MATHEMATICS PAPER IB

COORDINATE GEOMETRY(2D &3D) AND CALCULUS.

TIME : 3hrs **Max. Marks.75**

Note: This question paper consists of three sections A,B and C.

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS. 10X2 =20

1. Find the condition for the points (a, 0), (h, k) and (0,b)where $ab \neq 0$ to be collinear.

2. Fund the value of P, if the straight lines 3x + 7y – 1 = 0 and 7x – p y + 3 = 0 are mutually perpendicular.

3. Find the distance between the midpoint of the line segment $\overline{AB}$ and the point (3,–1, 2) where A = (6, 3, –4) and B = (–2, –1, 2).

4. Find the constant k so that the planes x – 2y + kz = 0 and 2x + 5y – z = 0 are at right angles.

5. $\underset{x \to 0}{Lt} \dfrac{\sin(a+bx) - \sin(a-bx)}{x}$

6. If f is given by $f(x) = \begin{cases} k^2x - k & \text{if} \quad x \geq 1 \\ 2 & \text{if} \quad x < 1 \end{cases}$ is a continuous function on R, then find the values of k.

7. $y = \log_7 \ \log x \quad x > 0$

8. If $x^4 + y^4 - a^2 \ xy = 0$,then find $\frac{dy}{dx}$

9. The time t of a complete oscillation of a simple pendulum of length *l is* given by the equation t = $\mathbf{2\pi}\sqrt{\frac{\mathbf{1}}{\mathbf{g}}}$ where g is gravitational constant. Find the approximate percentage error in the calculated g, corresponding to an error of 0.01 percent is the value of t.

10. Verify the Rolle's theorem for the function $(x^2 - 1)(x - 2)$ on [–1, 2]. Find the point in the interval where the derivate vanishes.

SECTION B

SHORT ANSWER TYPE QUESTIONS.

ANSWER ANY FIVE OF THE FOLLOWING **5 X 4 = 20**

11. A(5, 3) and B(3, –2) are two fixed points. Find the equation of locus of P, so that the area of triangle PAB is 9.

12. When the origin is shifted to the point (2,3), the transformed equation of a curve is $x^2 + 3xy - 2y^2 + 17x - 7y - 11 = 0$. Find the original equation of the curve.

13. A straight line parallel to the line $y = \sqrt{3}x$ passes through Q $2,3$ and cuts the line $2x + 4y - 27 = 0$ at P. Find the length of PQ.

14. Evaluate $\underset{x \to a}{Lt}\left[\frac{x\sin a - a\sin x}{x - a}\right]$

15. A balloon which always remains spherical on inflation is being inflated by pumping on 900 cubic centimeters of gas per second. Find the rate at which the radius of balloon increases when the radius in 15 cm.

16. Show that the curves $x^2+y^2= 2$ and $\mathbf{3x^2+x^2=4x}$ have a common tangent at the point (1, 1).

17. If f and g are differentiable functions at x and g(x)≠0 then the quotient function $\frac{f}{g}$ is differentiable at x and $\left(\frac{f}{g}\right)'(x)=\frac{g(x).f'(x)-f(x)g'(x)}{[g(x)]^2}$

SECTION C

LONG ANSWER TYPE QUESTIONS.

ANSWER ANY FIVE OF THE FOLLOWING 5 X 7 =35

18. If p and q are lengths of the perpendiculars from the origin to the straight lines $x\sec\alpha+y\operatorname{cosec}\alpha=a$ and $x\cos\alpha-y\sin\alpha=a\cos2\alpha$, prove that $4p^2+q^2=a^2$.

19. Write down the equation of the pair of straight lines joining the origin to the points of intersection of the lines 6x – y + 8 = 0 with the pair of straight lines $3x^2+4xy-4y^2-11x+2y+6=0$. Show that the lines so obtained make equal angles with the coordinate axes.

20. Show that the lines joining the origin to the points of intersection of the curve $x^2-xy+y^2+3x+3y-2=0$ and the straight line $x-y-\sqrt{2}=0$ are mutually perpendicular.

