{dt} = 12a\frac{da}{dt}$ …(2)

From (1), $\frac{dv}{dt} = 3a^2\frac{da}{dt}$

$8 = 3(144)\frac{da}{dt}$

$\frac{da}{dt} = \frac{8}{3(144)} cm/s$

$\frac{ds}{dt} = 12a\frac{da}{dt}$

$= 12(12)\frac{8}{3(144)} = 144 \times \frac{8}{3(144)} = \frac{8}{3} cm^2/s$

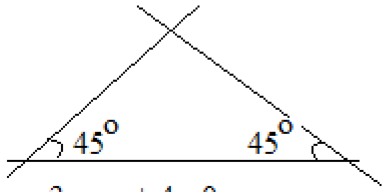
SECTION C

18. Find the equation of the straight lines passing through the point (-3,2) and making an angle of $45°$ with the straight line 3x – y + 4 =0.

Sol. Given point P (-3,2)

Given line 3x – y + 4 = 0 ------(1)

Slope $m_1 = -\frac{a}{b} = 3$



Let m be the slope of the required line.

Then $\tan 45° = \frac{m-3}{1+3m}$

$\Rightarrow \left|\frac{m-3}{1+3m}\right| = 1 \Rightarrow \frac{m-3}{1+3m} = 1$

$\Rightarrow$ m-3 = 1+ 3m $\Rightarrow$ 2m = -4 or m = -2

$\Rightarrow \frac{m-3}{1+3m}$ = - $\Rightarrow$ m – 3 = - 1 – 3m

$\Rightarrow$ 4m = 2 $\Rightarrow$ m= ½

case (1) m = - 2 and point (-3,2)

Equation of the line is

y -2 = -2(x + 3) = -2x – 6 $\Rightarrow$ 2x + y + 4 = 0

case (2) m = $\frac{1}{2}$,point (-3,2)

Equation of the line is

y – 2 = $\frac{1}{2}(x+3) \Rightarrow$ 2y – 4 = x + 3 $\Rightarrow$ x -2y + 7 = 0

19. ***The product of the perpendiculars from*** (α, β) ***to the pair of lines*** $ax^2 + 2hxy + by^2 = 0$ ***is*** $\frac{|a\alpha^2 + 2h\alpha\beta + b\beta^2|}{\sqrt{(a-b)^2 + 4h^2}}$

Proof: Let $ax^2 + 2hxy + by^2 = 0$ represent the lines $l_1x + m_1y = 0$ -- (1) and $l_2x + m_2y = 0$ -- (2).

Then $l_1l_2 = a$, $l_1m_2 + l_2m_1 = 2h$, $m_1m_2 = b$.

The lengths of perpendiculars from (α, β) to

the line (1) is p= $= \frac{|l_1\alpha + m_1\beta|}{\sqrt{l_1^2 + m_1^2}}$

and to the line (2) is q= $= \frac{|l_2\alpha + m_2\beta|}{\sqrt{l_2^2 + m_2^2}}$

$\therefore$ The product of perpendiculars is pq = $\left|\dfrac{l_1\alpha + m_1\beta}{\sqrt{l_1^2 + m_1^2}}\right| \cdot \left|\dfrac{l_2\alpha + m_2\beta}{\sqrt{l_2^2 + m_2^2}}\right|$

= $\left|\dfrac{l_1 l_2\alpha^2 + (l_1 m_2 + l_2 m_1)\alpha\beta + m_1 m_2\beta^2}{\sqrt{l_1^2 l_2^2 + l_1^2 m_2^2 + l_2^2 m_1^2 + m_1^2 m_2^2}}\right|$

= $\dfrac{|a\alpha^2 + 2h\alpha\beta + b\beta^2|}{\sqrt{(l_1 l_2 - m_1 m_2)^2 + 2l_1 l_2 m_1 m_2 + (l_1 m_2 + l_2 m_1)^2 - 2l_1 m_2 l_2 m_1}}$ = $\dfrac{|a\alpha^2 + 2h\alpha\beta + b\beta^2|}{\sqrt{(a-b)^2 + 4h^2}}$

20. Show that the straight lines represented by $3x^2 + 48xy + 23y^2 = 0$ and $3x - 2y + 13 = 0$ form an equilateral triangle of area $\dfrac{13}{\sqrt{3}}$ sq. units.

Sol. Equation of pair of lines is $3x^2 + 48xy + 23y^2 = 0$ ………….(1)

Equation of given line is $3x - 2y + 13 = 0$ ……(2)

$\Rightarrow$ slope = 3/2

$\therefore$ the line (2) is making an angle of $\tan^{-1}\dfrac{3}{2}$ with the positive direction of x-axis.

