MATHEMATICS PAPER IB

COORDINATE GEOMETRY(2D &3D) AND CALCULUS.

TIME : 3hrs **Max. Marks.75**

Note: This question paper consists of three sections A,B and C.

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS. 10X2 =20

1. Find the equation of the straight line passing through (-4, 5) and cutting off equal non- zero intercepts on the co- 0rdinate axes.

2. Find the value of k, the straight lines y – 3kx + 4 = 0 and (2k – 1) x – (8k – 1) y = 6 are perpendicular.

3. If (3, 2, –1), (4, 1, 1) and (6, 2, 5) are three vertices and (4, 2, 2) is the centroid of a tetrahedron, find the fourth vertex.

4. Verify the Rolle's theorem for the function $(x^2 - 1)(x - 2)$ on [–1, 2]. Find the point in the interval where the derivate vanishes.

5. Find the d.c.'s of the normal to the plane x + 2y + 2z – 4 = 0.

6. Evaluate $\underset{x \to \frac{\pi}{2}}{Lt} \dfrac{\cos x}{\left[x - \frac{\pi}{2}\right]}$

7. Evaluate $\underset{x \to 0}{Lt} \left[\dfrac{3^x - 1}{\sqrt{1+x} - 1}\right]$

8. Find the derivativie of $y = \cos \log \cot x$

9. Find the derivativie of Find the derivativie of $\tan^{-1} \frac{a - x}{1 + ax}$

10. The time t of a complete oscillation of a simple pendulum of length *l is* given by the equation t = $\mathbf{2\pi \sqrt{\frac{1}{g}}}$ where g gravitational constant. Find the approximate percentage error in the calculated g, corresponding to an error of 0.01 percent is the value of t.

SECTION B

SHORT ANSWER TYPE QUESTIONS.

ANSWER ANY FIVE OF THE FOLLOWING **5 X 4 = 20**

11. Find the value of k, if the angle between the straight lines 4x – y + 7 = 0 and kx – 5y – 9 = 0 is 45°.

12. A container is in the shape of an inverted cone has height 8 m and radius 6 m at the top. If it is filled with water at the rate of 2m^3/minute, how fast is the height of water changing when the level is 4 m?

13. If the transformed equation of a curve is $3x^2 + xy - y^2 - 7x + y - 7 = 0$ when the origin is shifted to the point (1,2) by translation of axes, find the original equation of the curve.

14. Find the equation of locus of a point, the difference of whose distances from (–5, 0) and (5, 0) is 8 units.

15. If f and g are two differentiable functions at x then the product function f.g is differentiable at x and (fg)' (x) = f '(x). g(x) + f(x) . g'(x)

16. Show that $f\ \ x \begin{cases} \dfrac{\cos ax - \cos bx}{x^2} & \text{if} \quad x \neq 0 \\ \dfrac{1}{2}\ b^2 - a^2 & \text{if} \quad x = 0 \end{cases}$

Where a and b are real constant, is continuous at 0.

17. **Find the length of normal and sub-normal at a point on the curve** $\mathbf{y = \frac{a}{2}\ e^{x/a} + e^{-x/a}}$ **.**

SECTION C

LONG ANSWER TYPE QUESTIONS.

ANSWER ANY FIVE OF THE FOLLOWING 5 X 7 = 35.

18. Find the circumcentre of the triangle whose sides are given by $x + y + 2 = 0$, $5x - y - 2 = 0$ and $x - 2y + 5 = 0$.

19. *The area of triangle formed by the lines* $ax^2 + 2hxy + by^2 = 0$ *and* $\boldsymbol{lx + my + n = 0}$ *is* $\dfrac{n^2\sqrt{h^2 - ab}}{\left|am^2 - 2h\ell m + b\ell^2\right|}$

20. Write down the equation of the pair of straight lines joining the origin to the points of intersection of the lines 6x – y + 8 = 0 with the pair of straight lines $3x^2 + 4xy - 4y^2 - 11x + 2y + 6 = 0$. Show that the lines so obtained make equal angles with the coordinate axes.