21. Show that the lines whose direction cosines are given by $l+m+n=0$ $2m+3nl-5lm=0$ are perpendicular to each other

22. If $y = x^{\tan x} + \sin x^{\cos x}$, find $\frac{dy}{dx}$

23. If the tangent at a point P on the curve $\mathbf{x^m y^n = a^{m+n} (mn \neq 0)}$ meets the coordinate axes in A, B. show that AP : PB is constant.

24. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

solutions

SECTION A

1. Find the condition for the points (a, 0), (h, k) and (0,b)where $ab \neq 0$ to be collinear.

Sol. A(a, 0), B(h, k), C(0, b) are collinear.

$\Rightarrow$ Slope of AB = Slope of AC

$$\frac{k-0}{h-a} = \frac{-b}{a} \Rightarrow ak = -bh + ab$$

$$bh + ak = ab \Rightarrow \frac{h}{a} + \frac{k}{b} = 1$$

2. Fund the value of P, if the straight lines 3x + 7y – 1 = 0 and 7x – p y + 3 = 0 are mutually perpendicular.

Sol. Given lines are 3x + 7y – 1 = 0, 7x – p y + 3 = 0

lines are perpendicular

$$\Rightarrow a_1a_2 + b_1b_2 = 0$$

$$\Rightarrow 3.7 + 7\ -p\ = 0 \Rightarrow 7p = 21 \Rightarrow p = 3$$

3. Find the distance between the midpoint of the line segment $\overrightarrow{AB}$ and the point (3,–1, 2) where A = (6, 3, –4) and B = (–2, –1, 2).

Sol. Given points are :

A= (6, 3, –4), B = (–2, –1, 2)

Coordinates of Q are :

$$\left(\frac{6-2}{2}, \frac{3-1}{2}, \frac{-4+2}{2}\right) = (2, 1, -1)$$

Coordinates of P are : (3, –1, 2)

$$PQ = \sqrt{(3-2)^2 + (-1-1)^2 + (2+1)^2}$$

$= \sqrt{1+4+9} = \sqrt{14}$ units.

4. Find the constant k so that the planes x – 2y + kz = 0 and 2x + 5y – z = 0 are at right angles.

Sol. Equations of the given planes are

x – 2y + kz = 0 and 2x + 5y – z = 0

since these planes are perpendicular , therefore

$1\cdot 2-2\cdot 5+k(-1)=0$

$2-10=k\Rightarrow k=-8$

5. $\underset{x\to 0}{Lt}\dfrac{\sin(a+bx)-\sin(a-bx)}{x}$

Sol : $\underset{x\to 0}{Lt}\dfrac{\sin(a+bx)-\sin(a-bx)}{x}=\underset{x\to 0}{Lt}\dfrac{2\cos a.\sin bx}{x}=\underset{x\to 1}{Lt}\,2\cos a.\ \underset{x\to 0}{Lt}\dfrac{\sin bx}{bx}.b$

$=2\cos a.\ \ b=2b.\cos a.$

6. If f is given by $f(x)=\begin{cases}k^2x-k & \text{if } x\geq 1\\ 2 & \text{if } x<1\end{cases}$ is a continuous function on R, then find the values of k.

Sol : $\underset{x\to 1-}{Lt}f(x)=\underset{x\to 1-}{Lt}(2)=2$

$\underset{x\to 1+}{Lt}f(x)=\underset{x\to 1+}{Lt}(kx^2-k)=k^2-k$ Given $f(x)$ is continuous at $x=0$

$\underset{x\to 1-}{Lt}f(x)=\underset{x\to 1+}{Lt}f(x)\ \ 2=k^2-k$

Given f is continuous on R, hence it is continuous at x=1.

