MATHEMATICS PAPER IB

COORDINATE GEOMETRY(2D &3D) AND CALCULUS.

TIME : 3hrs **Max:Marks.75**

Note: This question paper consists of three sections A,B and C.

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS. 10X2 =20

1 Find the value of k, if the straight lines $6x-10y+3=0$ and $kx-5y+8=0$ are parallel.

2. **The intercepts of a straight line on the axes of co-ordinates are a and b. If P is the length of the perpendicular drawn from the origin to this line. Write the value of P in terms of a and b.**

3. Find the fourth vertex of the parallelogram whose consecutive vertices are $2,4,-1$, $3,6,-1$ and $4,5,1$

4.Find the angle between the planes $2x-y+z=6$ and $x+y+2z=7$

5. Compute $\underset{x\to 0}{Lt} \frac{1-\cos 2mx}{\sin^2 nx}$ $m,n \in -2$

6. computute $\lim_{x\to\infty} \frac{8|x|+3x}{3|x|-2x}$.

7. If $y=\log \sin \log x$, find $\frac{dy}{dx}$

8. If the increase in the side of a square is 4% find the percentage of change in the area of the square.

9. Find the value of 'C' in the Rolle's theorem for the function $f\ x = x^2+4$ on $-3,3$

10. $Find\ \frac{dy}{dx}, if\ y = Cot^{-1}x^3\ ^2$

SECTION B

SHORT ANSWER TYPE QUESTIONS. **5X4 =20**

Note : Answer any FIVE questions. Each question carries 4 marks.

11. The ends of the hypotenuse of a right angled triangle are $0,6$ and $6,0$. Find the equation of locus of its third vertex.

12. When the axes are rotated through an angle 45^0, the transformed equation of curve is $17x^2 - 16xy + 17y^2 = 225$. Find the original equation of the curve.

13. If the straight line $ax + by + c = 0, bx + cy + a = 0$ and $cx + ay + b = 0$ are concurrent, then prove that $a^3 + b^3 + c^3 = 3abc$

14. Find derivative of the function $\sin 2x$ form the first principles w.r.to x.

15. Show that the length of the subnormal at any point on the curve $xy = a^2$ varies as the cube of the ordinate of the point.

16. A point P is moving on the curve $y = 2x^2$. The x co-ordinate of P is increasing at the rate of 4 units per second. Find the rate at which the y co-ordinate is increasing when the point is at $2,8$

17. Evaluate $\underset{x \to 0}{\mathrm{Lt}} \dfrac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$

SECTION C

LONG ANSWER TYPE QUESTIONS. **5X7 =35**

Note: Answer any Five of the following. Each question carries 7 marks.

18. Find the circumcentre of the triangle with the vertices $-2,3$, $2,-1$ and $4,0$

19. ***If θ is the angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$, then***

$$\cos\theta = \pm \frac{a+b}{\sqrt{(a-b)^2 + 4h^2}}$$

20. If the equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of distinct (ie., intersecting) lines, then the combined equation of the pair of bisectors of the angles between these lines is $h\ x^2 - y^2 = a-b\ xy$. $ax^2 + 2hxy + by^2 =$

21. Find the angle between the lines whose direction cosines are given by the equations $3l + m + 5n = 0$ and $6mn - 2nl + 5lm = 0$.

22 If $ax^2 + 2hxy + by^2 = 1$, **then prove that** $\frac{d^2y}{dx^2} = \frac{h^2 - ab}{hx + by\ ^3}$

23. Show that the curves $6x^2 - 5x + 2y = 0$ and $4x^2 + 8y^2 = 3$ touch each other at $\left(\frac{1}{2}, \frac{1}{2}\right)$.

24. Show that when the curved surface of right circular cylinder inscribed in a sphere of radius ‘r’ is maximum, then the height of the cylinder is $\sqrt{2}r$.