21. If $x^y = y^x$, show that $\dfrac{dy}{dx} = \dfrac{y\ \ x\log y - y}{x\ \ y\log x - x}$

22. Find the direction cosines of two lines which are connected by the relation $l - 5m + 3n = 0$ *and* $7l^2 + 5m^2 - 3n^2 = 0$

23. From a rectangular sheet of dimensions 30cm×80cm four equal squares of side x cm are removed at the corners and the sides are then tuned up so as to form an open rectangular box. What is the value of x, so that the volumes of the box is the greatest?

24. Find the angle between the curves xy = 2 and $x^2 + 4y = 0$.

SOLUTIONS

1. Find the equation of the straight line passing through (-4, 5) and cutting off equal non- zero intercepts on the co- 0rdinate axes.

Sol. Line is making equal intercepts, let the inter cepts be a,a

Equation of the line in the intercept from is

$\frac{x}{a}+\frac{y}{b}=1 \quad \Rightarrow \frac{x}{a}+\frac{y}{a}=1 \quad \Rightarrow x+y=a$

This line is passing through p (-4, 5) ⇒ -4 + 5 = a $\Rightarrow a=1$

Equation of the line is x + y = 1

2. Find the value of k, the straight lines y – 3kx + 4 = 0 and (2k – 1) x – (8k – 1) y = 6 are perpendicular.

Sol. Given lines are -3kx + y + 4 = 0

(2k – 1)x – (8k – 1)y – 6 = 0

These lines are perpendicular $\Rightarrow a_1a_2+b_1b_2=0$

$\Rightarrow -3k(2k-1)-1(8k-1)=0 \Rightarrow -6k^2+3k-8k+1=0$

$6k^2+5k-1=0 \Rightarrow (k+1)(6k-1)=0$

$k=-1$ or 1/6

3. If (3, 2, –1), (4, 1, 1) and (6, 2, 5) are three vertices and (4, 2, 2) is the centroid of a tetrahedron, find the fourth vertex.

Sol. A(3, 2, –1), B(4, 1, 1), C(6, 2, 5), let D=(x, y, z) are the vertices of the tetrahedron.

And centroid G = (4, 2, 2)

But G = $\left(\frac{3+4+6+x}{4}, \frac{2+1+2+y}{4}, \frac{-1+1+5+z}{4}\right)$

Therefore, $\left(\frac{13+x}{4},\frac{5+y}{4},\frac{5+z}{4}\right)=(4,2,2)$

$\Rightarrow \frac{13+x}{4}=4,\ \frac{5+y}{4}=2,\ \frac{5+z}{4}=2$

$\Rightarrow$ 13 + x = 16, 5 + y = 8, 5+ z = 8

$\Rightarrow$ x = 3, y = 3, z = 3

Coordinates of D are (3, 3, 3)

4. Verify the Rolle's theorem for the function $(x^2 - 1)(x - 2)$ on [–1, 2]. Find the point in the interval where the derivate vanishes.

Sol. Let $f(x) = (x^2 - 1)(x - 2) = x^3 - 2x^2 - x + 2$

f is continuous on [–1, 2]

since f(–1) = f(2) = 0 and

f is differentiable on [–1, 2]

$\therefore$ By Rolle's theorem $\exists c \in (-1, 2)$

Let f′(c) = 0

$f'(x) = 3x^2 - 4x - 1$

$3c^2 - 4c - 1 = 0$

$$c=\frac{4\pm\sqrt{16+12}}{6}=\frac{4\pm\sqrt{28}}{6}$$

$$\Rightarrow c=\frac{2\pm\sqrt{7}}{3}$$

5. Find the d.c.'s of the normal to the plane x + 2y + 2z – 4 = 0.

Sol. Equation of the plane is x + 2y + 2z – 4 = 0

d.r.'s of the normal are (1, 2, 2)

Dividing with $\sqrt{1+4+4}=3$

d.c.'s of the normal to the plane are $\left(\frac{1}{3},\frac{2}{3},\frac{2}{3}\right)$.