Therefore L.L =R.L

⇨ $k^2-k-2=0$

=> $(k-2)(k+1)=0$ => $k=2\ or\ -1$

7. $y = \log_7 \ \log x \quad x > 0$

sol : $y = \log_7 \ \log x \quad x > 0$

$$\frac{dy}{dx} = \frac{1}{\log_7}.\frac{1}{\log x}.\frac{1}{x}$$

$$= \frac{1}{x \ \log x \ \ \log_e^7} = \frac{\log_7^e}{x \log_e^x}$$

8. If $x^4 + y^4 - a^2 \ xy = 0$,thenfind $\frac{dy}{dx}$

sol : Differentiate w. r. to x

$$\frac{d}{dx} \ x^4 + y^4 - a^2 \ xy \ = 0$$

$$4x^3 + 4y^3.\frac{dy}{dx} - a^2 \ \ x.\frac{dy}{dx} + y.1 \ = 0$$

$$4x^3 + 4y^3.\frac{dy}{dx} - a^2 x\frac{dy}{dx} - a^2 \ \ y = 0$$

$$4y^3 - a^2 x \ \frac{dy}{dx} = a^2 y - 4x^3$$

$$\Rightarrow \frac{dy}{dx} = \frac{a^2 y - 4x^3}{4y^3 - a^2 x}$$

9. The time t of a complete oscillation of a simple pendulum of length *l is* given by the equation t = $\mathbf{2\pi}\sqrt{\frac{\mathbf{1}}{\mathbf{g}}}$ where g is gravitational constant. Find the approximate percentage error in the calculated g, corresponding to an error of 0.01 percent is the value of t.

Sol: percentage error in t is =0.01

Given t=

Taking logs on both sides

log t = log ( ) + {(log (l) – log g}

Taking differentials on both sides,

= 0 +

Multiplying with 100,

$\rightarrow$

Percentage error in g = -0.02

10. Verify the Rolle's theorem for the function $(x^2 - 1)(x - 2)$ on $[-1, 2]$. Find the point in the interval where the derivate vanishes.

Sol. Let $f(x) = (x^2 - 1)(x - 2) = x^3 - 2x^2 - x + 2$

f is continuous on $[-1, 2]$

since $f(-1) = f(2) = 0$ and

f is differentiable on $[-1, 2]$

$\therefore$ By Rolle's theorem $\exists c \in (-1, 2)$

Let $f'(c) = 0$

$f'(x) = 3x^2 - 4x - 1$

$3c^2 - 4c - 1 = 0$

$$c = \frac{4 \pm \sqrt{16+12}}{6} = \frac{4 \pm \sqrt{28}}{6}$$

$$\Rightarrow c = \frac{2 \pm \sqrt{7}}{3}$$

SECTION B

11. A(5, 3) and B(3, –2) are two fixed points. Find the equation of locus of P, so that the area of triangle PAB is 9.

Sol. Given points are A(5, 3), B(3, –2)

Let P(x, y) be a point in the locus.

Given, area of ΔAPB = 9.

$$\Rightarrow \frac{1}{2}\begin{vmatrix} x-5 & y-3 \\ 3-5 & -2-3 \end{vmatrix} = 9$$

$$\Rightarrow \begin{vmatrix} x-5 & y-3 \\ -2 & -5 \end{vmatrix} = 18$$

$$\Rightarrow |-5x+25+2y-6| = 18$$

$$\Rightarrow |-5x+2y+19| = 18$$

$$\Rightarrow -5x+2y+19 = \pm 18$$

$$\Rightarrow -5x+2y+19 = 18 \text{ or } -5x+2y+19 = 18$$

$$\Rightarrow 5x-2y-1 = 0 \text{ or } 5x-2y-37 = 0$$

∴ Locus of P is (5x – 2y – 1)(5x – 2y – 37) = 0.

12. When the origin is shifted to the point (2,3), the transformed equation of a curve is $x^2+3xy-2y^2+17x-7y-11=0$. Find the original equation of the curve.