Therefore no straight line which makes an angle of 60^0 with (2) is vertical.

Let m be the slope of the line passing through origin and making an angle of 60^0 with line (2).

$\therefore \tan 60^0 = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right| \Rightarrow \sqrt{3} = \left|\dfrac{\frac{3}{2} - m}{1 + \frac{3}{2}m}\right| \Rightarrow \sqrt{3} = \left|\dfrac{3 - 2m}{2 + 3m}\right|$

Squaring on both sides, $3 = \dfrac{(3 - 2m)^2}{(2 + 3m)^2} \Rightarrow 23m^2 + 48m + 3 = 0$, which is a quadratic equation in m.

Let the roots of this quadratic equation be m_1, m_2, which are the slopes of the lines.

Now, $m_1 + m_2 = \dfrac{-48}{23}$ and $m_1 . m_2 = \dfrac{3}{23}$.

The equation of the lines passing through origin and having slopes m_1, m_2 are $m_1x - y = 0$ and $m_2x - y = 0$.

Their combined equation is $(m_1x - y)(m_2x - y) = 0$

$$\Rightarrow m_1m_2x^2 - (m_1 + m_2)\,xyy^2 = 0$$

$$\Rightarrow \frac{3}{23}x^2 - \left(-\frac{48}{23}\right)xy + y^2 = 0$$

$$\Rightarrow 3x^2 + 48xy + 23y^2 = 0$$

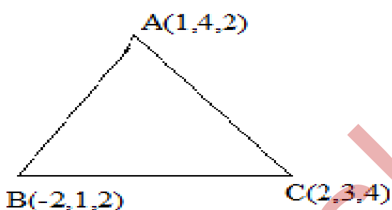
Which is the given pair of lines.

Therefore, given lines form an equilateral triangle.

$$\therefore \text{ Area of } \Delta = \left|\frac{n^2\sqrt{h^2-ab}}{am^2 - 2hlm + bl^2}\right| = \frac{169\sqrt{576-69}}{\left|3(-2)^2 - 48.3(-2) + 23(3)^2\right|}$$

$$= \frac{169\sqrt{507}}{|12+288+207|} = \frac{169.13\sqrt{3}}{507} = \frac{13\sqrt{3}}{3} = \frac{13}{\sqrt{3}} \text{ sq.units.}$$

21. The vertices of a triangle are A(1, 4, 2), B(-2, 1, 2) C(2, 3, -4). Find $\underline{|A}, \underline{|B}, \underline{|C}$

Sol: 

Vertices of the triangle are $A(1, 4, 2), B(-2, 1, 2), C(2, 3, -4)$

D.rs of AB are 3, 3,0 i.e., 1, 1, 0

D.rs of BC are -4, -2, 5 i.e., 2, 1, -3

D.rs of AC are -1, 1, 6

$$\cos\underline{|ABC} = \frac{|1.2 + 1.0 + 0(-3)|}{\sqrt{1+1}\sqrt{4+1+9}} = \frac{3}{\sqrt{28}} = \frac{3}{2\sqrt{7}} \qquad \therefore \underline{|B} = \cos^{-1}\left(\frac{3}{2\sqrt{7}}\right)$$

$$\cos\underline{|BCA} = \frac{1(-1) + 1.1 + (-3)6}{\sqrt{4+1+9}\sqrt{1+1+36}} = \frac{19}{\sqrt{19}\sqrt{28}} = \sqrt{\frac{19}{28}} \;\therefore \underline{|C} = \cos^{-1}\left(\sqrt{\frac{19}{28}}\right)$$

$$\cos\underline{|CAB} = \frac{|-1.1 + 1.1 + 6.0|}{\sqrt{1+1+36}\sqrt{1+1+0}} = 0 \Rightarrow \underline{|A} = \pi/2$$

22. If $y = \mathrm{Tan}^{-1}\left(\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$ for

$0 < |x| < 1$, find $\dfrac{dy}{dx}$.