6. Evaluate $\underset{x\to\frac{\pi}{2}}{Lt}\ \frac{\cos x}{\left[x-\frac{\pi}{2}\right]}$

Sol : Let $y = x-\frac{\pi}{2}$ so that as $x\to\frac{\pi}{2}, y\to 0$ and $x = y+\frac{\pi}{2}$

$$\therefore \underset{x\to\frac{\pi}{2}}{Lt}\ \frac{\cos x}{x-\frac{\pi}{2}} = \underset{y\to 0}{Lt}\ \frac{\cos\left(\frac{\pi}{2}+y\right)}{y}\ \underset{y\to 0}{Lt}\ \frac{-\sin y}{y} = -1$$

7. Evaluate $\underset{x\to 0}{Lt}\left[\frac{3^x-1}{\sqrt{1+x}-1}\right]$

Sol : $\underset{x\to 0}{Lt}\ \frac{3^x-1}{\sqrt{1+x}-1} = \underset{x\to 0}{Lt}\ \frac{3^x-1}{\sqrt{1+x}-1}\times\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$ *rationalise* Dr.

$$= \underset{x\to 0}{Lt}\ \frac{\left(3^x-1\right)\left(\sqrt{1+x}+1\right)}{1+x-1} = \underset{x\to 0}{Lt}\ \frac{3^x-1}{x}.\ \underset{x\to 0}{Lt}\ \left(\sqrt{1+x}+1\right)$$

$$= \log 3\left(\sqrt{1+0}+1\right) = 2.\log 3$$

8. Find the derivativie of $y = \cos\left(\log\left(\cot x\right)\right)$

sol : $\frac{dy}{dx} = \frac{d}{du}\cos\left(\log\left(\cot x\right)\right)$

$$= -\sin\left(\log\left(\cot x\right)\right).\frac{1}{\cot x}\left(-\operatorname{cosec}^2 x\right)$$

$$= \frac{1}{\sin^2 x}.\frac{1}{\cos x}.\sin \ \log \ \cot \ x$$

$$= \frac{\sin \ \log \ \cot x \quad .\cos ecx}{\cos x}$$

9. Find the derivativie of Find the derivativie of $\tan^{-1} \frac{a-x}{1+ax}$

sol : Put $a = \tan\alpha, x = \tan\theta$ then $\theta = \tan^{-1}x$ and $\alpha = \tan^{-1}a$

$$y = \tan^{-1} \frac{\tan\alpha - \tan\theta}{1+\tan\alpha\tan\theta}$$

$$= \tan^{-1} \ \tan \ \alpha - \theta \quad = \alpha - \theta$$

$$= \tan^{-1} a - \tan^{-1} x;$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = 0 - \frac{1}{1+x^2} = -\frac{1}{1+x^2}$$

10. The time t of a complete oscillation of a simple pendulum of length *l is* given by the equation t = $\mathbf{2\pi}\sqrt{\frac{\mathbf{1}}{\mathbf{g}}}$ where g gravitational constant. Find the approximate percentage error in the calculated g, corresponding to an error of 0.01 percent is the value of t.

Sol: percentage error in t is $\frac{\Delta t}{t} \times 100$ =0.01

Given t= $2\pi\sqrt{\frac{l}{g}}$

Taking logs on both sides log t = log (2π) + $\frac{1}{2}$ {(log (*l*) – log g}

Taking differentials on both sides, $\frac{1}{t}\ \Delta t\ = 0 + \frac{1}{2}\left\{o - \frac{1}{g}.\ \Delta g\ \right\}$

Multiplying with 100, $\frac{\Delta t}{t}\times 100 = -\frac{1}{2}\frac{\Delta g}{g}\times 100$

$\rightarrow \frac{\Delta t}{t}\times 100 = -\frac{1}{2}\frac{\Delta g}{g}\times 100 \qquad \Rightarrow 0.001 = -\frac{1}{2}\frac{\Delta g}{g}\times 100 \qquad \Rightarrow \frac{\Delta g}{g}\times 100 = -0.02$

$\therefore$ Percentage error in g = -0.02

SECTION B

ANSWER ANY 5 OF THE FOLLOWING. 5x4 =20

11. Find the value of k, if the angle between the straight lines 4x – y + 7 = 0 and kx – 5y – 9 = 0 is 45°.

Sol. $\cos\theta = \frac{|4k+5|}{\sqrt{16+1}\sqrt{k^2+25}}$

$\cos\theta = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\frac{1}{\sqrt{2}} = \frac{|4k+5|}{\sqrt{17}\sqrt{k^2+25}}$