Sol. New origin =(2,3) = (h,k)

Equations of transformation are

$X = x+h,\ y = Y+k \ \longrightarrow\ X = x-h = x-2,\ Y = y-k = y-3$

Transformed equation is

$x^2+3xy-2y^2+17x-7y-11=0$ (here x, y can be treated as upper case letters)

Original equation is

$$(x-2)^2+3(x-2)(y-3)-2(y-3)^2+17(x-2)-7(y-3)-11=0$$

$$x^2+4x+4+3xy-9x-6y+18-2y^2+12y-18+17x-34-7y+21-11=0$$

Therefore, original equation is $x^2+3xy-2y^2+4x-y-20=0$

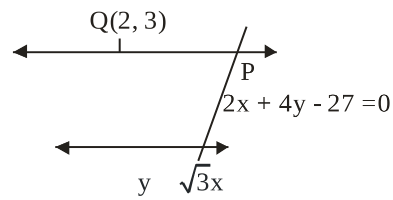
13. A straight line parallel to the line $y=\sqrt{3}x$ passes through $Q(2,3)$ and cuts the line $2x+4y-27=0$ at P. Find the length of PQ.

Sol: PQ is parallel to the straight line $y=\sqrt{3}x$

$\tan\alpha = \sqrt{3} = \tan 60^{\circ}$

$\alpha = 60^o$

$Q\ (2,3)$ is a given point



Co-ordinates of any point P are

$(x_1 + r\cos\alpha,\ y_1 + r\sin\alpha)$

$(2 + r\cos 60^o,\ 3 + r\sin 60^o)$

$$= P\left(2 + \frac{r}{2},\ 3 + \frac{\sqrt{3}}{2}r\right)$$

P is a point on the line $2x + 4y - 27 = 0$

$$2\left(2 + \frac{r}{2}\right) + 4\left(3 + \frac{\sqrt{3}}{2}r\right) - 27 = 0$$

$4 + r + 12 + 2\sqrt{3}r - 27 = 0$

$r\ (2\sqrt{3} + 1) = 27 - 16 = 11$

$$= \frac{11}{2\sqrt{3}+1}\cdot\frac{2\sqrt{3}-1}{2\sqrt{3}-1} = \frac{11\ (2\sqrt{3}-1)}{11}$$

14. $\underset{x\to a}{Lt}\left[\dfrac{x\sin a - a\sin x}{x-a}\right]$

Sol : $\underset{x\to a}{Lt}\dfrac{x\sin a - a\sin x}{x-a} = \underset{x\to a}{Lt}\dfrac{(x\sin a - a\sin a) - (a\sin x - a\sin a)}{x-a}$

(Adding and subtactin a sina in Nr.)

$$= \underset{x\to a}{Lt}\frac{(x-a)\sin a - a(\sin x - \sin a)}{x-a} = \underset{x\to a}{Lt}\frac{(x-a)\sin a}{x-a} - \underset{x\to a}{Lt}\ a\ \frac{\sin x - \sin a}{x-a}$$

$$= \sin a - a.\ \underset{x\to a}{Lt}\frac{2\cos\frac{x+a}{2}.\sin\frac{x-a}{2}}{x-a} = \sin a - a.\ \underset{x\to a}{Lt}\frac{\cos(x+a)}{2}\ \underset{x\to a}{Lt}\frac{\sin\left(\frac{x-a}{2}\right)}{\frac{x-a}{2}}$$

$= \sin a - a\cos a\,(1) = \sin a - a\cos a$

15. A balloon which always remains spherical on inflation is being inflated by pumping on 900 cubic centimeters of gas per second. Find the rate at which the radius of balloon increases when the radius in 15 cm.

Sol. $\frac{dv}{dt} = 900\,\text{c.c./sec}$

r = 15 cm

Volume of the sphere $v = \frac{4}{3}\pi r^3$

$$\frac{dv}{dt} = \frac{4}{3}\pi 3r^2 \frac{dr}{dt} \Rightarrow 900 = 4\pi(15)^2 \frac{dr}{dt}$$

$$\Rightarrow \frac{900}{4\times 225\pi} = \frac{dr}{dt} \Rightarrow \frac{900}{900\pi} = \frac{dr}{dt}$$

$$\frac{1}{\pi} = \frac{dr}{dt} \qquad \therefore \frac{dr}{dt} = \frac{1}{\pi}\,\text{cm/s}$$