SOLUTIONS

Section A - VSAQ’S

1. Find the value of k, if the straight lines
6x – 10y + 3 = 0 and k x – 5y + 8 = 0 are parallel.

Sol. Given lines are 6x – 10y + 3 = 0 and
k x – 5y + 8 = 0

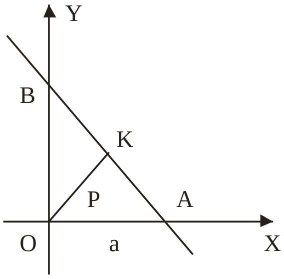
lines are parallel $\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\Rightarrow$ -30 = -10 k $\Rightarrow$ k = 3

2. **The intercepts of a straight line on the axes of co-ordinates are a and b. If P is the length of the perpendicular drawn from the origin to this line. Write the value of P in terms of a and b.**

Sol. Equation of the line in the intercept form is

$$\frac{x}{a}+\frac{y}{b}-1=0$$



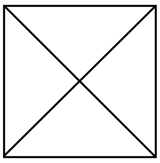
P = length of the perpendicular from origin $\frac{|0+0-1|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}} \Rightarrow \frac{1}{p}=\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}$

Square on both sides

$$\Rightarrow \frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}=\frac{a^2+b^2}{a^2b^2} \qquad \Rightarrow p^2=\frac{a^2b^2}{a^2+b^2} \Rightarrow p=\frac{|ab|}{\sqrt{a^2+b^2}}$$

3. Find the fourth vertex of the parallelogram whose consecutive vertices are (2, 4, –1), (3, 6, –1) are (4, 5, 1).

Sol.



ABCD is a parallelogram

where

A = (2, 4, –1), B = (3, 6, –1), C = (4, 5, 1)

Suppose D(x, y, z) is the fourth vertex

A B C D is a parallelogram

Mid point of AC = Mid point of BD

$$\left(\frac{2+4}{2},\frac{4+5}{7},\frac{-1+1}{2}\right)=\left(\frac{3+x}{2},\frac{6+y}{2},\frac{-1+z}{2}\right)$$

$$\frac{3+x}{2}=\frac{6}{2}\Rightarrow x=3$$

$$\frac{6+y}{2}=\frac{9}{2}\Rightarrow y=3$$

$$\frac{z-1}{2}=\frac{0}{2}\Rightarrow z=1$$

∴ Coordinates of the fourth vertex are :

D (3, 3, 1)

4. Find the angle between the planes 2x -y + z =6 and x + 2y + 2z = 7.

Sol. Equation of the plane are 2x -y + z =6 and x + 2y + 2z = 7.

Let θ be the angle between the planes, then

$$\cos\theta=\frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$= \frac{|2.1-1.2+1.2|}{\sqrt{4+1+1}\sqrt{1+1+4}} = \frac{2}{6} = \frac{1}{3}$$

$$\theta = \cos^{-1}\frac{1}{3}$$

5. $\underset{x\to 0}{Lt}\dfrac{1-\cos 2mx}{\sin^2 nx}$ $m, n \in -2$

Sol : $\underset{x\to 0}{Lt}\dfrac{1-\cos 2mx}{\sin^2 nx} = \underset{x\to 0}{Lt}\dfrac{2\sin^2 mx}{\sin^2 nx}$

$$= \underset{x\to 0}{Lt}\frac{2\sin^2 mx}{\sin^2 nx}\times\frac{x^2}{x^2} = \underset{x\to 0}{Lt}\frac{2\dfrac{\sin^2 mx}{x^2}}{\dfrac{\sin^2 nx}{x^2}} = 2\frac{\left(\underset{x\to 0}{Lt}\dfrac{\sin mx}{mx}\right)^2}{\left(\underset{x\to 0}{Lt}\dfrac{\sin nx}{nx}\right)^2}\times\frac{m^2}{n^2} = \frac{2m^2}{n^2}$$

6. $\underset{x\to 0}{Lt}\dfrac{8|x|+3x}{3|x|-2x}$

Sol : *as* $x\to\infty \Rightarrow |x| = x$ $\because$ *here* x is positive

$$\underset{x\to\infty}{Lt}\frac{8|x|+3x}{3|x|-2x} = \underset{x\to\infty}{Lt}\frac{8x+3x}{3x-2x} = \underset{x\to\infty}{Lt}\frac{11x}{x} = 11$$