Squaring and cross multiplying

$2(4k+5)^2 = 17(k^2+25)$

$2(16k^2+40k+25) = 17k^2+425$

$32k^2+80k+50 = 17k^2+425$

$15k^2+80k-375 = 0$

$3k^2+16k-75 = 0$

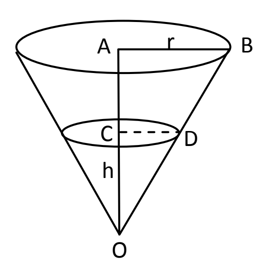
$(k-3)(3k+25) = 0$

$k = 3 \text{ or } -25/3$

12. A container is in the shape of an inverted cone has height 8 m and radius 6 m at the top. If it is filled with water at the rate of 2m^3/minute, how fast is the height of water changing when the level is 4 m?

Sol. h = 8 m = OC

r = 6 m = AB



$\frac{dv}{dt} = 2\,m^3/\text{minute}$

ΔOAB and OCD are similar angle then

$\frac{CD}{AB} = \frac{OC}{OA}$

$\frac{r}{6} = \frac{h}{8} \Rightarrow r = h\frac{3}{4}$

Volume of cone $v = \frac{1}{3}\pi r^2 h$

$v = \frac{1}{3}\pi h^2 \frac{9}{16} h$

$v = \frac{3}{16}\pi h^3$

$\frac{dv}{dt} = \frac{3}{16}\pi 3h^2 \frac{dh}{dt}\quad \because h = 16$

$2 = \frac{3}{16}\pi 3(16)\frac{dh}{dt} \Rightarrow \frac{2}{9\pi} = \frac{dh}{dt}$

13. If the transformed equation of a curve is $3x^2 + xy - y^2 - 7x + y - 7 = 0$ when the origin is shifted to the point (1,2) by translation of axes, find the original equation of the curve.

Sol. Given transformed equation is

$3x^2 + xy - y^2 - 7x + y - 7 = 0$ ....(1)

New origin=(h, k) =(1,2)

Let x, y be the old co-ordinates of X, Y

X= x - h = x - 1 and Y= y – k = y - 2

Therefore, original equation of (1) is

$\Rightarrow 3(x-1)^2 + (x-1)(y-2) - (y-2)^2 - 7(x-1) + (y-2) + 7 = 0$

$\Rightarrow 3(x^2 - 2x + 1) + (xy - 2y - y + 2) - (y^2 - 4y + 4) - 7(x-1) + (y-2) + 7 = 0$

$\Rightarrow 3x^2 - 6x + 3 + xy - 2x - y + 2 - y^2 + 4y - 4 - 7x + 7 + y - 2 + 7 = 0$

$\therefore 3x^2 + xy - y^2 - 15x + 4y + 13 = 0$ is the original equation.

14. Find the equation of locus of a point, the difference of whose distances from (–5, 0) and (5, 0) is 8 units.

Sol. Given points are A(5, 0), B(–5, 0)

Let P(x, y) be any point in the locus

Given |PA – PB| = 8

$\Rightarrow$ PA – PB = ± 8

$\Rightarrow$ PA = ±8 + PB

Squaring on both sides

$PA^2 = 64 + PB^2 \pm 16PB$

$\Rightarrow (x-5)^2 + y^2 - (x+5)^2 - y^2 - 64 = \pm 16PB$

$-4\cdot 5\cdot x - 64 = \pm 16PB$

$-5x - 16 = \pm 4PB$

Squaring on both sides

$25x^2 + 256 + 160x = 16(PB)^2$

$= 16[(x+5)^2 + y^2]$

$= 16x^2 + 400 + 160x + 16y^2$

$9x^2 - 16y^2 = 144$

Dividing with 144, locus of P is

$$\frac{9x^2}{144} - \frac{16y^2}{144} = 1 \Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$$

15. If f and g are two differentiable functions at x then the product function f.g is differentiable at x and (fg)' (x) = f '(x). g(x) + f(x) . g'(x)

Proof:

Since f and g are differentiable at x,therefore

f '(x)and g'(x) exist and

$$f'(x) = \underset{h\to 0}{Lt} \frac{f(x+h)-f(x)}{h}$$

and $\underset{h\to 0}{Lt} \frac{g(x+h)-g(x)}{h} = g'(x)$

$$\underset{h\to 0}{Lt} \frac{(fg)(x+h)-(fg)(x)}{h} =$$

$$= \underset{h\to 0}{Lt} \frac{f(x+h)g(x+h)-f(x)g(x)}{h}$$

$$= \underset{h\to 0}{Lt} \frac{1}{h}[f(x+h).g(x+h)-f(x)g(x+h)] + \underset{h\to 0}{Lt} \frac{1}{h}[f(x)g(x+h)-f(x)g(x)]$$

$= \underset{h\to 0}{Lt}\left[\frac{f(x+h)-f(x)}{h}\right]\underset{h\to 0}{Lt}\, g(x+h)$ + f(x). $\underset{h\to 0}{Lt}\left[\frac{g(x+h)-g(h)}{h}\right]$

$\therefore$ (fg)'(x)= f '(x) g(x) + f(x) g'(x)

16. Show that $f(x) = \begin{cases} \dfrac{\cos ax - \cos bx}{x^2} & \text{if } x \neq 0 \\ \dfrac{1}{2}(b^2 - a^2) & \text{if } x = 0 \end{cases}$

Where a and b are real constant, is continuous at 0.

Sol : $\underset{x\to 0}{Lt}\, f(x) = \underset{x\to 0}{Lt}\dfrac{\cos ax - \cos bx}{x^2}$

$= \underset{x\to 0}{Lt}\dfrac{2\sin\dfrac{(a+b)x}{x}\sin\dfrac{(b-a)x}{2}}{x^2}$

$= 2\underset{x\to 0}{Lt}\dfrac{\sin(a+b)\dfrac{x}{2}}{x}\underset{x\to 0}{Lt}\dfrac{\sin(b-a)\dfrac{x}{2}}{x}$

$= \dfrac{2(b+a)}{2}\dfrac{(b-a)}{2} = \dfrac{b^2-a^2}{2}$

Given $f(0) = \dfrac{b^2-a^2}{2}$. $\quad \therefore \underset{x\to 0}{Lt}\, f(x) = f(0)$

17. **Find the length of normal and sub-normal at a point on the curve** $\mathbf{y = \frac{a}{2}\left(e^{x/a} + e^{-x/a}\right)}$ **.**

Sol: Equation of the curve is $y = \frac{a}{2}\left(e^{x/a} + e^{-x/a}\right) \quad = a.\cosh\left(\frac{x}{a}\right)$

$\Rightarrow \frac{dy}{dx} = a.\sinh\left(\frac{x}{a}\right)\frac{1}{a} = \sinh\frac{x}{a}$ **=slope of tantent at any point =m**

Length of the normal $\left|y_1\sqrt{1+m^2}\right| \quad = \left|a.\cosh\frac{x}{a}\right|\sqrt{1+\sinh^2\frac{x}{a}}$

$= a.\cosh\frac{x}{a}.\cosh\frac{x}{a} = a.\cosh^2\frac{x}{a}$

Length of the sub-normal $|y_1 m| \quad = \left|a.\cosh\left(\frac{x}{a}\right).\sinh\left(\frac{x}{a}\right)\right|$

$= \left|\frac{a}{2}\left(2\sinh\frac{x}{a}.\cosh\frac{x}{a}\right)\right| \quad = \left|\frac{a}{2}.\sinh\frac{2x}{a}\right|$

SECTION C

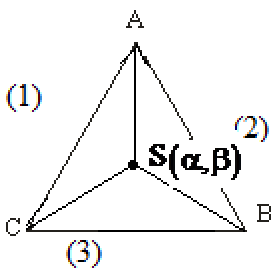
ANSWER ANY 5 OF THE FOLLOWING. 5x7 =35

18. Find the circumcentre of the triangle whose sides are given by $x+y+2=0$, $5x-y-2=0$ and $x-2y+5=0$.

Sol: Given lines are $x+y+2=0$ ---(1)

$5x-y-2=0$ ---(2)

$x-2y+5=0$ ---(3)