Sol. Put $x^2 = \cos 2\theta$

$$y = \mathrm{Tan}^{-1}\left(\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\theta\right)\right)$$

$$= \frac{\pi}{4}+\theta$$

$$= \frac{\pi}{4}+\frac{1}{2}\cos^{-1}(x^2)$$

$$\frac{dy}{dx} = \frac{1}{2}\frac{(-1)}{\sqrt{1-x^4}}\times 2x = \frac{-x}{\sqrt{1-x^4}}$$

23. Find the angle between the curves $\mathbf{x+y+2=0;\ x^2+y^2-10y=0}$

Sol: $x+y+2=0 \Rightarrow x = -\ \ y+2$ ----(1)

Equation of the curve $x^2+y^2-10y=0$ --(2)

Solving 1 and 2, $y+2\ ^2+y^2-10y=0 \ \Rightarrow y^2+4y+4+y^2-10y=0$

$\Rightarrow 2y^2-6y+4=0 \qquad \Rightarrow y^2-3y+2=0 \Rightarrow \ \ y+1 \ \ y-2 \ =0$

$\Rightarrow y=1$ or $y=2$

$x = -\ \ y+2$

$y=1 \Rightarrow x = -\ \ 1+2 \ =-3$

$y=2 \Rightarrow x=-(2+2)=-4$

The points of intersection are $P(-3,1)$ and $Q(-4,2)$,

equation of the curve is $x^2+y^2-10y=0$

Differente $x^2+y^2-10y=0$ w.r.to x.

$$\Rightarrow 2x+2y\frac{dy}{dx}-10\frac{dy}{dx}=0 \Rightarrow 2\frac{dy}{dx}(y-5)=-2x \Rightarrow \frac{dy}{dx}=-\frac{x}{y-5}$$

Equation of the line is $x+y+2=0$

Slope is m_2= -1.

Case (i):

$$\Rightarrow slope\ m_1=\frac{dy}{dx}atP=-\frac{-3}{1-5}=-\frac{3}{4}$$ and Slope is m_2 = -1.

Let θ be the angle between the curves, then $\mathbf{tan}\,\theta=\left|\frac{\boldsymbol{m_1-m_2}}{\boldsymbol{1+m_1m_2}}\right|$

$$=\left|\frac{-\frac{3}{4}+1}{1+\frac{3}{4}}\right|=\frac{1}{7}\Rightarrow\theta=\tan^{-1}\left(\frac{1}{7}\right)$$

Case (ii):

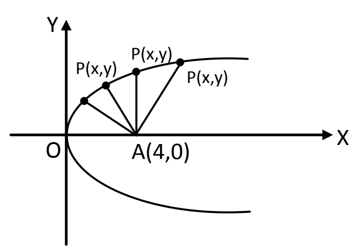
$$\Rightarrow slope\ m_1=\frac{dy}{dx}atQ=-\frac{4}{2-5}=-\frac{4}{3}$$ and Slope is m_2 = -1.

$$\Rightarrow \mathbf{tan}\,\theta=\left|\frac{\boldsymbol{m_1-m_2}}{\boldsymbol{1+m_1m_2}}\right|=\left|\frac{-\frac{4}{3}+1}{1+\frac{4}{3}}\right|=\frac{1}{7}$$

$$\Rightarrow\theta=\tan^{-1}\left(\frac{1}{7}\right)$$

24. Find the point on the graph $y^2 = x$ which is the nearest to the point (4, 0).

Sol.



Let P(x, y) be any point on $y^2 = x$ and A(4, 0). We have to find P such that PA is minimum. Suppose PA = D. The quantity to be minimized is D.

$$D=\sqrt{(x-4)^2+(y-0)^2} \qquad \text{...(1)}$$

P(x, y) lies on the curve, therefore

$$y^2 = x \qquad \text{...(2)}$$

from (1) and (2), we have

$$D=\sqrt{(x-4)^2+x}$$

$$D=\sqrt{(x^2-7x+16)} \qquad \text{...(3)}$$

Differentiating (3) w.r.t. x, we get

$$\frac{dD}{dx}=\frac{2x-7}{2}\cdot\frac{1}{\sqrt{x^2-7x+16}}$$

Now $\frac{dD}{dx}=0$

Gives x = 7/2. Thus 7/2 is a stationary point of the function D. We apply the first derivative test to verify whether D is minimum at x = 7/2

$$\left(\frac{dD}{dx}\right)_{x=3}=-\frac{1}{2}\cdot\frac{1}{\sqrt{9-12+16}}$$

and it is negative

$$\left(\frac{dD}{dx}\right)_{x=4}=\frac{1}{2}\cdot\frac{1}{\sqrt{16-28+16}}$$

and it is positive

$\frac{dD}{dx}$ changes sign from negative to positive. Therefore, D is minimum at x = 7/2. Substituting x = 7/2 in (2) we have $y^2 = 7/2$.

$$\therefore y=\pm\sqrt{\frac{7}{2}}$$

Thus the points $\left(\frac{7}{2},\sqrt{\frac{7}{2}}\right)$ and $\left(\frac{7}{2},-\sqrt{\frac{7}{2}}\right)$ are nearest to A(4, 0).