7. If $y = \log\ \sin\ \log x$, find $\dfrac{dy}{dx}$.

$y = \log\ \sin\ \log x$

$$\frac{dy}{dx} = \frac{d}{dx}\log\ \sin\ \log x == \frac{1}{\sin\ \log x}\frac{d}{dx}\sin\ \log x$$

$$= \frac{1}{\sin\ \log x}\cos\ \log x\ \frac{d}{dx}\log x$$

$$= \frac{1}{\sin\ \log x}\cos\ \log x\ \frac{1}{x}$$

8. Let x be the side and A be the area of the Square .

percentage error in x is $\dfrac{\delta x}{x}\times 100 = 4$

Area A= x^2

Applying logs on both sides Log A = 2 logx

Taking differentials on both sides

$$\frac{1}{A}\delta A = 2.\frac{1}{x}\delta x \Rightarrow \frac{\delta A}{A}\times 100 = 2.\frac{\delta x}{x}\times 100$$

= 2 x 4 =8.

There fore percentage error in A is 8%

9. Let $f(x) = x^2 + 4$.
f is continuous on [–3, 3]
since f(–3) = f(3) and
f is differentiable on [–3, 3]
$\therefore$ By Rolle's theorem $\exists c \in (-1,1)$ Such that $f'(c) = 0$
$f'(x) = 2x = 0$
$\therefore = f'(c) = 0$
$2c = 0 \Rightarrow c = 0$
The point $c = 0 \in (-3, 3)$

10. If $y = (\cot^{-1} x^3)^2$, find $\frac{dy}{dx}$.

Sol. $u = \cot^{-1} x^3, u = x^3, y = u^2$

$$\frac{du}{dv} = -\frac{1}{1+u^2}, \frac{du}{dx} = 3x^2$$

$$\frac{dy}{dx} = 2u = 2\cot^{-1}(x^3) = -\frac{1}{1+x^6}$$

$$\frac{dy}{dx} = \frac{dy}{dx} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}$$

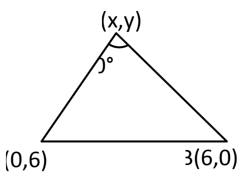
$$= 2\cot^{-1}(x^3)\left(-\frac{1}{1+x^6}\right)3x^2$$

$$= -\frac{6x^2}{1+x^6}\cot^{-1}(x^3)$$

SECTION B- SAQ'S

11. The ends of the hypotenuse of a right angled triangle are (0, 6) and (6, 0). Find the equation of locus of its third vertex.

ANS. Given points A(2, 3), B(–1, 5).
Let P(x, y) be any point in the locus.



Given condition is : $\angle APB = 90°$

$\Rightarrow$ (slope of $\overline{AP}$) (slope of $\overline{BP}$) = –1

$$\Rightarrow \frac{y-6}{x-0}\cdot\frac{y-0}{x-6} = -1$$

$$(y)(y-6)+(x)(x-6)=0$$

$$x^2+y^2-6x-6y=0$$

$\therefore$ Locus of P is $x^2+y^2-6x-6y=0$

12. When the axes are rotated through an angle $45°$, the transformed equation of a curve is $17x^2-16xy+17y^2=225$. Find the original equation of the curve.

Sol. Angle of rotation is $\theta = 45^0$. Let (X,Y) be the new coordinates of (x, y)

$$X = x\cos\theta + y\sin\theta = x\cos 45 + y\sin 45 \;= \frac{x+y}{\sqrt{2}}$$

$$Y = -x\sin\theta + y\cos\theta = -x\sin 45 + y\cos 45 \;= \frac{-x+y}{\sqrt{2}}$$

The original equation of $17X^2 - 16XY + 17Y^2 = 225$ is

$$\Rightarrow 17\left(\frac{x+y}{\sqrt{2}}\right)^2 - 16\left(\frac{x+y}{\sqrt{2}}\right)\left(\frac{-x+y}{\sqrt{2}}\right) + 17\left(\frac{-x+y}{\sqrt{2}}\right)^2 = 225$$

$$\Rightarrow 17\frac{x^2+y^2+2xy}{2} - 16\frac{y^2-x^2}{2} + 17\frac{x^2+y^2-2xy}{2} = 225$$

$$\Rightarrow 17x^2+17y^2+34xy-16y^2+16x^2+17x^2+17y^2-34xy=450$$

$\Rightarrow 50x^2+18y^2=450 \Rightarrow 25x^2+9y^2=225$ is the original equation