Point of intersection of (1) and (2) is $A=(0,-2)$

Point of intersection of (2) and (3) is $B=(1,3)$

Point of intersection of (1) and (3) is $C=(-3,1)$

Let $S=(\alpha,\beta)$ the orthocentre of ΔABC then $SA=SB=SC$

$\Rightarrow SA^2=SB^2=SC^2$

$\Rightarrow (\alpha-0)^2+(\beta+2)^2=(\alpha-1)^2+(\beta-3)^2=(\alpha+3)^2+(\beta-1)^2$

$\Rightarrow \alpha^2+\beta^2+4\beta+4=\alpha^2+\beta^2-2\alpha-6\beta+10$

$=\alpha^2+\beta^2+6\alpha-2\beta+10$

$SA^2=SB^2\Rightarrow \alpha^2+\beta^2+4\beta+4=\alpha^2+\beta^2-2\alpha-6\beta+10$

$\Rightarrow 2\alpha+10\beta-6=0\Rightarrow \alpha+5\beta-3=0$ ---(4)

$SA^2=SC^2\Rightarrow \alpha^2+\beta^2+4\beta+4=\alpha^2+\beta^2+6\alpha-2\beta+10$

$\Rightarrow 6\alpha-6\beta+6=0\Rightarrow \alpha-\beta+1=0$ ---(5)

From (4) and (5)

$$\begin{array}{cccc} & & 1 & \\ 5 & -3 & 1 & 5 \\ -1 & 1 & 1 & -1 \end{array}$$

$$\frac{\alpha}{5-3}=\frac{\beta}{-3-1}=\frac{1}{-1-5}\Rightarrow\frac{\alpha}{2}=\frac{\beta}{-4}=\frac{1}{-6}$$

$$\alpha=-\frac{2}{6}=-\frac{1}{3}$$

$$\beta=-\frac{4}{-6}=\frac{2}{3}$$

$\therefore$ Circumcentre $S=\left(-\frac{1}{3},\frac{2}{3}\right)$

19. *The area of triangle formed by the lines* $ax^2+2hxy+by^2=0$ *and* $\boldsymbol{lx+my+n=0}$ *is* $\dfrac{n^2\sqrt{h^2-ab}}{\left|am^2-2h\ell m+b\ell^2\right|}$

Let $ax^2+2hxy+by^2=0$ represent the lines $l_1x+m_1y=0$ -- (1) and $l_2x+m_2y=0$ -- (2).
Then $l_1l_2=a$, $l_1m_2+l_2m_1=2h$, $m_1m_2=b$.

The given straight line is $\boldsymbol{lx+my+n=0}$ -- (3) Clearly (1) and (2) intersect at the origin.

Let A be the point of intersection of (1) and (3). Then

$$\begin{array}{cccc} x & y & 1 & \\ m_1 & 0 & l_1 & m_1 \\ m & n & l & m \end{array}$$

$$\Rightarrow\frac{x}{m_1n-0}=\frac{y}{0-nl_1}=\frac{1}{l_1m-lm_1}$$

$$\Rightarrow x=\frac{m_1n}{l_1m-lm_1}\text{ and }y=\frac{-nl_1}{l_1m-lm_1}$$

$$\therefore A=\left(\frac{m_1n}{l_1m-lm_1},\frac{-l_1n}{l_1m-lm_1}\right)=(x_1,y_1)$$

$$B=\left(\frac{m_2n}{l_2m-lm_2},\frac{-l_2n}{l_2m-lm_2}\right)=(x_2,y_2)$$

$$\therefore\text{ The area of }\Delta OAB=\frac{1}{2}\left|x_1y_2-x_2y_1\right|$$

$$=\frac{1}{2}\left|\left(\frac{m_1n}{l_1m-lm_1}\right)\left(\frac{-l_2n}{l_2m-lm_2}\right)-\left(\frac{m_2n}{l_2m-lm_2}\right)\left(\frac{-nl_1}{l_1m-lm_1}\right)\right|$$

$$\frac{1}{2}\left|\frac{l_1m_2n^2-l_2m_1n^2}{l_1m-lm_1 \quad l_2m-lm_2}\right|$$

$$=\frac{n^2}{2}\left|\frac{(l_1m_2-l_2m_1)}{l_1l_2m^2-(l_1m_2+l_2m_1)lm+m_1m_2l^2}\right|=$$

$$=\frac{n^2}{2}\left|\frac{\sqrt{(l_1m_2+l_2m_1)^2-4l_1m_2l_2m_1}}{am^2-2hlm+bl^2}\right|$$

$$=\frac{n^2}{2}\frac{\sqrt{4h^2-4ab}}{\left|am^2-2hlm+bl^2\right|}=\frac{n^2\sqrt{h^2-ab}}{\left|am^2-2hlm+bl^2\right|}$$