16. Show that the curves $x^2+y^2= 2$ and $\mathbf{3x^2 + x^2 = 4x}$ have a common tangent at the point (1, 1).

Sol: Equation of the first curve is $x^2 + y^2 = 2$

Differentiating w, r, to x

$$\Rightarrow \quad 2x + 2y\frac{dy}{dx} = 0 \Rightarrow 2y\frac{dy}{dx} = -2x \qquad \Rightarrow \frac{dy}{dx} = -\frac{2x}{2y} = -\frac{x}{y}$$

At p (1, 1) slope of the tangent = - $\frac{-1}{1} = -1$

Equation of the second curve is $3x^2 + y^2 = 4y.$

Differentiating w. r. to x, $6x + 2y.\frac{dy}{dx} = 4 \;\Rightarrow 2y.\frac{dy}{dx} = 4 - 6x$

$$\Rightarrow \frac{dy}{dx} = \frac{4-6x}{2y} = \frac{2y-3x}{y}$$

At p(1, 1) slope of the tangent = $\frac{2-3}{1} \quad = -\frac{1}{1} = -1$

The slope of the tangents to both the curves at (1, 1) are same and pass through the same point (1, 1)

$\therefore$ The given curves have a common tangent p (1, 1)

17\.

If f and g are differentiable functions at x and g(x)≠0 then the quotient function $\frac{f}{g}$ is differentiable at x and $\left(\frac{f}{g}\right)'(x)=\frac{g(x).f'(x)-f(x)g'(x)}{[g(x)]^2}$

Proof: Since f and g are differentiable at x, therefore

$f'(x) = \underset{h\to 0}{Lt}\frac{f(x+h)-f(x)}{h}$

and $\underset{h\to 0}{Lt}\frac{g(x+h)-g(x)}{h} = g'(x)$

$\left(\frac{f}{g}\right)^1 = \underset{h\to 0}{Lt}\frac{(f/g)(x+h)-(f/g)(x)}{h}$

$= \underset{h\to 0}{Lt}\frac{1}{h}\left[\frac{f(x+h)}{g(x+h)}-\frac{f(x)}{g(x)}\right]$

$= \underset{h\to 0}{Lt}\frac{1}{h}\left[\frac{f(x+h)g(x)-g(x+h)f(x)}{g(x)g(x+h)}\right]$

$= \cdot\underset{h\to 0}{Lt}\frac{1}{g(x).g(x+h)}\left[\frac{f(x+h)g(x)-f(x)g(x)+f(x)g(x)-f(x)g(x+h)}{h}\right]$

$= \frac{1}{[g(x)]^2}\cdot\left\{g(x)\underset{h\to 0}{Lt}\left[\frac{f(x+h)-f(x)}{h}\right]-f(x)\underset{h\to 0}{Lt}\left[\frac{g(x+h)-g(x)}{h}\right]\right\}$

$= \frac{1}{[g(x)]^2}\left\{g(x)\underset{h\to 0}{Lt}\left[\frac{f(x+h)-f(x)}{h}\right]-f(x)\underset{h\to 0}{Lt}\left[\frac{g(x+h)-g(x)}{h}\right]\right\}$

$= \frac{1}{[g(x)]^2}\quad g(x)f'(x)-f(x)g'(x)$

$\therefore\left(\frac{f}{g}\right)'(x) = \frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$

SECTION C

18\. If p and q are lengths of the perpendiculars from the origin to the straight lines $x\sec\alpha+y\operatorname{cosec}\alpha=a$ and $x\cos\alpha-y\sin\alpha=a\cos 2\alpha$, prove that $4p^2+q^2=a^2$.