13. If the straight lines $ax+by+c=0$, $bx+cy+a=0$ and $cx+ay+b=0$ are concurrent, then prove that $a^3+b^3+c^3=3abc$.

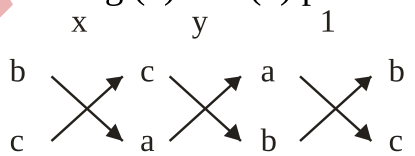
Sol: The equations of the given lines are

$ax+by+c=0$ ---(1)

$bx+cy+a=0$ ---(2)

$cx+ay+b=0$ ---(3)

Solving (1) and (2) points of intersection is got by



$$\frac{x}{ab-c^2} = \frac{y}{bc-a^2} = \frac{1}{ca-b^2}$$

Point of intersection is $\left(\frac{ab-c^2}{ca-b^2},\frac{bc-a^2}{ca-b^2}\right)$

$$c\left(\frac{ab-c^2}{ca-b^2}\right)+a\left(\frac{bc-a^2}{ca-b^2}\right)+b=0$$

$$c\left(ab-c^2\right)+a\left(bc-a^2\right)+b\left(ca-b^2\right)=0$$

$$abc-c^3+abc-a^3+abc-b^3=0$$

$$\therefore a^3+b^3+c^3=3abc.$$

14. $f^1(x)=\underset{h\to 0}{Lt}\frac{f(x+h)-f(x)}{h}=\underset{h\to 0}{Lt}\frac{SIN2(x+h)-\cos 2x}{h}$

$$=\underset{h\to 0}{Lt}\frac{2\cos\frac{2x+2h+2x}{2}.\sin\frac{2x+2h-2x}{2}}{h}$$

$$=\underset{h\to 0}{Lt}\frac{2\cos\left(2x+\frac{2h}{2}\right).\sin\left(\frac{2h}{2}\right)}{h}$$

$$\underset{h\to 0}{Lt}\,2\cos(2x+h)\;\underset{h\to 0}{Lt}\frac{\sin h}{h}$$

$$=2\cos 2x.1=2\cos 2x$$

15. Show that the length of sub-normal at any point on the curve $xy = a^2$ varies as the cube of the ordinate of the point.

Sol: Equation of the curve is $xy = a^2$.

$$\Rightarrow y=\frac{a^2}{x}\Rightarrow\frac{dy}{dx}=\frac{-a^2}{x^2}=m$$

Let P(x,y) be a point on the curve.

Length of the sub-normal $= |y_1.m|$

$$=\left|y\left(\frac{-a^2}{x^2}\right)\right|=\left|-a^2y\frac{y^2}{a^4}\right|=\left|\frac{y^3}{a^2}\right|\left(\because x=\frac{a^2}{y}\right)$$

$\therefore l.s.t\ \alpha\ y^3$ i.e. cube of the ordinate.

16. equation of the curve $y=2x^2$

Diff .w.r.t.t, $\frac{dy}{dt}=4x.\frac{dx}{dt}$

Given $x=2$ and $\frac{dx}{dt}=4.$

$$\frac{dy}{dt} = 4\ \ 2\ \ .4 = 32$$

y co-ordinate is increasing at the rate of 32 units/sec.