20. Write down the equation of the pair of straight lines joining the origin to the points of intersection of the lines 6x – y + 8 = 0 with the pair of straight lines $3x^2+4xy-4y^2-11x+2y+6=0$. Show that the lines so obtained make equal angles with the coordinate axes.

Sol. Given pair of line is $3x^2+4xy-4y^2-11x+2y+6=0$ …(1)

Given line is $6x-y+8=0 \Rightarrow \frac{6x-y}{-8}=1 \quad \Rightarrow \frac{y-6x}{8}=1$-----(2)

Homogenising (1) w.r.t (2)

$$3x^2+4xy-4y^2-\ 11x-2y\left(\frac{y-6x}{8}\right)+6\left(\frac{y-6x}{8}\right)^2=0$$

$$64\left[3x^2+4xy-4y^2\right]-8\left[11xy-66x^2-2y^2+12xy\right] \quad +6\left[y^2+36x^2-12xy\right]=0$$

$$\Rightarrow 936x^2+256xy-256xy-234y^2=0$$

$$\Rightarrow 468x^2-117y^2=0 \qquad \Rightarrow 4x^2-y^2=0 \text{ ----(3)}$$

is eq. of pair of lines joining the origin to the point of intersection of (1) and (2).

The eq. pair of angle bisectors of (3) is $h\ \ x^2-y^2\ \ -\ \ a-b\ \ xy=0$

$$\Rightarrow 0\ \ x^2-y^2\ \ -\ \ 4-1\ \ xy=0 \qquad \Rightarrow xy=0$$

x = 0 or y = 0 which are the eqs. is of co-ordinates axes

$\therefore$ The pair of lines are equally inclined to the co-ordinate axes

21. If $x^y = y^x$, show that $\frac{dy}{dx} = \frac{y \;\; x\log y - y}{x \;\; y\log x - x}$

sol : Given $x^y = y^x; \log x^y = \log y^x$

$y\log x = x\log y$

Differentiating w. r. to x

$$\frac{d}{dx} y\log x = \frac{d}{dx} x\log y$$

$$y.\frac{1}{x}\log x.\frac{dy}{dx} = x\frac{1}{y}.\frac{dy}{dx} + \log y$$

$$\therefore \log x.\frac{dy}{dx} - \frac{x}{y}.\frac{dy}{dx} = \log y - \frac{y}{x}$$

$$\left(\log x - \frac{x}{y}\right)\frac{dy}{dx} = \log y - \frac{y}{x}$$

$$\frac{y\log x - x}{y}.\frac{dy}{dx} = \frac{x\log y - y}{x}$$

$$\frac{dy}{dx} = \frac{y \;\; x\log y - y}{x \;\; y\log x - x}$$

22. Find the direction cosines of two lines which are connected by the relation $l - 5m + 3n = 0 \; and \; 7l^2 + 5m^2 - 3n^2 = 0$

Sol. Given $l - 5m + 3n = 0$

$\Rightarrow l = 5m - 3n ------ 1$

and $7l^2 + 5m^2 - 3n^2 = 0 --- 2$

Substituting the value of l in (2)

$7 \;\; 5m - 3n \;^2 + 5m^2 - 3n^2 = 0$

$\Rightarrow 7 \;\; 25^2 + 9n^2 - 30mn \;\; + 5m^2 - 3n^2 = 0$

$\Rightarrow 175m^2 + 63n^2 - 210mn + 5m^2 - 3n^2 = 0$

$\Rightarrow 180m^2 - 210mn + 60n^2 = 0$

$\Rightarrow 6m^2 - 7mn + 2n^2 = 0$

$\Rightarrow \;\; 3m - 2n \;\;\; 2m - n \;\; = 0$

Case (i) : $3m_1 = 2n_1 \Rightarrow \frac{m_1}{2} = \frac{n_1}{3}$

Then $m_1 = \frac{2}{3} n_1$

From 1 $l_1 = 5m_1 - 3n_1 = \frac{10}{3} n_1 - 3n_1$

$= \frac{10n_1 - 9n_1}{3} = \frac{n_1}{3}$

$\therefore \frac{l_1}{1} = \frac{m_1}{2} = \frac{n_1}{3}$

d.rs of the first line are (1, 2, 3)