Sol: Equation of AB is $x\sec\alpha+y\operatorname{cosec}\alpha=a$

$\frac{x}{\cos\alpha}+\frac{y}{\sin\alpha}=a$

$x\sin\alpha+y\cos\alpha=a\sin\alpha\cos\alpha$

$x\sin\alpha+y\cos\alpha-a\sin\alpha\cos\alpha=0$

p = length of the perpendicular from O on AB = $\dfrac{|0+0-a\sin\alpha\cos\alpha|}{\sqrt{\sin^2\alpha+\cos^2\alpha}}$

$= a\sin\alpha.\cos\alpha = a.\dfrac{\sin 2\alpha}{2} \Rightarrow$

$2p = a\sin 2\alpha$ ---(1)

Equation of CD is $x\cos\alpha - y\sin\alpha = a\cos 2\alpha$

$x\cos\alpha - y\sin\alpha - a\cos 2\alpha = 0$

q = Length of the perpendicular from O on CD $\dfrac{|0+0-a\cos 2\alpha|}{\sqrt{\cos^2\alpha+\sin^2\alpha}} = a\cos 2\alpha$ ---(2)

Squaring and adding (1) and (2)

$4p^2 + q^2 = a^2\sin^2 2\alpha + a^2\cos^2 2\alpha$

$= a^2\ \sin^2 2\alpha + \cos^2 2\alpha\ = a^2.1 = a^2$

19. Write down the equation of the pair of straight lines joining the origin to the points of intersection of the lines 6x – y + 8 = 0 with the pair of straight lines $3x^2+4xy-4y^2-11x+2y+6=0$. Show that the lines so obtained make equal angles with the coordinate axes.

Sol. Given pair of line is $3x^2+4xy-4y^2-11x+2y+6=0$ …(1)

Given line is $6x-y+8=0 \Rightarrow \dfrac{6x-y}{-8}=1 \quad \Rightarrow \dfrac{y-6x}{8}=1$-----(2)

Homogenising (1) w.r.t (2)

$$3x^2+4xy-4y^2-11x-2y\left(\frac{y-6x}{8}\right)+6\left(\frac{y-6x}{8}\right)^2=0$$

$$64\left[3x^2+4xy-4y^2\right]-8\left[11xy-66x^2-2y^2+12xy\right]\quad +6\left[y^2+36x^2-12xy\right]=0$$

$$\Rightarrow 936x^2+256xy-256xy-234y^2=0$$

$$\Rightarrow 468x^2-117y^2=0 \qquad \Rightarrow 4x^2-y^2=0 \text{ ----(3)}$$

is eq. of pair of lines joining the origin to the point of intersection of (1) and (2).

The eq. pair of angle bisectors of (3) is $h \; x^2 - y^2 \; - \; a - b \; xy = 0$

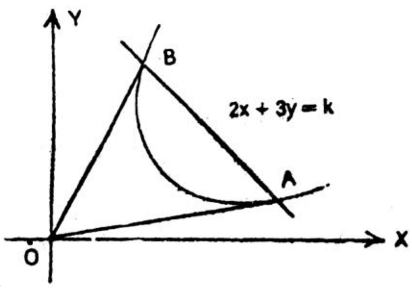
$\Rightarrow 0 \; x^2 - y^2 \; - \; 4 - 1 \; xy = 0 \quad \Rightarrow xy = 0$

x = 0 or y = 0 which are the eqs. is of co-ordinates axes

$\therefore$ The pair of lines are equally inclined to the co-ordinate axes

20. Show that the lines joining the origin to the points of intersection of the curve $x^2 - xy + y^2 + 3x + 3y - 2 = 0$ and the straight line $x - y - \sqrt{2} = 0$ are mutually perpendicular.

Sol.



Le t A,B the the points of intersection of the line and the curve.

Equation of the curve is $x^2 - xy + y^2 + 3x + 3y - 2 = 0$ …….(1)

Equation of the line AB is $x - y - \sqrt{2} = 0$

$\Rightarrow \quad x - y = \sqrt{2} \Rightarrow \dfrac{x - y}{\sqrt{2}} = 1$ ….(2)

Homogenising, (1) with the help of (2) combined equation of OA, OB is

$x^2 - xy + y^2 + 3x.1 + 3y.1 - 2.1^2 = 0$

$\Rightarrow x^2 - xy + y^2 + 3 \; x + y \; \dfrac{x - y}{\sqrt{2}} - 2 \dfrac{x - y^{\;2}}{2} = 0$

$\Rightarrow x^2 - xy + y^2 + \dfrac{3}{\sqrt{2}} \; x^2 - y^2 \; - \; x^2 - 2xy + y^2 \; = 0$