17. $\underset{x\to 0}{Lt} \dfrac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$

Sol. $\underset{x\to 0}{Lt} \dfrac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$

$$= \underset{x\to 0}{Lt} \frac{x\dfrac{\tan x}{1-\tan^2 x} - 2x\tan x}{(2\sin^2 x)^2}$$

$$= \underset{x\to 0}{Lt} \frac{2x\tan\left[\dfrac{1}{1-\tan^2 x} - 1\right]}{4\sin^2 x}$$

$$= \underset{x\to 0}{Lt} \frac{2x\tan x}{4\sin^2 x}\left[\frac{1-1+\tan^2 x}{1-\tan^4 x}\right]$$

$$= \underset{x\to 0}{Lt} \frac{2x\tan^3 x}{4\sin^4 x}$$

$$= \underset{x\to 0}{Lt} \frac{2x^4\tan^3 x}{x^3 4\sin^4 x} dx$$

$$= \frac{2}{4} \underset{x\to 0}{Lt} \frac{x^4}{\sin^4 x} \cdot \underset{x\to 0}{Lt} \frac{\tan^3 x}{x^3}$$

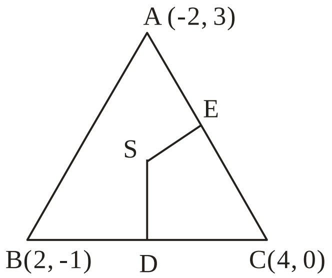
$$= \frac{1}{2}(1)(1) = \frac{1}{2}$$

SECTION C

18. A $-2,3$, B $2,-1$, C $4,0$ are the vertices of ΔABC.

Let S be the circumcentre of the ΔABC.

Let D be the midpoint of BC



$$\Rightarrow D = \left(\frac{2+4}{2}, \frac{-1+0}{2}\right) = \left(3, \frac{-1}{2}\right)$$

$$\Rightarrow \text{Slope of BC} = \frac{-1-0}{2-4} = \frac{-1}{-2} = \frac{1}{2}$$

$\Rightarrow$ SD is perpendicular to BC

Slope of SD $= -\frac{1}{m} = -2$

Equation of SD is $y + \frac{1}{2} = -2\ \ x - 3$

$\Rightarrow 2y + 1 = -4\ \ x - 3\ \ = -4x + 12$

$\Rightarrow 4x - 2y - 11 = 0$ ---(1)

Let E be the midpoint of AC

Co-ordinates of E are $\left(\frac{-2+4}{2}, \frac{3+0}{2}\right) = \left(1, \frac{3}{2}\right)$

Slope of $AC = \frac{3-0}{-2-4} = -\frac{3}{6} = -\frac{1}{2}$

$\Rightarrow$ SE is perpendicular to AC

$\Rightarrow$ Slope of SE = $-\frac{1}{m} = 2$

Equation of SE is $y - \frac{3}{2} = 2\ \ x - 1$

$\Rightarrow 2y - 3 = 4\ \ x - 1\ \ = 4x - 4$

$\Rightarrow 4x - 2y - 1 = 0$ ---(2)

$\Rightarrow 4x + 2y - 11 = 0$ ---(1)

Adding (1), (2) $\Rightarrow 8x - 12 = 0$

$8x = 12$

$\Rightarrow x = \frac{12}{8} = \frac{3}{2}$

Substitute this x in (1),

$2y = 11 - 4x = 11 - 4.\frac{3}{2} = 11 - 6 = 5$ $\Rightarrow y = \frac{5}{2}$

$\therefore$ Co-ordinates of S are $\left(\frac{3}{2}, \frac{5}{2}\right)$

19.***If*** θ ***is the angle between the lines represented by*** $ax^2 + 2hxy + by^2 = 0$***,then***

$$\cos\theta = \pm\frac{a+b}{\sqrt{(a-b)^2 + 4h^2}}$$

Proof: Let $ax^2 + 2hxy + by^2 = 0$ represent the lines $l_1x + m_1y = 0$ -- (1) and $l_2x + m_2y = 0$ -- (2).

Then $l_1l_2 = a$, $l_1m_2 + l_2m_1 = 2h$, $m_1m_2 = b$.