Dividing with $\sqrt{1+4+9} = \sqrt{14}$

d.cs of the first line are $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$

Case (ii) $2m_2 = n_2$

From (1) $l_2 - 5m_2 + 3n_2 = 0$

$\Rightarrow l_2 - 5m_2 + 6m_2 = 0$

$\Rightarrow -l_2 = m_2$

$\therefore \frac{l_2}{-1} = \frac{m_2}{1} = \frac{n_2}{2}$

d.rs of the second line are -1, 1, 2

Dividing with $\sqrt{1+1+4} = \sqrt{6}$

d.cs of the second line are $\left(\frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$

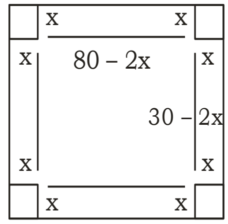
23. From a rectangular sheet of dimensions 30cm×80cm four equal squares of side x cm are removed at the corners and the sides are then tuned up so as to form an open rectangular box. What is the value of x, so that the volumes of the box is the greatest?

Sol: length of the sheet = 80 , breadth = 30.

Side of the square = x

Length of the box $= 80 - 2x = l$

Breadth of the box $= 30 - 2x = b$



Height of the box = x = h

Volume = $lbh = (80-2x)(30-2x).x$

$= x(2400-200x+4x^2)$

$f(x) = 4x^3 - 220x^2 + 2400x$

$\Rightarrow f'(x) = 12x^2 - 440x + 2400$

$= 4\left[3x^2 - 110x + 600\right]$ and $f'' = 4(6x - 110)$

For max / min $f'(x) = 0 \Rightarrow 3x^2 - 110x + 600 = 0$

$$x = \frac{110 \pm \sqrt{12100 - 7200}}{6}$$

$$= \frac{110 \pm 70}{6} = \frac{180}{6} \text{ or } \frac{40}{6} = 30 \text{ or } \frac{20}{3}$$

If $x = 30,\ b = 30 - 2x = 30 - 2(30) = -30 < 0$

$\Rightarrow x \neq 30$

$\therefore x = \frac{20}{3}$

$f''(x) = 24x - 440$

When $x = \frac{20}{3}, f''(x) = 24.\frac{20}{3} - 440$

$= 160 - 440 = -280 < 0$

$\Rightarrow$ f(x) is maximum when $x = \frac{20}{3}$

Volume of the box is maximum when $x = \frac{20}{3}\text{cm}$.

24. Find the angle between the curves xy = 2 and $x^2 + 4y = 0$.

Sol. Solving xy = 2 and $x^2 + 4y = 0$

$$y = \frac{-x^2}{4}$$

But xy = 2

$$\Rightarrow x\left(\frac{-x^2}{4}\right) = 2 \Rightarrow x^3 = -8 \Rightarrow x = -2$$

$$y = \frac{-x^2}{4} = -\frac{4}{4} = -1$$

Point of intersection is P(–2, –1)

$$xy = 2 \Rightarrow y = \frac{2}{x}$$

$$\frac{dy}{dx} = -\frac{2}{x^2}$$

$$m_1 = \left(\frac{dy}{dx}\right)_P = -\frac{2}{4} = -\frac{1}{2}$$

$$x^2 + 4y = 0 \Rightarrow y = -\frac{x^2}{4}$$

$$\frac{dy}{dx} = -\frac{2x}{4} = -\frac{x}{2}$$

$$m_2 = \left(\frac{dy}{dx}\right)_P = \frac{-2}{-2} = 1$$

Let ϕ be the angle between the given curves

$$\tan\phi = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{-\frac{1}{2} - 1}{1 + \left(-\frac{1}{2}\right)1}\right| = \left|\frac{-\frac{3}{2}}{\frac{1}{2}}\right| = 3$$

$$\phi = \tan^{-1}(3)$$