$\Rightarrow x^2 - xy + y^2 + \dfrac{3}{\sqrt{2}} x^2 - \dfrac{3}{\sqrt{2}} y^2 - x^2 + 2xy - y^2 = 0$

$\Rightarrow \dfrac{3}{\sqrt{2}} x^2 + xy - \dfrac{3}{\sqrt{2}} y^2 = 0$

$\Rightarrow$ coefficient of x^2 +coefficient of y^2 $= a + b = \frac{3}{\sqrt{2}} - \frac{3}{\sqrt{2}} = 0$

$\therefore$ OA, OB are perpendicular.

21. Show that the lines whose direction cosines are given by $l + m + n = 0$ $2m + 3nl - 5lm = 0$ are perpendicular to each other

Sol: Given equations are $l + m + n = 0 ------ 1$

$2mn + 3nl - 5lm = 0 ----- 2$

From (1), $l = - \ m + n$ Substituting in (2)

$\Rightarrow 2mn - 3n \ \ m + n \ \ + 5m \ \ m + n \ \ = 0$

$\Rightarrow 2mn - 3mn - 3n^2 + 5m^2 + 5mn = 0$

$\Rightarrow 5m^2 + \ 4mn - 3n^2 = 0$

$\Rightarrow 5\left(\frac{m}{n}\right)^2 + 4\frac{m}{n} - 3 = 0$

$\Rightarrow \frac{m_1 m_2}{n_1 n_2} = \frac{-3}{5} \Rightarrow \frac{m_1 m_2}{-3} = \frac{n_1 n_2}{5} ------ \ 3$

From (1), n = $- \ l + m$

Substituting in (2) , $-2m \ \ l + m \ \ - 3l \ \ l + m \ \ - 5lm = 0$

$\Rightarrow -2lm - 2m^2 - 3l^2 - 3lm - 5lm = 0$

$\Rightarrow 3l^2 + 10lm + 2m^2 = 0$

$\Rightarrow 3\left(\frac{l}{m}\right)^2 + 10\frac{l}{m} + 2 = 0$

$\frac{l_1 l_2}{m_1 m_2} = \frac{2}{3} \Rightarrow \frac{l_1 l_2}{2} = \frac{m_1 m_2}{3} ----- \ 4$

Form (3) and (4)

$\frac{l_1 l_2}{2} = \frac{m_1 m_2}{3} = \frac{n_1 n_2}{-5} = k \ \ say$

$l_1 l_2 = 2k, m_1 m_2 = 3k, n_1 n_2 = -5k$

$\therefore l_1 l_2 + m_1 m_2 + n_1 n_2 = 2k + 3k - 5k = 0$

The two lines are perpendicular

22. If $y = x^{\tan x} + \sin x^{\,\cos x}$, find $\frac{dy}{dx}$

Sol : Let $u = x^{\tan x} and\ v = \sin x^{\,\cos x}$

$$\log u = \log x^{\tan x} = \tan x \ \log x$$

$$\frac{1}{u}.\frac{dy}{dx} = \tan x \frac{1}{x} + \log x \ \sec^2 x.$$

$$\frac{du}{dx} = u \ \frac{\tan x}{x} + \log x \ .\sec^2 x$$

$$= x^{\tan x} \ \frac{\tan x}{x} + \log x \ .\sec^2 x$$

$$\log v = \log \ \sin x \ \cos x] = \cos x.\log \sin x$$

$$\frac{1}{v}.\frac{dv}{dx} = \cos x.\frac{1}{\sin x}\cos x + \log \sin x \ -\sin x$$

$$= \frac{\cos^2 x}{\sin x} - \sin x \log \ \sin^{x} \ ^{\cos x}$$

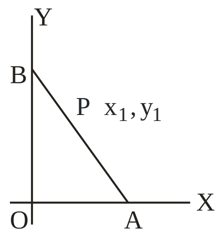
$$\frac{dv}{dx} = v\left(\frac{\cos^2 x}{\sin x} - \sin x \log \sin x\right)$$

$$= \ \sin x^{\,\cos x}\left(\frac{\cos^2}{\sin x} - \sin x \log \ \sin x\right)$$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} = x^{\tan x}$$

$$\frac{\tan x}{x} + \log x \ \sec^2 x \ + \sin x^{\,\cos x}\left(\frac{\cos^2 x}{\sin x} - \sin x.\log \ \sin x\right)$$