Let θ be the angle between the lines (1) and (2). Then $\cos\theta = \pm\frac{l_1l_2 + m_1m_2}{\sqrt{l_1^2 + m_1^2\ \ l_2^2 + m_2^2}}$

$$= \pm\frac{l_1l_2 + m_1m_2}{\sqrt{l_1^2l_2^2 + m_1^2m_2^2 + l_1^2m_2^2 + l_2^2m_1^2}}$$

$$= \pm \frac{l_1l_2 + m_1m_2}{\sqrt{(l_1l_2)^2 + (m_1m_2)^2 + 2l_1l_2m_1m_2 + (l_1m_2)^2 + (l_2m_1)^2 - 2l_1m_2l_2m_1}}$$

$$= \pm \frac{l_1l_2 + m_1m_2}{\sqrt{(l_1l_2 - m_1m_2)^2 + (l_1m_2 + l_2m_1)^2}}$$

$$= \pm \frac{a+b}{\sqrt{(a-b)^2 + 4h^2}}$$

20. Let $ax^2 + 2hxy + by^2 = 0$ represent the lines $l_1x + m_1y = 0$ -- (1) and $l_2x + m_2y = 0$ -- (2).

Then $l_1l_2 = a$, $l_1m_2 + l_2m_1 = 2h$, $m_1m_2 = b$.

The equations of bisectors of angles between (1) and (2) are $\frac{l_1x + m_1y}{\sqrt{l_1^2 + m_1^2}} = \pm \frac{l_2x + m_2y}{\sqrt{l_2^2 + m_2^2}}$

$$\Rightarrow \frac{l_1x + m_1y}{\sqrt{l_1^2 + m_1^2}} - \frac{l_2x + m_2y}{\sqrt{l_2^2 + m_2^2}} = 0 \text{ and}$$

$$\frac{l_1x + m_1y}{\sqrt{l_1^2 + m_1^2}} + \frac{l_2x + m_2y}{\sqrt{l_2^2 + m_2^2}} = 0$$

The combined equation of the bisectors is

$$\left(\frac{l_1x + m_1y}{\sqrt{l_1^2 + m_1^2}} - \frac{l_2x + m_2y}{\sqrt{l_2^2 + m_2^2}}\right)\left(\frac{\ell_1x + m_1y}{\sqrt{\ell_1^2 + m_1^2}} + \frac{\ell_2x + m_2y}{\sqrt{\ell_2^2 + m_2^2}}\right) = 0$$

$$\Rightarrow \left(\frac{l_1x + m_1y}{\sqrt{l_1^2 + m_1^2}}\right)^2 - \left(\frac{l_2x + m_2y}{\sqrt{l_2^2 + m_2^2}}\right)^2 = 0$$

$$\Rightarrow (l_2^2 + m_2^2)(l_1x + m_1y)^2 - (l_1^2 + m_1^2)(l_2x + m_2y)^2 = 0$$

$$\Rightarrow x^2\left[l_1^2(l_2^2 + m_2^2) - l_2^2(l_1^2 + m_1^2)\right] y^2\left[m_2^2(l_1^2 + m_1^2) - m_1^2(l_2^2 + m_2^2)\right]$$

$$-2xy\left[l_2m_2(l_1^2 + m_1^2) - l_1m_1(l_2^2 + m_2^2)\right] = 0$$

$$\Rightarrow x^2(l_1^2l_2^2 + l_1^2m_2^2 - l_1^2l_2^2 - l_2^2m_1^2) - y^2(l_1^2m_2^2 + m_1^2m_2^2 - m_1^2l_2^2 - m_1^2m_2^2) - 2xy\left(l_2m_2l_1^2 + l_2m_2m_1^2 - l_1m_1l_2^2 - l_1m_1m_2^2\right) = 0$$

$$\Rightarrow x^2(l_1^2m_2^2 - l_2^2m_1^2) - y^2(l_1^2m_2^2 - l_2^2m_2^2) = 2xy\,(l_1l_2(l_1m_2 - l_2m_1) - m_1m_2(l_1m_2 - l_2m_1))$$

$$\Rightarrow (x^2 - y^2)(l_1^2m_2^2 - l_2^2m_1^2) = 2xy(l_1l_2 - m_1m_2)(l_1m_2 - l_2m_1) \Rightarrow (x^2 - y^2)(l_1m_2 + l_2m_1) = 2xy(l_1l_2 - m_1m_2)$$

$$\Rightarrow 2h(x^2 - y^2) = 2xy(a-b)$$

$$\therefore h(x^2 - y^2) = (a-b)xy \qquad \text{OR} \qquad \frac{x^2 - y^2}{a-b} = \frac{xy}{h}$$

21. Given $3l + m + 5n = 0$

$6mn - 2nl + 5lm = 0$

From (1), $m = -\ 3l + 5n$

Substituting in (2)

$\Rightarrow -6n\ \ 3l + 5n\ \ -2nl - 5l\ \ 3l + 5n\ \ = 0$

$\Rightarrow -18\ln - 30n^2 - 2nl - 15l^2 - 25\ln = 0$

$\Rightarrow -15l^2 - 45\ln - 30n^2 = 0$

$\Rightarrow l^2 + 3\ln + 2n^2 = 0$

$\Rightarrow\ \ l + 2n\ \ \ l + n\ \ = 0$

$\Rightarrow l + 2n = 0 \ or \ l + n = 0$

Case (i) :

$l_1 + n_1 = 0 \Rightarrow n_1 = -l_1; \Rightarrow n_1 = -l_1; \Rightarrow \frac{l_1}{1} = \frac{n_1}{-1}$

But $m_1 = -\ \ 3l_1 + 5n_1\ \ = -\ \ -3n_1 + 5n_1\ \ = -2n_1$

$\therefore \frac{m_1}{+2} = \frac{n_1}{-1}$

$\therefore \frac{l_1}{1} = \frac{m_1}{2} = \frac{n_1}{-1}$

D.rs of the first line l_1 *are* $1, 2, -1$

Case (ii) : $l_2 + 2n_2 = 0$

$\Rightarrow l_2 = -2n_2 \Rightarrow \frac{l_2}{-2} = \frac{n_2}{1}$

$\Rightarrow m_2 = -\ \ 3l_2 + 5n_2\ \ = -\ \ -6n_2 + 5n_2\ \ = n_2$

$\frac{m_2}{1} = \frac{n_2}{1}$

$\therefore \frac{l_2}{-2} = \frac{m_2}{1} = \frac{n_2}{1}$

D.rs of the second line l_2 *are* $-2, 1, 1$

Suppose $'\theta'$ is the angle between the lines l_1 *and* l_2

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\frac{|1\ \ -2\ \ + 2.1 + \ \ -1\ \ .1|}{\sqrt{1+4+1}\sqrt{4+1+1}}$$

$= \frac{1}{6} \Rightarrow \theta = \cos^{-1}\ \ 1/6$

22. **If** $ax^2 + 2hxy + by^2 = 1$, **then prove that** $\frac{d^2y}{dx^2} = \frac{h^2 - ab}{hx + by\ ^3}$

sol : **Given** $ax^2 + 2hxy + by^2 = 1$

Differentiating w. r. to x

$$\frac{d}{dx}\left(ax^2+2hxy+by^2\right)=0$$

$$\Rightarrow a.2x+2h\left(x.\frac{dy}{dx}+y\right)+b.2y\frac{dy}{dx}=0$$

$$\Rightarrow 2ax+2hx.\frac{dy}{dx}+2hy+2by.\frac{dy}{dx}=0$$

$$\Rightarrow 2\left(hx+by\right).\frac{by}{dx}=-2\left(ax+hy\right)$$

$$\frac{dy}{dx}=\frac{-2\left(ax+hy\right)}{2\left(hx+by\right)}=-\frac{ax+hy}{hx+by} \quad \ldots\ldots (1)$$

Differentiating again w. r. to x,

$$\frac{d^2y}{dx^2}=-\frac{d}{dx}\left(\frac{ax+hy}{hx+by}\right)$$

$$\frac{d^2y}{dx^2}$$

$$=\frac{\left[\left(hx+by\right)\left(a+h\frac{dy}{dx}\right)-\left(ax+hy\right)\left(h+b.\frac{by}{dx}\right)\right]}{\left(hx+by\right)^2}$$

$$=\frac{\left(ax+hy\right)\left[h-b.\frac{ax+hy}{hx+by}\right]-\left(hx+by\right)\left[\frac{ax+hy}{hx+by}\right]}{\left(hx+by\right)^2}\ \left(ax+hy\right)\left(h^2x+bhy-abx-bhy\right)$$