23. If the tangent at a point P on the curve $\mathbf{x^m y^n = a^{m+n} (mn \neq 0)}$ meets the coordinate axes in A, B. show that AP : PB is constant.



Sol: Equation of the curve is $x^m.y^n = a^{m+n}$

let P(x_1,y_1) be a point on the curve. Then $x_1^m.y_1^n = a^{m+n}$

Differente given curve w. r. to x, $x^m.ny^{n-1}.\frac{dy}{dx} + y^n.mx^{m-1} = 0$

$$nx^m.y^{n-1}\frac{dy}{dx} = -m.x^{m-1}.y^n \Rightarrow \frac{dy}{dx} = \frac{-m.x^{m-1}.y^n}{n.x^m y^{n-1}} = -\frac{my}{nx}$$

Slope of the tangent at $P\ x_1, y_1 = -\frac{my_1}{nx_1}$

Equation of the tangent at P is $y - y_1 = -\frac{my_1}{nx_1}\ x - x_1$

$$\Rightarrow nx_1y - nx_1y_1 = -my_1x + mx_1y_1$$

$$\Rightarrow my_1x + nx_1y = mx_1y_1 + nx_1y_1 = \ m+n\ \ x_1y_1$$

$$\Rightarrow \frac{my_1}{m+n\ \ x_1y_1}.x + \frac{nx_1}{m+n\ \ x_1y_1}.y = 1$$

$$\Rightarrow \frac{x}{\frac{m+n}{m}.x_1} + \frac{y}{\frac{m+n}{n}.y_1} = 1$$

$$\Rightarrow OA = \frac{m+n}{m}.x_1.OB = \frac{m+n}{n}$$

co-ordinates of A are $\left[\frac{m+n}{m}.x_1, 0\right]$ and B are $\left[0, \frac{m+n}{n}.y_1\right]$

the ratio in which P divides AB is

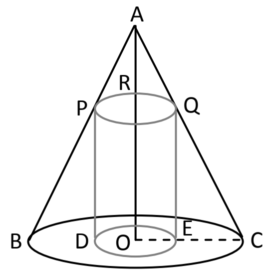
$$\frac{AP}{PB} = \frac{X - X_1}{X_1 - 0} = \frac{\frac{m+n}{m} - x_1}{x_1} = \frac{nx_1}{mx_1} = \frac{n}{m}$$

24. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

Sol. Let O be the center of the circular base of the cone and its height be h. Let r be the radius of the circular base of the cone.

Then AO = h, OC = r

Let a cylinder with radius x(OE) be inscribed in the given cone. Let its height be u.



i.e. RO = QE = PD = u

Now the triangles AOC and QEC are similar.

Therefore, $\frac{QE}{OA} = \frac{EC}{OC}$

i.e., $\frac{u}{h} = \frac{r-x}{r}$

$\therefore u = \frac{h(r-x)}{r}$

Let S denote the curved surface area of the chosen cylinder. Then

$S = 2\pi xu$

As the cone is fixed one, the values of r and h are constants. Thus S is function of x only.

Now, $\frac{dS}{dx} = 2\pi h(r-2x)/r$ and $\frac{d^2S}{dx^2} = -\frac{4\pi h}{r}$

The stationary point of S is a root of

$\frac{dS}{dx} = 0$

i.e., $\pi(r-2x)/r = 0$

i.e., $x = \frac{r}{2}$

$\frac{d^2S}{dx^2} < 0$ for all x, therefore $\left(\frac{d^2S}{dx^2}\right)_{x=r/2} < 0$

Hence, the radius of the cylinder of greatest curved surface area which can be inscribed in a given cone is r/2.