$$=\frac{-\left(hx+by\right)\left(ahx+aby-ahx-h^2y\right)}{\left(hx+by\right)^3}$$

$$=\frac{\left(h^2-ab\right)x\left(ax+hy\right)+\left(h^2-ab\right)y\left(hx+by\right)}{\left(hx+by\right)^3}=\frac{\left(h^2-ab\right)\left(x\left(ax+hy\right)+y\left(hx+by\right)\right)}{\left(hx+by\right)^3}$$

$$=\frac{\left(h^2-ab\right)\left[ax^2+2hxy+by^2\right]}{\left(hx+by\right)^3}$$

$$=\frac{h^2-ab}{\left(hx+by\right)^3}\left[\because ax^2+2hxy+by^2=1\right]$$

23. Show that the curves $\mathbf{6x^2-5x+2y=0}$ and $\mathbf{4x^2+8y^2=3}$ touch each other at $\left(\frac{1}{2},\frac{1}{2}\right)$.

Sol: Equation of the first curve is $6x^2-5x+2y=0$

$\Rightarrow 2y = 5x - 6x^2 \Rightarrow \quad 2.\frac{dy}{dx} = 5 - 12x \quad \Rightarrow \frac{dy}{dx} = \frac{5-12x}{2}$

$m_1 = \left(\frac{dy}{dx}\right)_{atP\left(\frac{1}{2},\frac{1}{2}\right)} = \frac{5-12.\frac{1}{2}}{2} = \frac{5-6}{2} = -\frac{1}{2}$

Equation of the second curve is $4x^2 + 8y^2 = 3$

$\Rightarrow 8x + 16y.\frac{dy}{dx} = 0 \Rightarrow 16y.\frac{dy}{dx} = -8x$

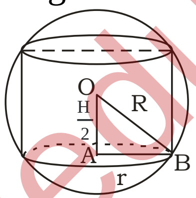
$\Rightarrow \frac{dy}{dx} = \frac{-8x}{16y} = -\frac{x}{2y}$

$m_2 = \left(\frac{dy}{dx}\right)_{atP\left(\frac{1}{2},\frac{1}{2}\right)} = \frac{-\frac{1}{2}}{2\left(\frac{1}{2}\right)} = -\frac{1}{2}$

$\therefore m_1 = m_2$

The given curves touch each other at $P\left(\frac{1}{2},\frac{1}{2}\right)$

24.let r be the radius and h be the height of the cylinder.



From ΔOAB, $OA^2 + AB^2 = OB^2$

$\Rightarrow r^2 + \frac{h^2}{4} = R^2; \; r^2 = R^2 - \frac{h^2}{4}$

Curved surface area $= 2\pi rh$

$= 2\pi\sqrt{R^2 - \frac{h^2}{4}}.h$

$= \pi h\sqrt{4R^2 - h^2}$

Let $f(h) = \pi h\sqrt{4R^2 - h^2}$

$f'(h) = \pi\left[h.\frac{1}{2\sqrt{4R^2 - h^2}}(-2h) + \sqrt{4R^2 - h^2}.1\right]$

$= \pi.\frac{-h^2 + 4R^2 - h^2}{\sqrt{4R^2 - h^2}} = \frac{2\pi(2R^2 - h^2)}{\sqrt{4R^2 - h^2}}$

For max or min $f'(h) = 0$

$\Rightarrow \frac{2\pi(2R^2 - h^2)}{\sqrt{4R^2 - h^2}} = 0$

$\therefore 2R^2 - h^2 = 0$

$\Rightarrow h^2 = 2R^2 \Rightarrow h = \sqrt{2}R$

$\Rightarrow \sqrt{4R^2 - h^2} \;\; -2h \;\; + \;\; 2R^2 - h^2$

$$f'' \text{ when } h = \sqrt{2}R \;\; = 2\pi \frac{\frac{d}{dh}\sqrt{4R^2 - h^2}}{4R^2 - h^2}$$

$$= -\frac{4\pi h + 0}{\sqrt{4r^2 - h^2}} < 0$$

f h is greatest when $h = \sqrt{2}R$

i.e., Height of the cylinder $= \sqrt{2}R$